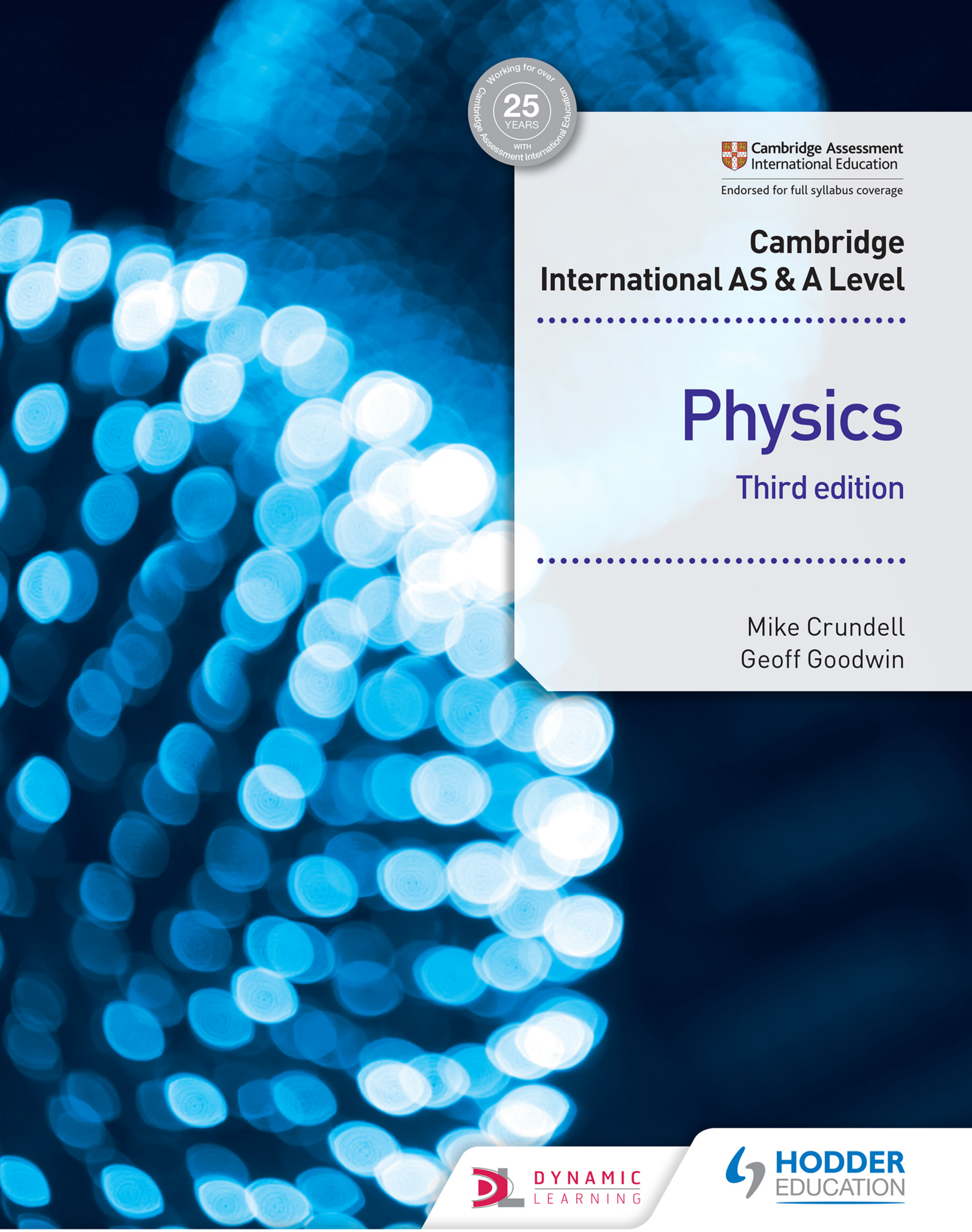

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Cambridge International AS & A Level

Physics

Third edition

Mike Crundell
Geoff Goodwin

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Introduction

Welcome to the third edition of *Cambridge International AS & A Level Physics*. This textbook has been revised to comprehensively cover the Cambridge International AS & A Level Physics syllabus (9702) for first examination in 2022.

This textbook is part of a suite of resources, which includes a Practical Skills Workbook, Teacher's Resources and a Study and Revision Guide.

How to use this book

This textbook, endorsed by Cambridge Assessment International Education, has been designed to make your study of Physics as successful and rewarding as possible.

Organisation

The book is divided into two parts. Topics 1–11 cover the Cambridge International AS Level Physics syllabus content and Topics 12–25 cover the extra content required by students studying the full Cambridge International A Level Physics course. The titles of the topics in this book exactly match those in the syllabus. In almost all cases, the subheadings within the topics also match those used in the syllabus. Topic 26 is a standalone chapter and provides information about practical work.

Numerical answers to questions are included at the back of the book.

Features to help you learn

Each topic contains a number of features designed to help you effectively navigate the syllabus content.

At the start of each topic, there is a blue box that provides a summary of the syllabus points to be covered in that topic. These are the exact learning outcomes listed in the syllabus.

Learning outcomes

By the end of this topic, you will be able to:

1.1 Physical quantities

1. understand that all physical quantities consist of a numerical magnitude and a unit
2. make reasonable estimates of physical quantities included within the syllabus

1.2 SI units

1. recall the following SI base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K)
2. express derived units as products or quotients of the SI base units and use the derived units for quantities listed in this syllabus as appropriate
3. use SI base units to check the homogeneity of physical equations
4. recall and use the following prefixes and their symbols to indicate decimal submultiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)

1.3 Errors and uncertainties

1. understand and explain the effects of systematic errors (including zero errors) and random errors in measurements
2. understand the distinction between precision and accuracy
3. assess the uncertainty in a derived quantity by simple addition of absolute or percentage uncertainties

1.4 Scalars and vectors

1. understand the difference between scalar and vector quantities and give examples of scalar and vector quantities included in the syllabus
2. add and subtract coplanar vectors
3. represent a vector as two perpendicular components

Each topic also has a number of *Starting points*, key bits of information that it may be useful to remind yourself of before you begin to read.

Starting points

- ★ Accurate measurement is very important in the development of physics.
- ★ Physicists begin by observing, measuring and collecting data.
- ★ The data items are analysed to discover whether they fit into a pattern.
- ★ If there is a pattern and this pattern can be used to explain other events, it becomes a theory.
- ★ The process is known as the *scientific method* (see Figure 1.1).

Key points and definitions are highlighted in blue panels throughout the book so that they can be easily identified and referred back to.

Every object continues in its state of rest, or with uniform velocity, unless acted on by a resultant force.

There are also a number of boxes labelled *Maths Note* to guide you through some of the mathematical skills required.

MATHS NOTE

There are actually *two* solutions to the defining equation of simple harmonic motion, $a = -\omega^2 x$, depending on whether the timing of the oscillation starts when the particle has zero displacement or is at its maximum displacement. If at time $t = 0$ the particle is at its maximum displacement, $x = x_0$, the solution is $x = x_0 \cos \omega t$ (not shown in Figure 17.5). The two solutions are identical apart from the fact that they are out of phase with each other by one quarter of a cycle or $\pi/2$ radians.

The variation of velocity with time is sinusoidal if the cosinusoidal displacement solution is taken:

$$v = -v_0 \sin \omega t \text{ when } x = x_0 \cos \omega t$$

Each topic features a number of *Worked Examples*, which show you how to answer the kinds of questions you may be asked about the content contained within that topic.

WORKED EXAMPLE 1A

Calculate the number of micrograms in 1.0 milligram.

Answer

$$1.0\,\text{g} = 1.0 \times 10^3\,\text{mg}$$

$$\text{and } 1.0\,\text{g} = 1.0 \times 10^6 \text{ micrograms } (\mu\text{g})$$

$$\text{so, } 1.0 \times 10^3\,\text{mg} = 1.0 \times 10^6\,\mu\text{g}$$

$$\text{and } 1.0\,\text{mg} = (1.0 \times 10^6)/(1.0 \times 10^3) = \mathbf{1.0 \times 10^3\,\mu g}$$

Practice questions are included to give you opportunities to test your understanding of the topic and to use the skills and techniques demonstrated in the *Worked Examples*.

Questions

Use the information in Tables 1.1 and 1.6 to determine the base units of the following quantities.

8 density = mass/volume

9 pressure = force/area

Material that goes beyond the requirements of the Cambridge International AS & A Level Physics syllabus, but which may be of interest, especially to those of you planning to study Physics at a higher level, is clearly labelled in *Extension* boxes.

EXTENSION

In astronomy, the wavelength tends to be measured rather than the frequency. If the measured wavelength of the emitted light (see Topic 25) is less than that measured for a stationary source, then the distance between the source (star) and detector is decreasing (blue shift). If the measured wavelength is greater than the value of a stationary source, then the source is moving away from the detector (red shift). The blue and red shifts are referred to in this way as red has the longest wavelength in the visible spectrum and blue the shortest.

At the end of each topic, there is a *Summary* of the key points that have been covered.

SUMMARY

- When a force moves its point of application in the direction of the force, work is done.
- Work done = $Fx\cos\theta$, where θ is the angle between the direction of the force F and the displacement x.
- Energy is needed to do work; energy is the ability to do work.
- Energy cannot be created or destroyed. It can only be converted from one form to another.
- Efficiency = useful energy output/total energy input
- Power is defined as the rate of doing work or work done per unit time:
  power = work done/time taken, $P = W/t$.
- The unit of power is the watt (W).
  1 watt = 1 joule per second
- Power = force × velocity
- Potential energy is the energy stored in an object due to its position or shape; examples are elastic potential energy and gravitational potential energy.
- When an object of mass m moves vertically through a distance Δh in a uniform gravitational field, then the change in gravitational potential energy is given by: $\Delta E_p = mg\Delta h$ where g is the acceleration of free fall.
- Kinetic energy is the energy stored in an object due to its motion.
- For an object of mass m moving with speed v, the kinetic energy is given by: $E_k = \frac{1}{2}mv^2$.

Finally, each topic ends with a set of *End of topic questions*, some are exam-style questions written by the authors, others are taken from Cambridge International AS & A Level Physics (9702) past examination papers.

END OF TOPIC QUESTIONS

1 State how the centripetal force is provided in the following examples:
- **a** a planet orbiting the Sun,
- **b** a child on a playground roundabout,
- **c** a train on a curved track,
- **d** a passenger in a car going round a corner.

2 NASA's 20-G centrifuge is used for testing space equipment and the effect of acceleration on humans. The centrifuge consists of an arm of length 17.8 m, rotating at constant speed and producing an acceleration equal to 20 times the acceleration of free fall. Calculate:
- **a** the angular speed required to produce a centripetal acceleration of $20g$,
- **b** the rate of rotation of the arm ($g = 9.81\,\text{m}\,\text{s}^{-2}$).

Assessment

If you are following the Cambridge International AS Level Physics course, you will take three examination papers:

- Paper 1 Multiple-choice (1 hour 15 minutes)
- Paper 2 AS Level Structured Questions (1 hour 15 minutes)
- Paper 3 Advanced Practical Skills (2 hours)

If you are studying the Cambridge International A Level Physics course, you will take five examination papers: Papers 1, 2 and 3 and also:

- Paper 4 A Level Structured Questions (2 hours)
- Paper 5 Planning, Analysis and Evaluation (1 hour 15 minutes)

Command words

The table below, taken from the syllabus, includes command words used in the assessment for this syllabus. The use of the command word will relate to the subject context. Make sure you are familiar with these.

command word	what it means
calculate	work out from given facts, figures or information
comment	give an informed opinion
compare	identify/comment on similarities and/or differences
define	give precise meaning
determine	establish an answer using the information available
explain	set out purposes or reasons/make the relationships between things evident/provide why and/or how and support with relevant evidence
give	produce an answer from a given source or recall/memory
identify	name/select/recognise
justify	support a case with evidence/argument
predict	suggest what may happen based on available information
show (that)	provide structured evidence that leads to a given result
sketch	make a simple freehand drawing showing the key features
state	express in clear terms
suggest	apply knowledge and understanding to situations where there are a range of valid responses in order to make proposals

Notes for teachers

Key concepts

These are the essential ideas that help learners to develop a deep understanding of the subject and to make links between the different topics. Although teachers are likely to have these in mind at all times when they are teaching the syllabus, the following icons are included in the textbook at points where the key concepts particularly relate to the text:

Models of physical systems

Physics is the science that seeks to understand the behaviour of the Universe. The development of models of physical systems is central to physics. Models simplify, explain and predict how physical systems behave.

Testing predictions against evidence

Physical models are usually based on prior observations, and their predictions are tested to check that they are consistent with the behaviour of the real world. This testing requires evidence, often obtained from experiments.

Mathematics as a language and problem-solving tool

Mathematics is integral to physics, as it is the language that is used to express physical principles and models. It is also a tool to analyse theoretical models, solve quantitative problems and produce predictions.

Matter, energy and waves

Everything in the Universe comprises matter and/or energy. Waves are a key mechanism for the transfer of energy and are essential to many modern applications of physics.

Forces and fields

The way that matter and energy interact is through forces and fields. The behaviour of the Universe is governed by fundamental forces with different magnitudes that interact over different distances. Physics involves study of these interactions across distances ranging from the very small (quantum and particle physics) to the very large (astronomy and cosmology).

The information in this introduction and the learning outcomes throughout the book are taken from the Cambridge International syllabus for examination from 2022. You should always refer to the appropriate syllabus document for the year of your students' examination to confirm the details and for more information. The syllabus document is available on the Cambridge International website at www.cambridgeinternational.org.

Additional support

A number of other Hodder Education resources are available to help teachers deliver the Cambridge International AS & A Level Physics syllabus.

- The *Cambridge International AS & A Level Physics Practical Skills Workbook* is a write-in resource designed to be used throughout the course and provides students with extra opportunities to test their understanding of the practical skills required by the syllabus.
- The *Cambridge International AS & A Level Physics Teacher's Resources* include an introduction to teaching the course, a scheme of work and additional teaching resources.
- The *Cambridge International AS & A Level Physics Study and Revision Guide* is a stand-alone resource that is designed to be used independently by students at the end of their course of study as they prepare for their examinations. This title has not been through the Cambridge International endorsement process.

1 Physical quantities and units

Learning outcomes

By the end of this topic, you will be able to:

1.1 Physical quantities

1. understand that all physical quantities consist of a numerical magnitude and a unit
2. make reasonable estimates of physical quantities included within the syllabus

1.2 SI units

1. recall the following SI base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K)
2. express derived units as products or quotients of the SI base units and use the derived units for quantities listed in this syllabus as appropriate
3. use SI base units to check the homogeneity of physical equations
4. recall and use the following prefixes and their symbols to indicate decimal submultiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)

1.3 Errors and uncertainties

1. understand and explain the effects of systematic errors (including zero errors) and random errors in measurements
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3. assess the uncertainty in a derived quantity by simple addition of absolute or percentage uncertainties

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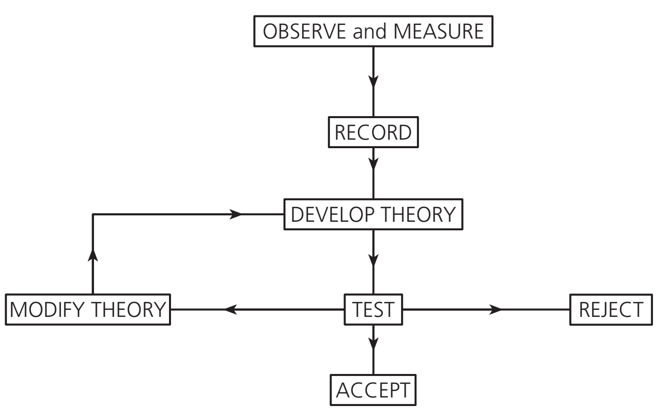
1. understand the difference between scalar and vector quantities and give examples of scalar and vector quantities included in the syllabus
2. add and subtract coplanar vectors
3. represent a vector as two perpendicular components

Starting points

★ Accurate measurement is very important in the development of physics.
★ Physicists begin by observing, measuring and collecting data.
★ The data items are analysed to discover whether they fit into a pattern.
★ If there is a pattern and this pattern can be used to explain other events, it becomes a theory.
★ The process is known as the *scientific method* (see Figure 1.1).



▲ **Figure 1.1** Block diagram to illustrate the scientific method

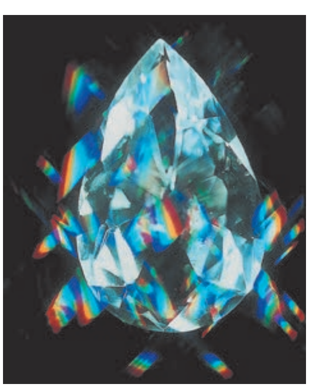
1.1 Physical quantities

▲ **Figure 1.2** Tycho Brahe (1546–1601) measured the elevations of stars; these days a modern theodolite is used for measuring angular elevation.

A **physical quantity** is a feature of something which can be measured, for example, length, mass or the time interval for a particular event. Every physical quantity has a numerical value and a unit; for example, the length of this page is 27.5 cm, the mass of an apple is 450 g, the time to run 100 m is 12 s. If someone says they are able to run a distance of 1500 in 200 s, they could be very fast or very slow depending on whether the measurement of distance is in metres or centimetres! Take care – it is vital to give the unit of measurement whenever a quantity is measured or written down.

Large and small quantities are usually expressed in scientific notation, i.e. as a simple number multiplied by a power of ten. For example, 0.00034 would be written as 3.4×10^{-4} and 152000000 as 1.52×10^{8}. There is far less chance of making a mistake with the number of zeros.

1.2 SI quantities and base units



▲ **Figure 1.3** The mass of this jewel could be measured in kilograms, pounds, carats, grains, etc.

In very much the same way that languages have developed in various parts of the world, many different systems of measurement have evolved. Just as languages can be translated from one to another, units of measurement can also be converted between systems. Although some conversion factors are easy to remember, some are very difficult. It is much better to have just one system of units. For this reason, scientists around the world use the **Système Internationale** (SI), which is based on the metric system of measurement.

If a quantity is to be measured accurately, the unit in which it is measured must be defined as precisely as possible.

SI is founded upon seven fundamental or **base units**. The base units each have a unique definition agreed at world conventions.

The base quantities and the units with which they are measured are listed in Table 1.1. For completeness, the candela has been included, but this unit will not be used in the Cambridge International AS & A Level Physics course. The mole will only be used in the Cambridge International A Level Physics course.

Tables 1.2, 1.3 and 1.4 give some examples of length, mass and time intervals which may be met in the Cambridge International AS & A Level Physics course.

base quantity	base unit	symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere (amp)	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

▲ **Table 1.1** The base quantities and units

quantity	length/m
from Earth to edge of observable Universe	4×10^{26}
diameter of a galaxy	1×10^{21}
from Earth to the Sun	2×10^{11}
radius of the Earth	6×10^{6}
from London to Paris	3×10^{5}
length of a car	2
diameter of a hair	5×10^{-4}
wavelength of light	5×10^{-7}
diameter of an atom	3×10^{-10}
diameter of a nucleus	6×10^{-15}

▲ **Table 1.2** Some values of length given to one significant figure

object	mass/kg
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
container ship	5×10^{8}
elephant	6×10^{3}
car	2×10^{3}
football	4×10^{-1}
grain of sand	4×10^{-10}
hydrogen atom	2×10^{-27}
electron	9×10^{-31}
electron neutrino	4×10^{-36}

▲ **Table 1.3** Some values of mass given to one significant figure

time interval	time interval/s
age of the universe	5×10^{17}
human life expectancy	2×10^{9}
time for the Earth to orbit the Sun	3×10^{7}
orbit period of the Moon	2×10^{6}
time to run a marathon (42 km)	9×10^{3}
time between human heartbeats	1
period of a musical note (middle C)	4×10^{-3}
time for light to travel 1 m	3×10^{-9}
lifetime of a bottom quark	1×10^{-12}
mean lifetime of a Higgs boson	2×10^{-24}

▲ **Table 1.4** Some values of time intervals given to one significant figure

MATHS NOTE

Significant figures

- All non-zero digits are considered significant. For example, 25 has two significant figures (2 and 5), while 123.45 has five significant figures (1, 2, 3, 4 and 5).
- Zeros appearing anywhere between two non-zero digits are significant: 20.052 has five significant figures: 2, 0, 0, 5, 2.
- Zeros to the left of the significant figures are not significant. For example, 0.00034 has two significant figures: 3 and 4.

For example,

- 6 is quoted to 1 significant figure
- 63 is quoted to 2 significant figures
- 634 is quoted to 3 significant figures
- 6.345 is quoted to 4 significant figures.

A problem arises when there are zeros at the end of the number. If the number is 600, then has this number been quoted to one, two, or three significant figures? This problem is overcome by using scientific notation.

For example,

- 6×10^2 is quoted to 1 significant figure
- 6.0×10^2 is quoted to 2 significant figures
- 6.00×10^2 is quoted to 3 significant figures.

Where a number of zeros are given before a number they do not count as significant figures. The number 0.00063 has two significant figures.

Each quantity has just one unit and this unit can have **multiples** and **sub-multiples** to cater for larger or smaller values. The unit is given a **prefix** to denote the multiple or sub-multiple (see Table 1.5). For example, one thousandth of a metre is known as a millimetre (mm) and 1.0 millimetre equals 1.0×10^{-3} metres (m).

prefix	symbol	multiplying factor
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

▲ **Table 1.5** The more commonly used prefixes

Beware when converting units for lengths, areas and volumes!

$$1\,\text{mm} = 10^{-3}\,\text{m}$$

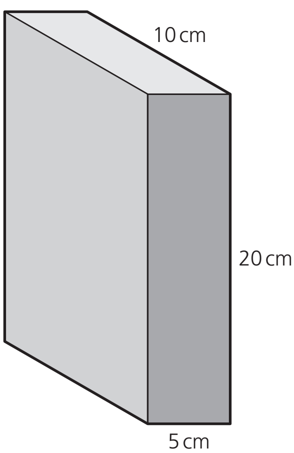
Squaring both sides $1\,\text{mm}^2 = (10^{-3})^2\,\text{m}^2 = 10^{-6}\,\text{m}^2$

and $1\,\text{mm}^3 = (10^{-3})^3\,\text{m}^3 = 10^{-9}\,\text{m}^3$

Note also that $1\,\text{cm} = 10^{-2}\,\text{m}$

$$1\,\text{cm}^2 = (10^{-2})^2\,\text{m}^2 = 10^{-4}\,\text{m}^2$$

and $1\,\text{cm}^3 = (10^{-2})^3\,\text{m}^3 = 10^{-6}\,\text{m}^3$



▲ **Figure 1.4** This box has a volume of $1.0 \times 10^3\,\text{cm}^3$.

The box in Figure 1.4 has a volume of $1.0 \times 10^3\,\text{cm}^3$ or $1.0 \times 10^6\,\text{mm}^3$ or $1.0 \times 10^{-3}\,\text{m}^3$.

A distance of 30 metres should be written as 30 m and not 30 ms or 30 m s. The letter s is *never* included in a unit for the plural. If a space is left between two letters, the letters denote different units. So, 30 m s means 30 metre seconds and 30 ms means 30 milliseconds.

WORKED EXAMPLE 1A

Calculate the number of micrograms in 1.0 milligram.

Answer

$1.0\,\text{g} = 1.0 \times 10^3\,\text{mg}$

and $1.0\,\text{g} = 1.0 \times 10^6$ micrograms (μg)

so, $1.0 \times 10^3\,\text{mg} = 1.0 \times 10^6\,\mu\text{g}$

and $1.0\,\text{mg} = (1.0 \times 10^6)/(1.0 \times 10^3) = \mathbf{1.0 \times 10^3\,\mu g}$

Questions

1 Calculate the area, in cm^2, of the top of a table with sides of 1.2 m and 0.9 m.

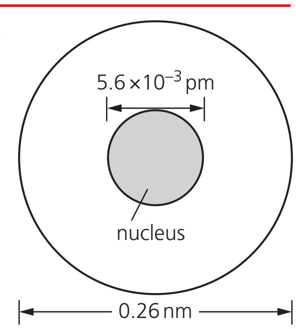
2 Determine the number of cubic metres in one cubic kilometre.

3 Calculate the volume in m^3 of a wire of length 75 cm and diameter 0.38 mm.

4 Write down, using scientific notation, the values of the following quantities:

a 6.8 pF b 32 μC c 60 GW

5 How many electric fires, each rated at 2.5 kW, can be powered from a generator providing 2.0 MW of electric power?



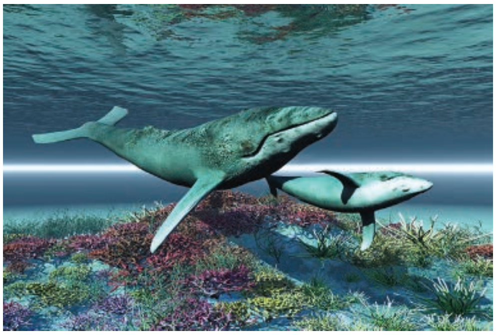
▲ **Figure 1.5** Atom of gold

6 An atom of gold, Figure 1.5, has a diameter of 0.26 nm and the diameter of its nucleus is 5.6×10^{-3} pm. Calculate the ratio of the diameter of the atom to that of the nucleus.

Order of magnitude of quantities

It is often useful to be able to estimate the size, or **order of magnitude**, of a quantity. Strictly speaking, the order of magnitude is the power of ten to which the number is raised. The ability to estimate is particularly important in a subject like physics where quantities have such widely different values. A *short* distance for an astrophysicist is a light-year (about 9.5×10^{15} m) whereas a *long* distance for a nuclear physicist is 6×10^{-15} m (the approximate diameter of a nucleus)!

The ability to *estimate* orders of magnitude is valuable when planning and carrying out experiments or when suggesting theories. Having an idea of the expected result provides a useful check that an error has not been made. This is also true when using a calculator. For example, the acceleration of free fall at the Earth's surface is about $10\,\mathrm{m\,s^{-2}}$. If a value of $9800\,\mathrm{m\,s^{-2}}$ is calculated, then this is obviously wrong and a simple error in the power of ten is likely to be the cause. Similarly, a calculation in which the journey time for a car travelling 400 km at $20\,\mathrm{m\,s^{-1}}$ is found to be several seconds instead of several hours may indicate that the distance has been assumed to be in metres rather than in kilometres.



▲ **Figure 1.6** The ratio of the mass of the humpback whale to the mass of the mouse is about 10^4, or 4 orders of magnitude. That is minute compared to the ratio of the mass of the Sun to the mass of an electron (10^{61} or 61 orders of magnitude)!

The approximate values of common objects or physical quantities should be known. For example, a carton of orange juice has a volume of $1000\,\mathrm{cm^3}$ (1 litre), the mass of a large car is about 2×10^3 kg and an adult about 1×10^2 kg. You could use the more familiar values for the masses of a car and an adult to make a reasonable estimate of the mass of an elephant or a jumbo jet – see Figure 1.7.

▲ **Figure 1.7** The elephant has a mass that is large in comparison with the boy but small compared with the jumbo jet.

WORKED EXAMPLE 1B

Estimate to 1 significant figure:

a the mass of jar of peanut butter in g

b the volume of an orange in cm^3.

Answers

a 5×10^2 g

b 3×10^2 cm^3

Question

7 Estimate to 1 significant figure the following quantities:

a the mass of an orange in g

b the mass of an adult human in kg

c the height of a room in a house in m

d the diameter of a pencil in cm

e the thickness of this page in mm

f the volume of a grain of rice in m^3

g the volume of a human head in m^3

h the maximum speed of a human in $m\,s^{-1}$

i the speed of a jumbo jet in $m\,s^{-1}$

j the kinetic energy of a ocean liner at cruising speed in GJ

k the change in gravitational potential energy of a child climbing two flights of stairs in a house in kJ.

Derived units

All quantities, apart from the base quantities, can be expressed in terms of **derived units**.

Derived units consist of some combination of the base units. The base units may be multiplied together or divided by one another, but never added or subtracted.

See Table 1.6 for examples of derived units. Some quantities have a named unit. For example, the unit of force is the newton, symbol N, but the newton can be expressed in terms of base units. Quantities which do not have a named unit are expressed in terms of other units. For example, moment of a force is measured in newton metre (N m) or $kg\,m^2\,s^{-2}$.

quantity	unit	derived unit
frequency	hertz (Hz)	s^{-1}
velocity	$m\,s^{-1}$	$m\,s^{-1}$
acceleration	$m\,s^{-2}$	$m\,s^{-2}$
force	newton (N)	$kg\,m\,s^{-2}$
momentum	newton second (N s)	$kg\,m\,s^{-1}$
energy	joule (J)	$kg\,m^2\,s^{-2}$
power	watt (W)	$kg\,m^2\,s^{-3}$
electric charge	coulomb (C)	A s
potential difference	volt (V)	$kg\,m^2\,s^{-3}\,A^{-1}$
electrical resistance	ohm (Ω)	$kg\,m^2\,s^{-3}\,A^{-2}$

▲ **Table 1.6** Some examples of derived units

WORKED EXAMPLE 1C

What are the base units of speed?

Answer

Speed is defined as distance/time and so the unit is m/s.

Division by a unit is shown using a negative index that is s^{-1}.

The base units of speed are $\mathbf{m\,s^{-1}}$.

Questions

Use the information in Tables 1.1 and 1.6 to determine the base units of the following quantities.

8 density = mass/volume

9 pressure = force/area

Checking equations

It is possible to work out the total number of oranges in two bags if one bag contains four and the other five (the answer is nine!). This exercise would, of course, be nonsense if one bag contained three oranges and the other four mangoes. In the same way, for any equation to make sense, each term involved in the equation must have the same base units. A term in an equation is a group of numbers and symbols, and each of these terms (or groups) is added to, or subtracted from, other terms. For example, in the equation

$$v = u + at$$

the terms are v, u and at.

In any equation where each term has the same base units, the equation is said to be **homogeneous** or 'balanced'.

In the example above, each term has the base units $m\,s^{-1}$. If the equation is not homogeneous, then it is incorrect and is not valid.

Note the checking an equation to see if it is balanced does not guarantee that the equation is correct. There may be missing or incorrect pure numbers or the equation may not be valid.

WORKED EXAMPLE 1D

Use base units to show that the following equation is homogeneous.

work done = gain in kinetic energy + gain in gravitational potential energy

Answer

The terms in the equation are work, (gain in) kinetic energy and (gain in) gravitational potential energy.

work done = force × distance moved in the direction of the force

and so the base units are $kg\,m\,s^{-2} \times m = \mathbf{kg\,m^2\,s^{-2}}$.

kinetic energy = $\frac{1}{2}$ mass × (speed)^2

Since any pure number such as $\frac{1}{2}$ has no unit, the base units are $kg \times (m\,s^{-1})^2 = \mathbf{kg\,m^2\,s^{-2}}$.

potential energy = mass × acceleration of free fall g × distance

The base units are $kg \times m\,s^{-2} \times m = \mathbf{kg\,m^2\,s^{-2}}$.

Conclusion: All terms have the same base units and the equation is homogeneous.

When an equation is known to be homogeneous, then the balancing of base units provides a means of finding the units of an unknown quantity.

WORKED EXAMPLE 1E

The drag force F acting on a sky diver is given by the equation

$$F = \frac{1}{2}C\rho A v^2$$

where C is a constant, ρ is the density of air, A is the cross-sectional area of the diver and v is the speed of fall. Show that C has no base units.

Answer

The base units of force are **kg m s^{-2}**.

The base units of $\rho A v^2$ are kg m^{-3} × m^2 × (m s^{-1})^2 = **kg m s^{-2}**.

Conclusion: C does not have any base units.

Questions

10 Use base units to check whether the following equations are balanced:

a pressure = depth × density × acceleration of free fall

b energy = mass × (speed of light)^2

11 The work done stretching a spring by extension x is given by the equation

$$W = \frac{1}{2}kx^2$$

where k is a constant. Determine the base units of k.

12 Use base units to check whether the following equations are balanced:

a power = potential difference × electric current,

b electrical energy = (electric current)^2 × resistance × time

13 Show that the left-hand side of the equation

pressure + $\frac{1}{2}$ × density × (speed)^2 = constant

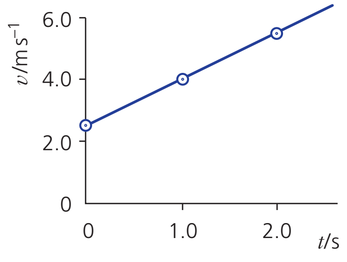
is homogeneous and find the base units of the constant on the right-hand side.

Conventions for symbols and units

You may have noticed that when symbols and units are printed, they appear in different styles of type. The symbol for a physical quantity is printed in *italic* (sloping) type, whereas its unit is in roman (upright) type. For example, velocity v is italic, but its unit m s^{-1} is roman. Of course, you will not be able to make this distinction in handwriting.

At Cambridge International AS & A Level and beyond, there is a special convention for labelling columns of data in tables and graph axes. The symbol is printed first (in italic), separated by a forward slash (the printing term is a solidus) from the unit (in roman). Then the data is presented in a column, or along an axis, as pure numbers. This is illustrated in Figure 1.8, which shows a table of data and the resulting graph for the velocity v of a particle at various times t.

t/s	v/m s^{-1}
0	2.5
1.0	4.0
2.0	5.5



▲ **Figure 1.8** The convention for labelling tables and graphs

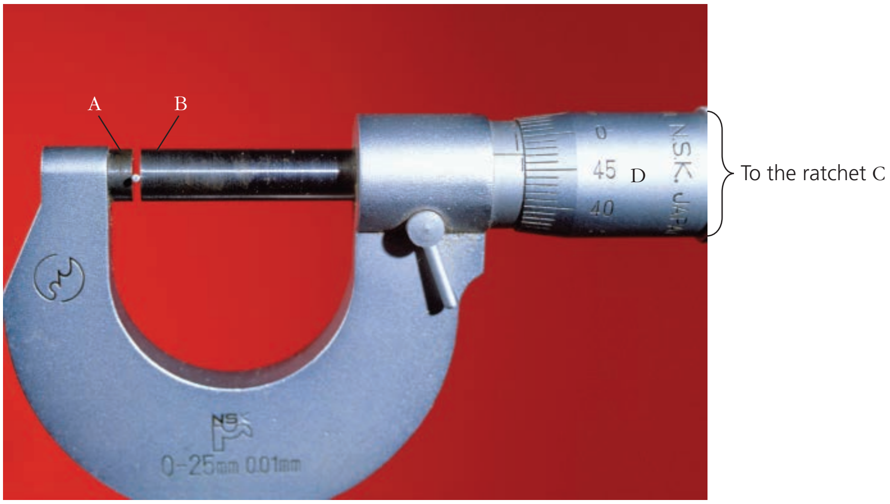
If you remember that a physical quantity contains a pure number and a unit, the reason for this style of presentation becomes clear. By dividing a physical quantity such as time (a number and a unit) by the appropriate unit, you are left with a pure number. It is then algebraically correct for the data in tables, and along graph axes, to appear as pure numbers.

You may also see examples in which the symbol for the physical quantity is followed by the slash, and then by a power of 10, and then the unit, for example, $t/10^2$ s. This means that the column of data has been divided by 100, to save repeating lots of zeros in the table. If you see a table or graph labelled $t/10^2$ s and the figures 1, 2, 3 in the table column or along the graph axis, this means that the experimental data was obtained at values of t of 100 s, 200 s, 300 s.

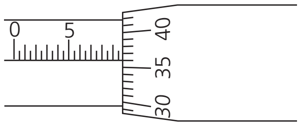
Try to get out of the habit of heading table columns and graphs in ways such as '*t* in s', '*t*(s)' or even of recording each reading in the table as 1.0 s, 2.0 s, 3.0 s.

1.3 Errors and uncertainties

If we want to measure the diameter of a steel sphere or a marble, we could use a metre rule, or a vernier caliper, or a micrometer screw gauge. The choice of measuring instrument would depend on the number of significant figures appropriate or required for the length being measured. For example, the metre rule could be used to measure to the nearest millimetre, the vernier caliper to the nearest tenth of a millimetre, and the micrometer screw gauge to the nearest one-hundredth of a millimetre.



▲ **Figure 1.9** A micrometer screw gauge. The object to be measured is placed between A and B. B is screwed down on to the object, using the ratchet C, until the ratchet slips.

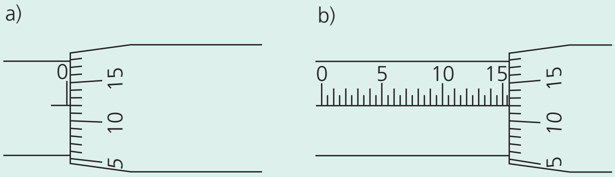


▲ **Figure 1.10** This screw gauge shows a reading of 9.5 mm on the divisions on the barrel plus 0.36 mm on the divisions on the thimble, 9.86 mm in total. You can easily read to the nearest division on the thimble; that is, to the nearest 0.01 mm.

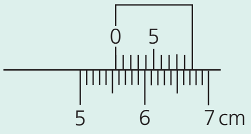
WORKED EXAMPLE 1F

1 Figure 1.11a shows the scale of a micrometer screw gauge when the two faces are moved to make contact with each other (this checks the so-called uncertainty in the zero reading), and Figure 1.11b shows the scale when the gauge is tightened on an object.
What is the length of the object?

2 Figure 1.12 shows the scale of a vernier caliper. What is the reading?



▲ **Figure 1.11** a) and b)



▲ **Figure 1.12**

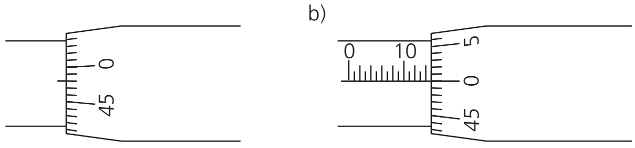
Answers

1 From Figure 1.11a, the initial reading is +0.12 mm. The reading in Figure 1.11b is 15.62 mm.
The length of the object is thus (15.62 − 0.12) mm = **15.50 mm**.

2 The zero of the vernier scale is between the 5.5 cm and 5.6 cm divisions of the fixed scale. There is coincidence between the third graduation of the vernier scale and one of the graduations of the fixed scale. The reading is thus **5.53 cm or 55.3 mm**.

Question

14 Figures 1.13a and 1.13b show the scales of a micrometer screw gauge when the zero is being checked, and again when measuring the diameter of an object. What is the diameter?



▲ **Figure 1.13**

Absolute and percentage uncertainty

We could show the readings for the diameter of a steel sphere measured with a metre rule, or a vernier caliper, or a micrometer screw gauge as follows:

metre rule: 12 ± 1 mm

vernier caliper: 12.3 ± 0.1 mm

micrometer screw gauge: 12.34 ± 0.01 mm

In the list above, each of the measurements is shown with its **uncertainty**.
For example, using the metre rule, the measurement of the diameter is 12 mm with an uncertainty of 1 mm.

The uncertainty in the measurement decreases as we move from the metre rule to the vernier caliper and finally to the micrometer screw gauge.

As we shall see in the section on accuracy and precision, in reality, uncertainty is not the only factor affecting the accuracy of the measurement.

The total range of values within which the measurement is likely to lie is known as its uncertainty.

For example, a measurement of 46.0 ± 0.5 cm implies that the most likely value is 46.0 cm, but it could be as low as 45.5 cm or as high as 46.5 cm. The **absolute uncertainty** in the measurement is ±0.5 cm. The **percentage uncertainty** in the measurement is ±(0.5/46) × 100% = ±1%.

It is important to understand that, when writing down measurements, the number of significant figures of the measurement indicates its uncertainty. Some examples of uncertainty are given in Table 1.7.

instrument	uncertainty	typical reading
top-pan balance	±0.01 g	17.35 g
stop-watch with 0.1 s divisions	±0.1 s	16.2 s
thermometer with 1 deg C intervals	±0.5 °C	22.5 °C
ammeter with 0.1 A divisions	±0.1 A	2.1 A

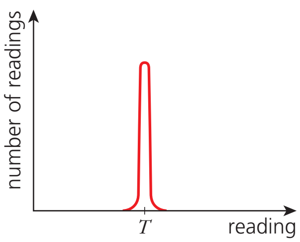
▲ **Table 1.7** Examples of uncertainty

The uncertainty in a measurement should be stated to one significant figure. The value for the quantity should be stated to the same number of decimal places as the uncertainty. For example, the reading for the time should not be stated as 16 s or 16.23 s where the uncertainty is ±0.1 s.

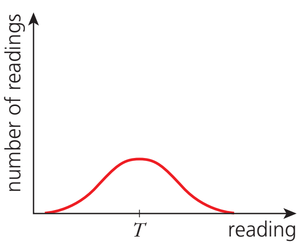
Remember that the uncertainty in a reading is not wholly confined to the reading of its scale or to the skill of the experimenter. Any measuring instrument has a built-in uncertainty. For example, a metal metre rule expands as its temperature rises. At only one temperature will readings of the scale be precise. At all other temperatures, there will be an uncertainty due to the expansion of the scale. Knowing by how much the rule expands would enable this uncertainty to be removed and hence improve precision.

Manufacturers of digital meters quote the uncertainty for each meter. For example, a digital voltmeter may be quoted as ±1% ±2 digits. The ±1% applies to the total reading shown on the scale and the ±2 digits is the uncertainty in the final display figure. This means that the uncertainty in a reading of 4.00 V would be (±4.00 × 1/100) ± 0.02 = ±0.06 V. This uncertainty would be added to any further uncertainty due to a fluctuating reading.

The uncertainty in a measurement is sometimes referred to as being its *error*. This is not strictly true. Error would imply that a mistake has been made. There is no mistake in taking the measurement, but there is always some doubt or some uncertainty as to its value.



a) precise and accurate



b) imprecise but accurate

▲ **Figure 1.14**

Accuracy and precision

Accuracy refers to the closeness of a measured value to the 'true' or 'known' value.

Accuracy depends on the equipment used, the skill of the experimenter and the techniques used. Reducing systematic error or uncertainty (described further below) in a measurement improves its accuracy.

Precision refers to how close a set of measured values are to each other.

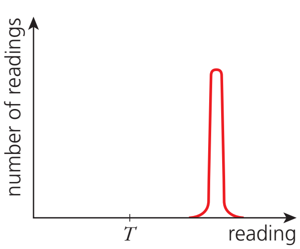
The precision of a set of measured values depends on the range of values. The smaller the range the better the precision.

Reducing the random error or uncertainty in a measurement improves its precision.

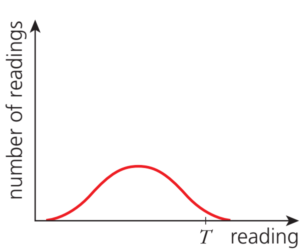
The experimenter may choose different measuring instruments and may use them with different levels of skill, thus affecting the precision of measurement.

When a measurement is repeated many times and the readings are all close together, as shown in Figure 1.14a, the measurement is precise. If there is a greater spread of readings, as shown in Figure 1.14b, the measurement is imprecise.

A set of measurements for a given quantity may be very precise but the measured value may not be accurate. Accuracy is concerned with how close the measured value is to the true value. For example, a micrometer screw gauge can be read to ±0.01 mm but, if there is a large zero error (described in the section on systematic errors below), then the readings from the scale for the diameter of a sphere or marble would not be accurate. The distinction between precision and accuracy is illustrated in Figure 1.15. On each of the graphs the value T is the true value of the quantity.



a) precise but not accurate



b) imprecise and not accurate

▲ **Figure 1.15**

WORKED EXAMPLE 1G

A student takes a large number of imprecise readings for the current in a wire. He uses an ammeter with a zero error of $-\Delta I$, meaning that all scale readings are too small by ΔI. The true value of the current is I. Sketch a distribution curve of the number of readings plotted against the measured value of the current. Label any relevant values.

Answer

This is the case illustrated in Figure 1.15b. The peak of the curve is centred on a value of $I - \Delta I$.

Questions

15 A large number of precise readings for the diameter D of a wire are made using a micrometer screw gauge. The gauge has a zero error $+E$, which means that all readings are too large. Sketch a distribution curve of the number of readings plotted against the measured value of the diameter.

16 The manufacturer of a digital ammeter quotes its uncertainty as ±1.5% ± 2 digits.

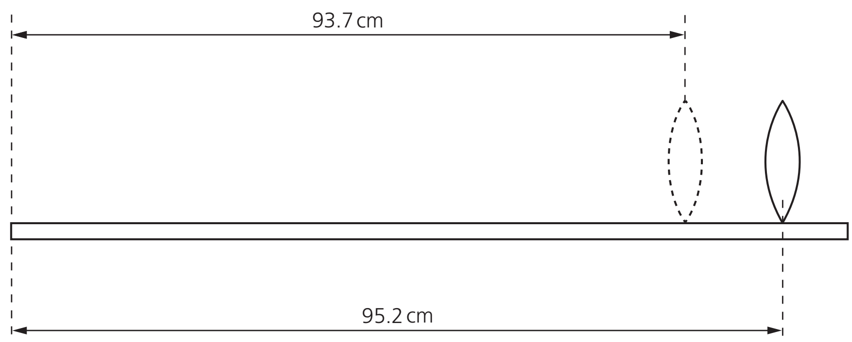
a Determine the uncertainty in a constant reading of 2.64 A.

b The meter is used to measure the current from a d.c. power supply. The current is found to fluctuate randomly between 1.98 A and 2.04 A. Determine the most likely value of the current, with its uncertainty.

Choice of instruments

The choice of an instrument required for a particular measurement is related to the measurement being made. Obviously, if the diameter of a hair is being measured, a micrometer screw gauge is required, rather than a metre rule, as the metre rule can only read to the nearest 0.5 mm so the uncertainty of ±1 mm is much greater than the diameter of the hair. Similarly, a galvanometer should be used to measure currents of the order of a few milliamperes, rather than an ammeter. Choice is often fairly obvious where single measurements are being made, but care has to be taken where two readings are subtracted. Consider the following example.

The distance of a lens from a fixed point is measured using a metre rule. The distance is 95.2 cm (see Figure 1.16). The lens is now moved closer to the fixed point and the new distance is 93.7 cm. How far has the lens moved? The answer is obvious: (95.2 – 93.7) = 1.5 cm. But what is the uncertainty in the measurement?



▲ **Figure 1.16**

The smallest division on the metre rule is 1 mm. If you are careful you should be able to estimate a reading to about 0.5 mm. If you are measuring the length of an object by taking a reading at each end, the uncertainties add to give a total uncertainty of ±1 mm (in this case $\frac{1}{2}$ mm at the zero end of the rule plus $\frac{1}{2}$ mm when finding the position of the centre of the lens). This means that each separate measurement of length has an uncertainty of about (1/940 × 100)%, i.e. about 0.1%. That appears to be good! However, the uncertainty in the distance moved is ±2 mm, because both distances have an uncertainty, and when finding the difference between these distances these uncertainties add up (see the section on Combining uncertainties), so the percentage uncertainty is ±(2/15 × 100)% = ±13%. This uncertainty is, quite clearly, unacceptable. Another means by which the distance moved could be measured must be devised to reduce the uncertainty.

During your Cambridge International AS & A Level Physics course, you will meet with many different measuring instruments. You must learn to recognise which instrument is most appropriate for particular measurements. A stop-watch may be suitable for measuring the period of oscillation of a pendulum but you would have difficulty using it to find the time taken for a stone to fall vertically from rest through a distance of 1 m. Choice of appropriate instruments is likely to be examined when you are planning experiments.

WORKED EXAMPLE 1H

Suggest appropriate instruments for the measurement of the dimensions of a single page of this book.

Answer

The obvious instrument to measure the height and width of a page is a 30 cm ruler, which can be read to ±1 mm. The width, the smaller dimension, is about 210 mm, so the actual uncertainty is 210 mm ±1 mm and the percentage uncertainty is about ±0.5%. It is not sensible to try to measure the thickness of a single page, even with a micrometer screw gauge, as the percentage error will be very high. Instead, use the screw gauge to measure the thickness of a large number of pages (but don't include the covers!). 400 pages are about 18 mm thick. The uncertainty in this measurement, using a screw gauge, is ±0.01 mm, giving a percentage uncertainty of about ±0.05% in the thickness of all 400 pages. This is also the percentage uncertainty in the thickness of a single page. If an uncertainty of ±0.5% is acceptable, a vernier caliper should be used instead of the screw gauge.

Question

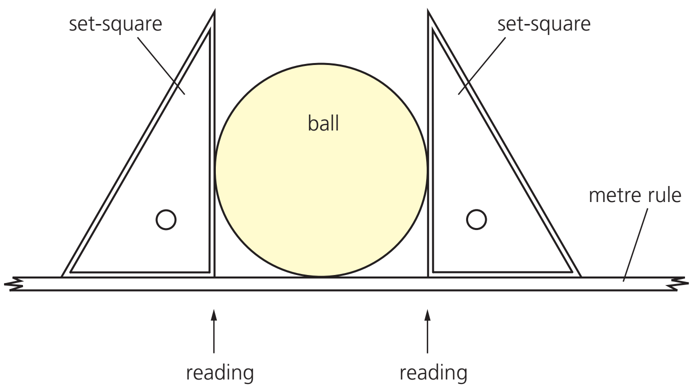
17 The diameter of a ball is measured using a metre rule and a set square, as illustrated in Figure 1.17. The readings on the rule are 16.8 cm and 20.4 cm. Each reading has an uncertainty of ±1 mm.

Calculate, for the diameter of the ball:

a its actual uncertainty

b its percentage uncertainty.

Suggest an alternative, but more precise, method by which the diameter could be measured.



▲ **Figure 1.17**

Systematic and random errors

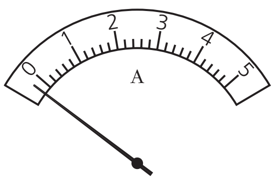
Not only is the choice of instrument important so that any measurement is made with acceptable percentage uncertainty but, also, the techniques of measurement must optimise accuracy. That is, your experimental technique must reduce as far as possible any uncertainties in readings. These uncertainties may be due to either *systematic* or *random* errors.

Systematic error

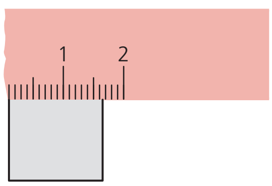
A **systematic error** will result in all readings being either above or below the true value. The shift from the true value is by a fixed amount and is in the same direction each time the measurement is taken. The uncertainty in the reading cannot be eliminated by repeat readings and then averaging. Instead systematic error can be reduced only by improving experimental techniques. This error affects the accuracy of the measurement.

Examples of systematic uncertainty are:

- **zero error on an instrument**
  The scale reading is not zero before measurements are taken – see Figure 1.18. Check before starting the experiment. Another example of a zero error is when the end of a rule is worn– see Figure 1.19. The length of the object is clearly not 1.65 cm. For this reason, it is bad practice to place the zero end of the rule against one end of the object to be measured and to take the reading at the other end. You should place the object against the rule so that a reading is made at each end of the object. The length of the object is then obtained by subtraction of the two readings.
- **wrongly calibrated scale**
  In school laboratories we assume that measuring devices are correctly calibrated (have no systematic error), and would not be expected to check the calibration in an experiment. However, if you have doubts, you can check the calibration of an ammeter by connecting several in series in the circuit, or of a voltmeter by connecting several in parallel. A metre rule can be checked by laying several of them alongside each other. Thermometers can be checked by placing several in well-stirred water. These checks will not enable you to say which of the instruments are calibrated correctly, but they will show you if there is a discrepancy.
- **reaction time of experimenter**
  When timings are carried out manually, it must be accepted that there will be a delay between the experimenter observing the event and starting the timing device. This delay, called the reaction time, may be as much as a few tenths of a second. To reduce the effect, you should arrange that the intervals you are timing are much greater than the reaction time. For example, you should time sufficient swings of a pendulum for the total time to be of the order of at least ten seconds, so that a reaction time of a few tenths of a second is less important.



▲ **Figure 1.18** This ammeter has a zero error of about −0.2 A.



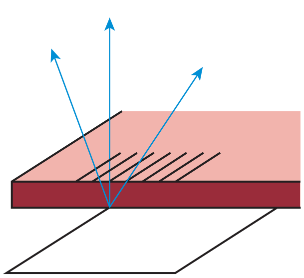
▲ **Figure 1.19** Zero error with a metre rule

Random error

Random error results in readings being scattered around the accepted value. Random error may be reduced by repeating a reading and averaging, and by plotting a graph and drawing a best-fit line. Random error affects the precision of the measurement.

Examples of random errors are:

- reading a scale, particularly if this involves the experimenter's judgement about interpolation between scale readings
- timing oscillations without the use of a reference marker, so that timings may not always be made to the same point of the swing
- taking readings of a quantity that varies with time, involving the difficulty of reading both a timer scale and another meter simultaneously
- reading a scale from different angles introduces a variable **parallax error** – see Figure 1.20. (In contrast, if a scale reading is always made from the same non-normal angle, this will introduce a systematic error.)



▲ **Figure 1.20** Parallax error with a metre rule

Parallax error may be reduced by arranging the rule so that there is no gap between the scale and the object. Parallax error is also important in reading any instrument in which a needle moves over a scale. A rather sophisticated way of eliminating parallax error is to place a mirror alongside the scale. When the needle and scale are viewed directly, the needle and its image in the mirror coincide. This ensures that the scale reading is always taken at the same viewing angle.

WORKED EXAMPLE 1I

The current in a resistor is to be measured using an analogue ammeter. State one source of:

a a systematic error

b a random error.

In both cases, suggest how the error may be reduced.

Answers

a A systematic error could be a zero error on the meter, or a wrongly calibrated scale. This can be reduced by checking for a zero reading before starting the experiment, or using two ammeters in series to check that the readings agree.

b A random error could be a parallax error caused by taking readings from different angles. This can be reduced by the use of a mirror behind the scale and viewing normally.

Questions

18 The length of a pencil is measured with a 30 cm rule. Suggest one possible source of:

a a systematic error

b a random error.

In each case, suggest how the error may be reduced.

19 The diameter of a wire is to be measured to a precision of ±0.01 mm.

a Name a suitable instrument.

b Suggest a source of systematic error.

c Explain why it is good practice to average a set of diameter readings, taken spirally along the length of the wire.

Combining uncertainties

In many situations, in order to obtain the value of a physical quantity, several other quantities are measured. Each of these measured quantities has an uncertainty and these uncertainties must be combined in order to determine the uncertainty in the value of the physical quantity.

There are two simple rules for obtaining an estimate of the overall uncertainty in a final result for a derived quantity. The rules are:

1. For quantities which are added or subtracted to give a final result, add the absolute uncertainties.
2. For quantities which are multiplied together or divided to give a final result, add the fractional or percentage uncertainties.

Suppose that we wish to obtain the value of a physical quantity x by measuring two other quantities, y and z. The relation between x, y and z is known, and is

$$x = y + z$$

If the uncertainties in y and z are Δy and Δz respectively, the uncertainty Δx in x is given by

$$\Delta x = \Delta y + \Delta z$$

If the quantity x is given by

$$x = y - z$$

the uncertainty in x is again given by

$$\Delta x = \Delta y + \Delta z$$

WORKED EXAMPLE 1J

1 I_1 and I_2 are two currents coming into a junction in a circuit. The current I going out of the junction is given by

$I = I_1 + I_2$

In an experiment, the values of I_1 and I_2 are determined as 2.0 ± 0.1 A and 1.5 ± 0.2 A respectively. What is the value of I? What is the uncertainty in this value?

2 In an experiment, a liquid is heated electrically, causing the temperature to change from 20.0 ± 0.2°C to 21.5 ± 0.5°C. Find the change of temperature, with its associated uncertainty.

Answers

1 Using the given equation, the value of I is given by $I = 2.0 + 1.5 =$ **3.5 A**.
The rule for combining the uncertainties gives $\Delta I = 0.1 + 0.2 = 0.3$ A.
The result for I is thus **(3.5 ± 0.3) A**.

2 The change of temperature is $21.5 - 20.0 =$ **1.5°C**. The rule for combining the uncertainties gives the uncertainty in the temperature change as $0.2 + 0.5 = 0.7$°C.
The result for the temperature change is thus **(1.5 ± 0.7)°C**.

Note that this second example shows that a small difference between two quantities may have a large uncertainty, even if the uncertainty in measuring each of the quantities is small. This is an important factor in considering the design of experiments, where the difference between two quantities may introduce an unacceptably large error.

Question

20 Two set-squares and a ruler are used to measure the diameter of a cylinder. The cylinder is placed between the set-squares, and the set-squares are aligned with the ruler, in the manner of the jaws of calipers. The readings on the ruler at opposite ends of a diameter are 4.15 cm and 2.95 cm. Each reading has an uncertainty of ±0.05 cm.

a What is the diameter of the cylinder?

b What is the uncertainty in the diameter?

Now suppose that we wish to find the uncertainty in a quantity x, whose relation to two measured quantities, y and z, is

$$x = Ayz$$

where A is a constant. The uncertainty in the measurement of y is $\pm\Delta y$, and that in z is $\pm\Delta z$. The **fractional uncertainty** in x is given by adding the fractional uncertainties in y and z:

$$\Delta x/x = \Delta y/y + \Delta z/z$$

and the **percentage uncertainty** in x is given by adding the percentage uncertainties in y and z:

$$\Delta x/x \times 100 = \Delta y/y \times 100 + \Delta z/z \times 100$$

To combine the uncertainties when the quantities are raised to a power, for example,

$$x = Ay^a z^b$$

where A is a constant, the rule is to multiply the fractional uncertainties by the power, so

$$\Delta x/x = a(\Delta y/y) + b(\Delta z/z)$$

Similarly the percentage uncertainty in x is given by

$$\Delta x/x \times 100 = a(\Delta y/y) \times 100 + b(\Delta z/z) \times 100$$

WORKED EXAMPLE 1K

The volume of a cylinder is determined by measuring its diameter and length. The diameter d was measured as 2.5 ± 0.1 cm and the length l measured as 7.6 ± 0.1 cm. Determine the volume with its absolute uncertainty in cm^3.

Answer

Volume = $(\pi d^2 l)/4 = \pi \times (2.5)^2 \times 7.6/4 = 37.31\ cm^3$

The percentage uncertainties are $(0.1/2.5) \times 100\% = 4.0\%$ for d and $(0.1/7.6) \times 100\% = 1.3\%$ in l.

The percentage uncertainty in the volume is:

2 × percentage uncertainty in $\boldsymbol{d}$ + percentage uncertainty in $\boldsymbol{l}$ = 9.3%

The absolute uncertainty in the volume is 9.3% of 37.31 = $3.47\ cm^3$.

The volume with its absolute uncertainty is **37 ± 3 cm³**.

WORKED EXAMPLE 1L

A value of the acceleration of free fall g was determined by measuring the period of oscillation T of a simple pendulum of length l. The relation between g, T and l is

$$g = 4\pi^2\left(\frac{l}{T^2}\right)$$

In the experiment, l was measured as 0.55 ± 0.02 m and T was measured as 1.50 ± 0.02 s. Find the value of g, and the fractional and percentage uncertainties in this value.

Answer

Substituting in the equation, $g = 4\pi^2(0.55/1.502) = 9.7\ m\,s^{-2}$. The fractional uncertainties are $\Delta l/l = 0.020/0.55 = 0.036$ and $\Delta T/T = 0.02/1.50 = 0.013$.

Applying the rule to find the fractional uncertainty in g

$$\Delta g/g = \Delta l/l + 2\Delta T/T = 0.036 + 2 \times 0.013 = 0.062$$

The actual uncertainty in g is given by (value of g) × (fractional uncertainty in g) = $9.7 \times 0.062 = 0.60\ m\,s^{-2}$. The experimental value of g, with its uncertainty, is thus $\mathbf{(9.7 \pm 0.6)\ m\,s^{-2}}$.

The percentage uncertainties are 3.6% for l and 1.3% for T. The percentage uncertainty in g is given by $3.6\% + 2 \times 1.3\% = 6.2\%$.

The absolute uncertainty in g is 6.2% of 9.7 also giving $\mathbf{(9.7 \pm 0.6)\ m\,s^{-2}}$.

Note that it is not good practice to determine g from the measurement of the period of a pendulum of fixed length. It would be much better to take values of T for a number of different lengths l, and to draw a graph of T^2 against l. The gradient of this graph is $4\pi^2/g$.

Questions

21 A value of the volume V of a cylinder is determined by measuring the radius r and the length L. The relation between V, r and L is

$$V = \pi r^2 L$$

In an experiment, r was measured as 3.30 ± 0.05 cm, and L was measured as 25.4 ± 0.4 cm. Find the value of V, and the absolute uncertainty in this value.

22 The mass and dimensions of a metal rectangular block are measured. The values obtained are: mass = 1.50 ± 0.01 kg, length =70 ± 1 mm, breadth 60 ± 1 mm and depth 40 ± 1 mm. Determine the density of the metal and its absolute uncertainty in $kg\,m^{-3}$.

If you find it difficult to deal with the fractional uncertainty rule, you can easily estimate the uncertainty by substituting extreme values into the equation. For $x = Ay^a z^b$, taking account of the uncertainties in y and z, the lowest value of x is given by

$$x_{low} = A(y - \Delta y)^a (z - \Delta z)^b$$

and the highest by

$$x_{high} = A(y + \Delta y)^a (z + \Delta z)^b$$

If x_{low} and x_{high} are worked out, the uncertainty in the value of x is given by

$$(x_{high} - x_{low})/2$$

WORKED EXAMPLE 1M

Apply the extreme value method to the data for the simple pendulum experiment in Worked example 1L.

Answer

Because of the form of the equation for g, the lowest value for g will be obtained if the lowest value of l and the highest value for T are substituted. This gives

$$g_{low} = 4\pi^2(0.53/1.52^2) = 9.1\,\text{m s}^{-2}$$

The highest value for g is obtained by substituting the highest value for l and the lowest value for T. This gives

$$g_{high} = 4\pi^2(0.57/1.48^2) = 10.3\,\text{m s}^{-2}$$

The uncertainty in the value of g is thus $(g_{high} - g_{low})/2 = (10.3 - 9.1)/2 = 0.6\,\text{m s}^{-2}$, as before.

Question

23 Apply the extreme value method to the data for the volume of the cylinder, in question 21.

If the expression for the quantity under consideration involves combinations of products (or quotients) and sums (or differences), then the best approach is the extreme value method.

1.4 Scalars and vectors

All physical quantities have a magnitude and a unit. For some quantities, magnitude and units do not give us enough information to fully describe the quantity. For example, if we are given the time for which a car travels at a certain speed, then we can calculate the distance travelled. However, we cannot find out how far the car is from its starting point unless we are told the direction of travel. In this case, the speed and direction must be specified.

Figure 1.21 Although the athlete runs 5 km in the race, his final distance from the starting point may well be zero!

A quantity which can be described fully by giving its magnitude and unit is known as a **scalar quantity**. They can be added algebraically.

A **vector quantity** has magnitude, unit and direction. They may not be added algebraically.

Some examples of scalar and vector quantities are given in Table 1.8.

quantity	scalar	vector
mass	✓	
weight		✓
speed	✓	
velocity		✓
acceleration		✓
force		✓
momentum		✓
energy	✓	
power	✓	
pressure	✓	
temperature	✓	

▲ **Table 1.8** Some scalars and vectors

Note: It may seem that electric current should be treated as a vector quantity. We give current a direction when we deal with, for example, the motor effect (see Topic 20) and when we predict the direction of the magnetic field due to current-carrying coils and wires. However, electric current does not follow the laws of vector addition and should be treated as a scalar quantity.

WORKED EXAMPLE 1N

A 'big wheel' at a theme park has a diameter of 14 m and people on the ride complete one revolution in 24 s. Calculate:

a the distance a rider moves in 3.0 minutes

b the distance of the rider from the starting position.

Answers

a In 3.0 minutes, the rider completes (3.0 × 60)/24 = 7.5 revolutions.

distance travelled = 7.5 × circumference of wheel

= 7.5 × 2π × 7.0

= **330 m**

b 7.5 revolutions completed. Rider is $\frac{1}{2}$ revolution from starting point. The rider is at the opposite end of a diameter of the big wheel. So, the distance from starting position = **14 m**.

Questions

24 State whether the following quantities are scalars or vectors:

a time of departure of a train

b acceleration due to free fall

c density of a liquid.

25 State whether the following quantities are scalars or vectors:

a temperature

b frequency of vibration

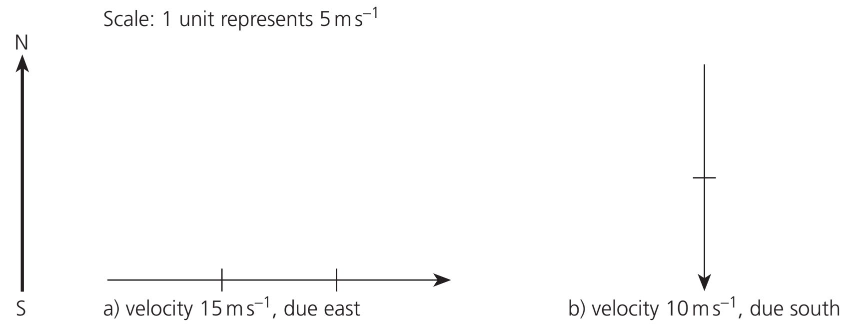
c flow of water in a pipe.

26 Speed and velocity have the same units. Explain why speed is a scalar quantity whereas velocity is a vector quantity.

27 A student states that a bag of sugar has a weight of 10 N and that this weight is a vector quantity. Discuss whether the student is correct when stating that weight is a vector.

Vector representation

When you hit a tennis ball, you have to judge the direction you want it to move in, as well as how hard to hit it. The force you exert is, therefore, a vector quantity and cannot be represented by magnitude (size) alone. One way to represent a vector is by means of an arrow. The direction of the arrow is the direction of the vector quantity. The length of the arrow, drawn to scale, represents its magnitude. This is illustrated in Figure 1.22.

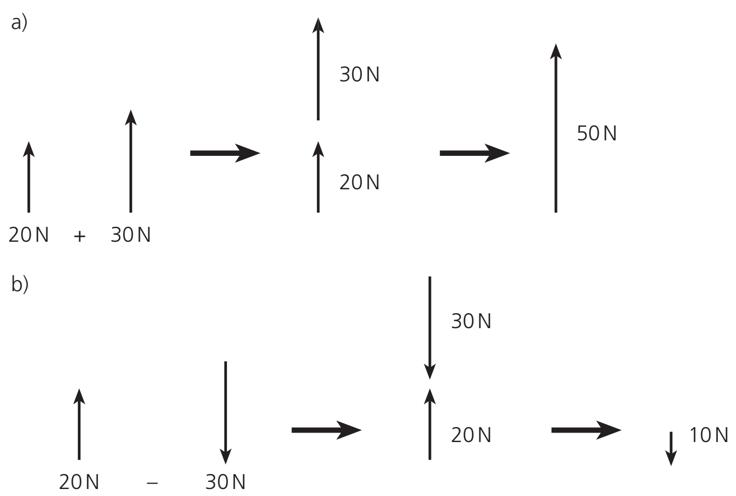


▲ **Figure 1.22** Representation of a vector quantity

Addition of vectors

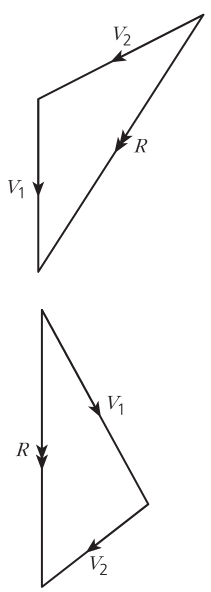
The addition of two scalar quantities which have the same unit is no problem. The quantities are added using the normal rules of addition. For example, a beaker of volume 250 cm^3 and a bucket of volume 9.0 litres have a total volume of 9250 cm^3.

Adding together two vectors is more difficult because they have direction as well as magnitude. If the two vectors are in the same direction, then they can simply be added together. Two objects of weight 50 N and 40 N have a combined weight of 90 N because both weights act in the same direction (vertically downwards). Figure 1.23 shows the effect of adding two forces of magnitudes 30 N and 20 N which act in the same direction or in opposite directions. The angle between the forces is 0° when they act in the same direction and 180° when they are in opposite directions. For all other angles between the directions of the forces, the combined effect, or **resultant**, is some value between 10 N and 50 N.

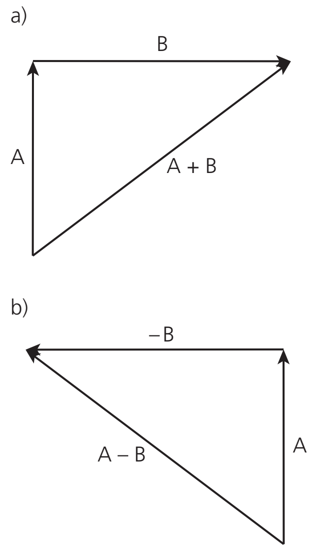


▲ **Figure 1.23** Vector addition a) and vector subtraction b)

In cases where the two vectors do not act in the same or opposite directions, the resultant is found by means of a **vector triangle**. Each one of the two vectors V_1 and V_2 is represented in magnitude and direction by the side of a triangle. Note that both vectors must be in either a clockwise or an anticlockwise direction (see Figure 1.24). The combined effect, or resultant R, is given in magnitude and direction by the third side of the triangle. It is important to remember that, if V_1 and V_2 are drawn clockwise, then R is anticlockwise; if V_1 and V_2 are anticlockwise, R is clockwise.



▲ **Figure 1.24** Vector triangles



▲ **Figure 1.25**

The resultant may be found by means of a scale diagram. Alternatively, having drawn a sketch of the vector triangle, the problem may be solved using trigonometry (see the Maths Note at the end of Topic 1).

The subtraction of vectors obeys the same rules as the addition of the vectors. To subtract a vector B from a vector A, the vector B in the opposite direction is added to vector A.

A – B = A + (–B) where –B is a vector of the same magnitude as B but has opposite direction. For example, if vector A is a displacement of 3 m due north and vector B is a displacement of 4 m due east, then A + B is shown in Figure 1.25a and A – B is shown in Figure 1.25b.

WORKED EXAMPLE 10

A ship is travelling due north with a speed of 12 km h^{-1} relative to the water. There is a current in the water flowing at 4.0 km h^{-1} in an easterly direction relative to the shore. Determine the velocity of the ship relative to the shore by:

a scale drawing

b calculation.

Answers

a By scale drawing (Figure 1.26):
Scale: 1 cm represents 2 km h^{-1}
resultant *R*
The velocity relative to the shore is:
6.3 × 2 = **12.6 km h^{-1}** in a direction **18° east of north**.

b By calculation:
Referring to the diagram (Figure 1.27) and using Pythagoras' theorem,

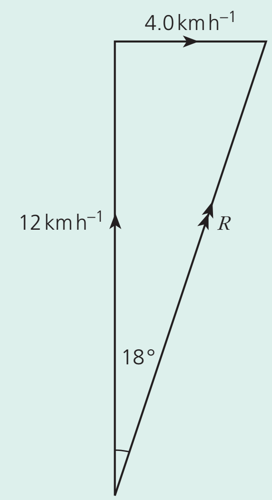
$$R^2 = 12^2 + 4^2 = 160$$

$$R = \sqrt{160} = 12.6$$

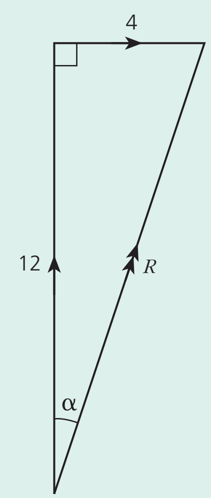
$$\tan\alpha = \frac{4}{12} = 0.33$$

$$\alpha = 18.4°$$

The velocity of the ship relative to the shore is **12.6 km h^{-1}** in a direction **18.4° east of north**.



▲ **Figure 1.26**



▲ **Figure 1.27**

Questions

28 Explain how an arrow may be used to represent a vector quantity.

29 Two forces are of magnitude 450 N and 240 N respectively. Determine:

a the maximum magnitude of the resultant force

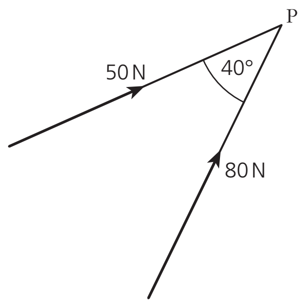
b the minimum magnitude of the resultant force

c the resultant force when the forces act at right angles to each other.

Use a vector diagram and then check your result by calculation.

30 A boat can be rowed at a speed of $7.0\,\mathrm{km\,h^{-1}}$ in still water. A river flows at a constant speed of $1.5\,\mathrm{km\,h^{-1}}$. Use a scale diagram to determine the angle to the bank at which the boat must be rowed in order that the boat travels directly across the river.

31 Two forces act at a point P as shown in Figure 1.28. Draw a vector diagram, to scale, to determine the resultant force. Check your work by calculation.



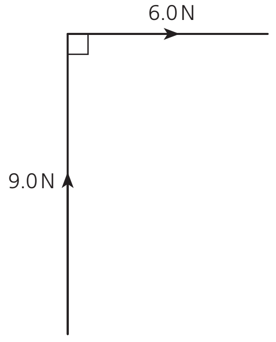
▲ **Figure 1.28**

32 A swimmer who can swim in still water at a speed of $4\,\mathrm{km\,h^{-1}}$ is swimming in a river. The river flows at a speed of $3\,\mathrm{km\,h^{-1}}$. Calculate the speed of the swimmer relative to the river bank when she swims:

a downstream

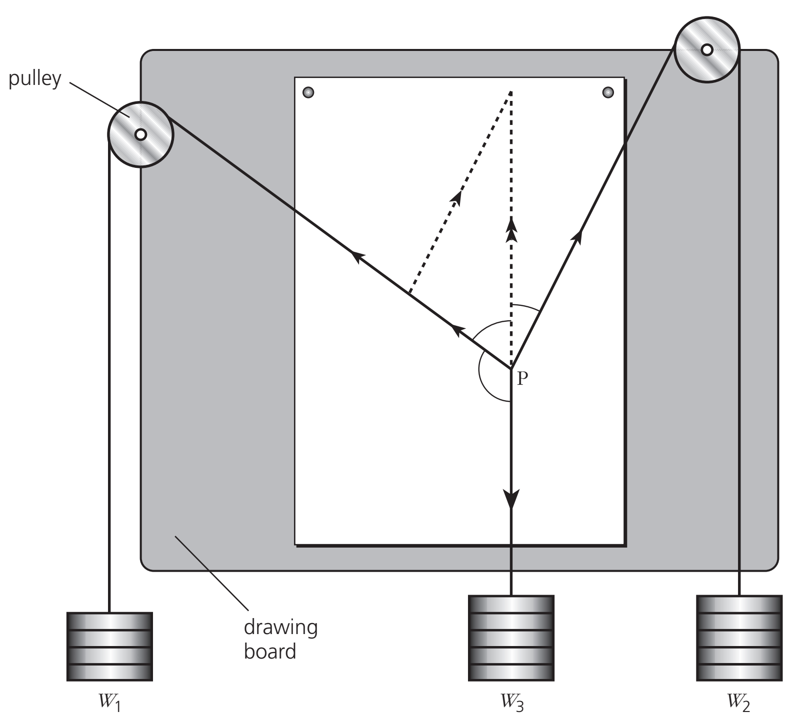
b upstream.

33 Draw to scale a vector triangle to determine the resultant of the two forces shown in Figure 1.29. Check your answer by calculating the resultant.

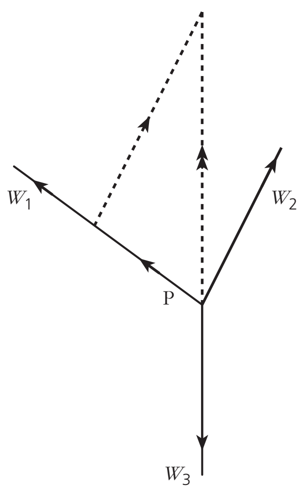


▲ **Figure 1.29**

The use of a vector triangle for finding the resultant of two vectors can be demonstrated by means of a simple laboratory experiment. A weight is attached to each end of a flexible thread and the thread is then suspended over two pulleys, as shown in Figure 1.30. A third weight is attached to a point P near the centre of the thread. The string moves over the pulleys and then comes to rest. The positions of the threads are marked on a piece of paper held on a board behind the threads. This is easy to do if light from a small lamp is shone at the board. Having noted the sizes W_1 and W_2 of the weights on the ends of the thread, a vector triangle can then be drawn on the paper, as shown in Figure 1.31. The resultant of W_1 and W_2 is found to be equal in magnitude but opposite in direction to the weight W_3. If this were not so, there would be a resultant force at P and the thread and weights would move. The use of a vector triangle is justified. The three forces W_1, W_2 and W_3 are in equilibrium. The condition for the vector diagram of these forces to represent the equilibrium situation is that the three vectors should form a **closed triangle**.



▲ **Figure 1.30** Apparatus to check the use of a vector triangle



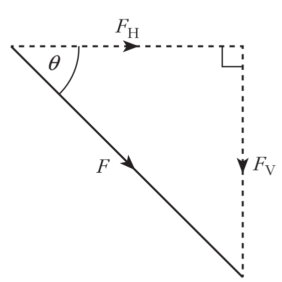
▲ **Figure 1.31** The vector triangle

We have considered only the addition of two vectors. When three or more vectors need to be added, the same principles apply, provided the vectors are **coplanar** (all in the same plane). The vector triangle then becomes a vector polygon: the resultant forms the missing side to close the polygon.

To subtract two vectors, reverse the direction (that is, change the sign) of the vector to be subtracted, and add.

Resolution of vectors

Earlier in this section we saw that two vectors may be added together to produce a single resultant. This resultant behaves in the same way as the two individual vectors. It follows that a single vector may be split up, or **resolved**, into two vectors, or **components**. The combined effect of the components is the same as the original vector. In later topics, we will see that resolution of a vector into two perpendicular components is a very useful means of solving certain types of problem.



▲ **Figure 1.32** Resolving a vector into components

Consider a force of magnitude F acting at an angle of θ below the horizontal (see Figure 1.32. A vector triangle can be drawn with a component F_H in the horizontal direction and a component F_V acting vertically. Remembering that F, F_H and F_V form a right-angled triangle, then

$$F_H = F\cos\theta$$

and $F_V = F\sin\theta$

The force F has been resolved into two perpendicular components, F_H and F_V. The example chosen is concerned with forces, but the method applies to all types of vector quantity.

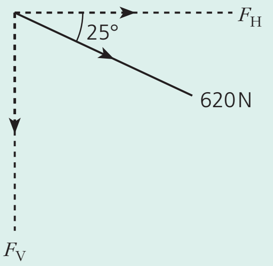
WORKED EXAMPLE 1P

A glider is launched by an aircraft with a cable, as shown in Figure 1.33. At one particular moment, the tension in the cable is 620 N and the cable makes an angle of 25° with the horizontal (see Figure 1.34). Calculate:

a the force pulling the glider horizontally

b the vertical force exerted by the cable on the nose of the glider.

▲ **Figure 1.33**



▲ **Figure 1.34**

Answers

a horizontal component $F_H = 620 \cos 25° =$ **560 N**

b vertical component $F_V = 620 \sin 25° =$ **260 N**

Questions

34 An aircraft is travelling 35° east of north at a speed of 310 km h^{-1}. Calculate the speed of the aircraft in:

a the northerly direction

b the easterly direction.

35 A cyclist is travelling down a hill at a speed of 9.2 m s^{-1}. The hillside makes an angle of 6.3° with the horizontal. Calculate, for the cyclist:

a the vertical speed

b the horizontal speed.

SUMMARY

- All physical quantities have a magnitude (size) and a unit.
- The SI base units of mass, length, time, electric current, thermodynamic temperature and amount of substance are the kilogram, metre, second, ampere, kelvin and mole respectively.
- Units of all mechanical, electrical, magnetic and thermal quantities may be derived in terms of these base units.
- Physical equations must be homogeneous (balanced). Each term in an equation must have the same base units.
- The convention for printing headings in tables of data, and for labelling graph axes, is the symbol for the physical quantity (in *italic*), followed by a forward slash, followed by the abbreviation for the unit (in roman). In handwriting, one cannot distinguish between italic and roman type.
- The order of magnitude of a number is the power of ten to which the number is raised. The order of magnitude can be used to make a check on whether a calculation gives a sensible answer.
- Accuracy refers to the closeness of a measured value to the 'true' or 'known' value.
- Precision is determined by the size of the random error and is the part of accuracy which can be controlled by the experimenter. Precision refers to how close a set of measured values are to each other.
- Uncertainty indicates the range of values within which a measurement is likely to lie.

- A systematic uncertainty (or systematic error) is often due to instrumental causes, and results in all readings being either above or below the true value. It cannot be eliminated by averaging.
- A random uncertainty (or random error) is due to the scatter of readings around the true value. It may be reduced by repeating a reading and averaging, or by plotting a graph and taking a best-fit line.
- Combining uncertainties:
  - for expressions of the form $x = y + z$ or $x = y - z$, the overall uncertainty is $\Delta x = \Delta y + \Delta z$
  - for expressions of the form $x = yz$ or $x = y/z$ the overall uncertainty is given by adding the fractional uncertainties or the percentage uncertainties
  - for expressions of the form $x = Ay^a z^b$, the overall fractional uncertainty is $\Delta x/x = a(\Delta y/y) + b(\Delta z/z)$ or the overall percentage uncertainty $(\Delta x/x) \times 100 = a[(\Delta xy/y) \times 100 + b[(\Delta z/z) \times 100$.
- A scalar quantity has magnitude and unit only.
- A vector quantity has magnitude, unit and direction.
- A vector quantity may be represented by an arrow, with the length of the arrow drawn to scale to give the magnitude.
- The combined effect of two (or more) vectors is called the resultant.
- Coplanar vectors may be added (or subtracted) using a vector diagram.
- The resultant may be found using a scale drawing of the vector diagram, or by calculation.
- A single vector may be divided into two separate components.
- The dividing of a vector into components is known as the resolution of the vector.
- In general, a vector is resolved into two components at right angles to each other.
  - The resolved components of a vector of magnitude V acting at an angle θ to the horizontal are $V\cos\theta$ horizontally and $V\sin\theta$ vertically.

END OF TOPIC QUESTIONS

1 Which of the following is an SI base unit?

A ampere **B** coulomb **C** newton **D** joule

2 Which of the following is not a SI base quantity?

A current **B** energy **C** mass **D** temperature

3 Which of the following is a vector?

A electric charge **B** momentum **C** power **D** work

4 Which order of magnitude is represented by the prefix p (pico)?

A 10^{-6} **B** 10^{-9} **C** 10^{-12} **D** 10^{-15}

5 The speed of a sound wave through a gas of pressure P and density ρ is given by the equation

$$v = \sqrt{\frac{kP}{\rho}}$$ where k is a constant.

An experiment is performed to determine k. The percentage uncertainties in v, P and ρ are ±4%, ±2% and ±3% respectively. Which of the following gives the percentage uncertainty in k?

A ±5% **B** ±9% **C** ±13% **D** ±21%

6 A girl walks at a speed of 1.5 m s^{-1} for 1.0 minutes in a direction of 35° north of east. How far east does she walk?

A 52 m **B** 63 m **C** 74 m **D** 90 m

7 **a** **i** Explain what is meant by a *base unit*.

ii Give four examples of base units.

b State what is meant by a *derived unit*.

c **i** For any equation to be valid, it must be homogeneous. Explain what is meant by a *homogeneous* equation.

ii The pressure P of an ideal gas of density ρ is given by the equation

$$P = \frac{1}{3}\rho \langle c^2 \rangle$$

where $\langle c^2 \rangle$ is the mean-square-speed (i.e. it is a quantity measured as $(\text{speed})^2$). Use base units to show that the equation is homogeneous.

8 The period T of a pendulum of mass M is given by the expression

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

where g is the acceleration of free fall and h is a length.

Determine the base units of the quantity I.

9 **a** Determine the base units of:

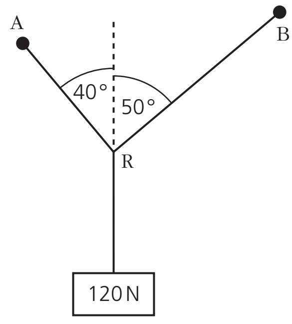
i work done,

ii the moment of a force.

b Explain why your answers to **a** mean that caution is required when the homogeneity of an equation is being tested.

10 Distinguish between *accuracy* and *precision*.

11 The mass of a coin is measured to be 12.5 ± 0.1 g. The diameter is 2.8 ± 0.1 cm and the thickness 2.1 ± 0.1 mm. Calculate the average density of the material from which the coin is made with its uncertainty. Give your answer in kg m^{-3}.



▲ **Figure 1.35**

12 **a** Distinguish between a *scalar* and a *vector* quantity.

b A mass of weight 120 N is hung from two strings as shown in Fig. 1.35. Determine, by scale drawing or by calculation, the tension in:

i RA,

ii RB.

c Use your answers in **b** to determine the horizontal component of the tension in:

i RA,

ii RB.

Comment on your answer.

13 A fielder in a cricket match throws the ball to the wicket-keeper. At one moment of time, the ball has a horizontal velocity of 16 m s^{-1} and a velocity in the vertically upward direction of 8.9 m s^{-1}.

a Determine, for the ball:

i its resultant speed,

ii the direction in which it is travelling relative to the horizontal.

b During the flight of the ball to the wicket-keeper, the horizontal velocity remains unchanged. The speed of the ball at the moment when the wicket-keeper catches it is 19 m s^{-1}. Calculate, for the ball just as it is caught:

i its vertical speed,

ii the angle that the path of the ball makes with the horizontal.

c Suggest with a reason whether the ball, at the moment it is caught, is rising or falling.

14 **a** The spacing between two atoms in a crystal is 3.8 × 10^{-10} m. State this distance in pm. **[1]**

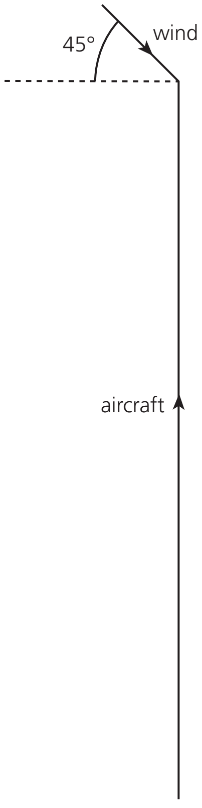
b Calculate the time of one day in Ms. **[1]**

c The distance from the Earth to the Sun is 0.15 Tm. Calculate the time in minutes for light to travel from the Sun to the Earth. **[2]**

d Identify all the vector quantities in the list below.

distance energy momentum weight work **[1]**

e The velocity vector diagram for an aircraft heading due north is shown to scale in Fig. 1.36. There is a wind blowing from the north-west. The speed of the wind is 36 m s^{-1} and the speed of the aircraft is 250 m s^{-1}.



▲ **Figure 1.36**

i Make a copy of Fig. 1.36. Draw an arrow to show the direction of the resultant velocity of the aircraft. [1]

ii Determine the magnitude of the resultant velocity of the aircraft. [2]

Cambridge International AS and A Level Physics (9702) Paper 23 Q1 Oct/Nov 2012

15 a i State the SI base units of volume. [1]

ii Show that the SI base units of pressure are kg m^{-1} s^{-2}. [1]

b The volume V of liquid that flows through a pipe in time t is given by the equation $\frac{V}{t} = \frac{\pi P r^4}{8Cl}$

where P is the pressure difference between the ends of the pipe of radius r and length l. The constant C depends on the frictional effects of the liquid. Determine the base units of C. [3]

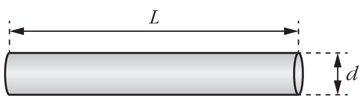
Cambridge International AS and A Level Physics (9702) Paper 21 Q1 May/June 2012

16 a Make estimates of:

i the mass, in kg, of a wooden metre rule, [1]

ii the volume, in cm^3, of a cricket ball or a tennis ball. [1]

b A metal wire of length L has a circular cross-section of diameter d, as shown in Fig. 1.37.



▲ **Figure 1.37**

The volume V of the wire is given by the expression

$$V = \frac{\pi d^2 L}{4}$$

The diameter d, length L and mass M are measured to determine the density of the metal of the wire. The measured values are:

$d = 0.38 \pm 0.01$ mm,

$L = 25.0 \pm 0.1$ cm,

$M = 0.225 \pm 0.001$ g.

Calculate the density of the metal, with its absolute uncertainty. Give your answer to an appropriate number of significant figures. [5]

Cambridge International AS and A Level Physics (9702), Paper 21 Q1 May/June 2016

17 a i Define *pressure*. [1]

ii Show that the SI base units of pressure are kg m^{-1} s^{-2}. [1]

b Gas flows through the narrow end (nozzle) of a pipe. Under certain conditions, the mass m of gas that flows through the nozzle in a short time t is given by

$$\frac{m}{t} = kC\sqrt{\rho P}$$

where k is a constant with no units,

C is a quantity that depends on the nozzle size,

ρ is the density of the gas arriving at the nozzle,

P is the pressure of the gas arriving at the nozzle.

Determine the base units of C. [3]

Cambridge International AS and A Level Physics (9702) Paper 22 Q1 Oct/Nov 2016

18 a An analogue voltmeter is used to take measurements of a constant potential difference across a resistor.

For these measurements, describe **one** example of:

i a systematic error, [1]

ii a random error. [1]

b The potential difference across a resistor is measured as 5.0 V ± 0.1 V. The resistor is labelled as having a resistance of 125 Ω ± 3%.

i Calculate the power dissipated by the resistor. [2]

ii Calculate the percentage uncertainty in the calculated power. [2]

iii Determine the value of the power, with its absolute uncertainty, to an appropriate number of significant figures. [2]

Cambridge International AS and A Level Physics (9702) Paper 23 Q1 May/June 2018

MATHS NOTE

Sine rule

For any triangle (Figure 1.38),

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

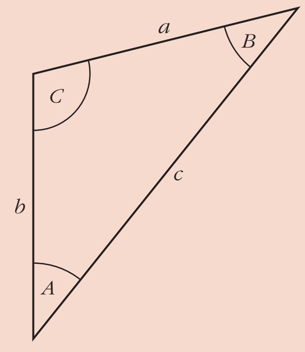
Cosine rule

For any triangle,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



▲ **Figure 1.38**

Pythagoras' theorem

For a right-angled triangle (Figure 1.39),

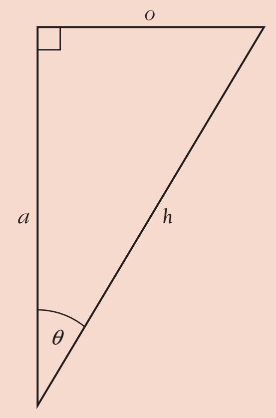
$$h^2 = o^2 + a^2$$

Also for a right-angled triangle:

$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

$$\tan\theta = \frac{o}{a}$$



▲ **Figure 1.39**

2 Kinematics

Learning outcomes

By the end of this topic, you will be able to:

2.1 Equations of motion

1 define and use distance, displacement, speed, velocity and acceleration
2 use graphical methods to represent distance, displacement, speed, velocity and acceleration
3 determine displacement from the area under a velocity–time graph
4 determine velocity using the gradient of a displacement–time graph
5 determine acceleration using the gradient of a velocity–time graph
6 derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line
7 solve problems using equations that represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance
8 describe an experiment to determine the acceleration of free fall using a falling object
9 describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction

Starting points

- Kinematics is a description of how objects move.
- The motion of objects can be described in terms of quantities such as position, speed, velocity and acceleration.

2.1 Equations of motion

Distance, displacement, speed, velocity and acceleration

Distance and displacement

The **distance** moved by a particle is the length along the actual path travelled from the starting point to the finishing point. Distance is a scalar quantity.

The **displacement** of a particle is its change of position. The displacement is the length travelled in a straight line in a specified direction from the starting point to the finishing point. Displacement is a vector quantity.

Consider a cyclist travelling 500 m due east along a straight road, and then turning round and coming back 300 m. The total distance travelled is 800 m, but the displacement is only 200 m due east, since the cyclist has ended up 200 m from the starting point.

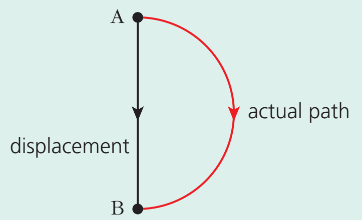
WORKED EXAMPLE 2A

A particle moves from point A to point B along the path of a circle of radius 5.0 m as shown in Figure 2.1.

What is

a the distance moved by the particle

b the displacement of the particle?



▲ **Figure 2.1** The movement of a particle from A to B

Answers

a The actual path of the particle along the circumference of the circle = $\pi \times 5$ = **16 m**.

b The displacement of the particle is the straight line from A to B along the diameter of the circle = **10 m in the direction downwards**.

Average speed

When talking about motion, we shall discuss the way in which the position of a particle varies with time. Think about a particle moving its position. In a certain time, the particle will cover a certain distance. The average speed of the particle is defined as the distance moved along the actual path divided by the time taken. Written as a word equation, this is

$$\text{average speed} = \frac{\text{distance moved along actual path}}{\text{time taken}}$$

The unit of speed is the metre per second ($m\,s^{-1}$).

One of the most fundamental of physical constants is the speed of light in a vacuum. It is important because it is used in the definition of the metre, and because, according to the theory of relativity, it defines an upper limit to attainable speeds. The range of typical speeds that you are likely to come across is enormous; some are summarised in Table 2.1.

	speed/$m\,s^{-1}$
light	3.0×10^8
electron around nucleus	2.2×10^6
Earth around Sun	3.0×10^4
jet airliner	2.5×10^2
typical car speed (80 km per hour)	22
sprinter	1.0×10^1
walking speed	1.5
snail	1.0×10^{-3}

▲ **Table 2.1** Examples of speeds

It is important to recognise that speed has a meaning only if it is quoted relative to a fixed reference. In most cases, speeds are quoted relative to the surface of the Earth, which – although it is moving relative to the Solar System – is often taken to be fixed. Thus, when we say that a bird can fly at a certain average speed, we are relating its speed to the Earth. However, a passenger on a ferry may see that a seagull, flying parallel to the boat, appears to be practically stationary. If this is the case, the seagull's speed relative to the boat is zero. However, if the speed of the boat through the water is $8\,m\,s^{-1}$, then the speed of the seagull relative to Earth is also $8\,m\,s^{-1}$. When talking about relative speeds we must also be careful about directions. It is easy if the speeds are in the same direction, as in the example of the ferry and the seagull. If the speeds are not in the same direction the addition of the motions should follow those introduced for vectors as considered in Topic 1.4.

WORKED EXAMPLE 2B

1 The radius of the Earth is 6.4×10^6 m; one revolution about its axis takes 24 hours (8.6×10^4 s).
Calculate the average speed of a point on the Equator relative to the centre of the Earth.

2 How far does a cyclist travel in 11 minutes if his average speed is 22 km h^{-1}?

3 A train is travelling at a speed of 25 m s^{-1} along a straight track. A boy walks along the corridor in a carriage towards the rear of the train, at a speed of 1 m s^{-1} relative to the train. What is his speed relative to Earth?

Answers

1 In 24 hours, the point on the equator completes one revolution and travels a distance of $2\pi \times$ the Earth's radius, that is $2\pi \times 6.4 \times 10^6 = 4.0 \times 10^7$ m.
The average speed is (distance moved)/(time taken), or $4.0 \times 10^7/8.6 \times 10^4$ $=$ **4.7×10^2 m s^{-1}**.

2 First convert the average speed in km h^{-1} to a value in m s^{-1}.
22 km (2.2×10^4 m) in 1 hour (3.6×10^3 s) is an average speed of 6.1 m s^{-1}.
11 minutes is 660 s.
Since average speed is (distance moved)/(time taken), the distance moved is given by (average speed) $\times$ (time taken), or $6.1 \times 660 =$ **4000 m**.
Note the importance of working in consistent units: this is why the average speed and the time were converted to m s^{-1} and s respectively.

3 In one second, the train travels 25 m forwards along the track. In the same time the boy moves 1 m towards the rear of the train, so he has moved 24 m along the track. His speed relative to Earth is thus $25 - 1 =$ **24 m s^{-1}**.

Questions

1 The speed of an electron in orbit about the nucleus of a hydrogen atom is 2.2×10^6 m s^{-1}. It takes 1.5×10^{-16} s for the electron to complete one orbit. Calculate the radius of the orbit.

2 The average speed of an airliner on a domestic flight is 220 m s^{-1}. Calculate the time taken to fly between two airports on a flight path 700 km long.

3 Two cars are travelling in the same direction on a long, straight road. The one in front has an average speed of 25 m s^{-1} relative to Earth; the other's is 31 m s^{-1}, also relative to Earth. What is the speed of the second car relative to the first when it is overtaking?

Speed and velocity

In ordinary language, there is no difference between the terms *speed* and *velocity*. However, in physics there is an important distinction between the two. **Velocity** is used to represent a vector quantity: the magnitude of how fast a particle is moving, and the direction in which it is moving. **Speed** does not have an associated direction. It is a scalar quantity (see Topic 1.4).

So far, we have talked about the total distance travelled by an object along its actual path. Like speed, distance is a scalar quantity, because we do not have to specify the direction in which the distance is travelled. However, in defining velocity we use the quantity **displacement**.

The **average velocity** is defined as the displacement divided by the time taken.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

Because distance and displacement are different quantities, the average speed of motion will sometimes be different from the magnitude of the average velocity. If the time taken for the cyclist's trip in the example at the start of this Topic is 120 s, the average speed is 800/120 = 6.7 m s^{-1}, whereas the magnitude of the average velocity is 200/120 = 1.7 m s^{-1}. This may seem confusing, but the difficulty arises only when the motion involves a change of direction and we take an average value. If we are interested in describing the motion of a particle at a particular moment in time, the speed at that moment is the same as the magnitude of the velocity at that moment.

▲ **Figure 2.2**

We now need to define average velocity more precisely, in terms of a mathematical equation, instead of our previous word equation. Suppose that at time t_1 a particle is at a point x_1 on the x-axis (Figure 2.2). At a later time t_2, the particle has moved to x_2. The displacement (the change in position) is $(x_2 - x_1)$, and the time taken is $(t_2 - t_1)$.

The average velocity $\bar{v}$ is then

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

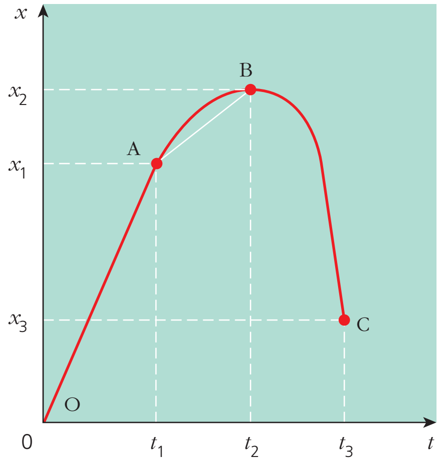
The bar over v is the symbol meaning 'average'. As a shorthand, we can write $(x_2 - x_1)$ as Δx, where Δ (the Greek capital letter delta) means 'the change in'. Similarly, $t_2 - t_1$ is written as Δt. This gives us

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

If x_2 were less than x_1, $(x_2 - x_1)$ and Δx would be negative. This would mean that the particle had moved to the left, instead of to the right as in Figure 2.2. The sign of the displacement gives the direction of particle motion. If Δx is negative, then the average velocity v is also negative. The sign of the velocity, as well as the sign of the displacement, indicates the direction of the particle's motion. This is because both displacement and velocity are vector quantities.

Describing motion by graphs

Position–time graphs

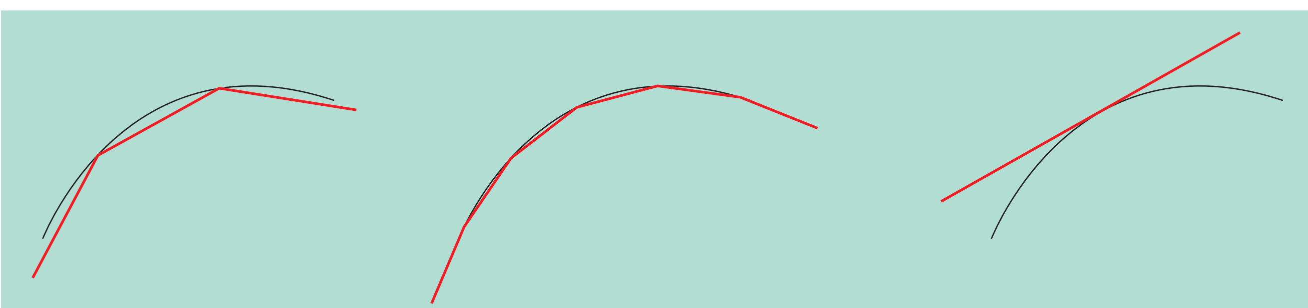


▲ **Figure 2.3**

Figure 2.3 is a graph of position x against time t for a particle moving in a straight line.

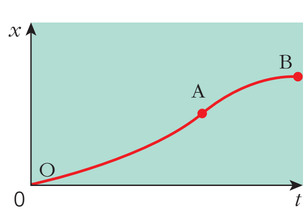
This curve gives a complete description of the motion of the particle. We can see from the graph that the particle starts at the origin O (at which $x = 0$) at time $t = 0$. From O to A the graph is a straight line: the particle is covering equal distances in equal periods of time. This represents a period of *uniform* (constant) velocity. The average velocity during this time is $(x_1 - 0)/(t_1 - 0)$. Clearly, this is the gradient of the straight-line part of the graph between O and A. Between A and B the particle is slowing down, because the distances travelled in equal periods of time are getting smaller. The average velocity during this period is $(x_2 - x_1)/(t_2 - t_1)$. On the graph, this is represented by the gradient of the straight line joining A and B. At B, for a moment, the particle is at rest, and after B it has reversed its direction and is heading back towards the origin. Between B and C the average velocity is $(x_3 - x_2)/(t_3 - t_2)$. Because x_3 is less than x_2, this is a negative quantity, indicating the reversal of direction.

Calculating the average velocity of the particle over the relatively long intervals t_1, $(t_2 - t_1)$ and $(t_3 - t_2)$ will not, however, give us the complete description of the motion. To describe the motion exactly, we need to know the particle's velocity at every instant. We introduce the idea of **instantaneous velocity**. To define instantaneous velocity we make the intervals of time over which we measure the average velocity shorter and shorter. This has the effect of approximating the curved displacement–time graph by a series of short straight-line segments. The approximation becomes better the shorter the time interval, as illustrated in Figure 2.4. Eventually, in the case of extremely small time intervals (mathematically we would say 'infinitesimally small'), the straight-line segment has the same direction as the tangent to the curve. This limiting case gives the instantaneous velocity as the gradient of the tangent to the displacement–time curve.

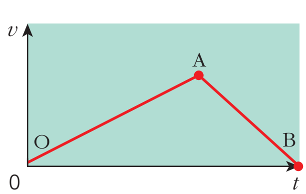


▲ **Figure 2.4**

Displacement–time and velocity–time graphs



▲ **Figure 2.5**



▲ **Figure 2.6**

Figure 2.5 is a sketch graph showing how the displacement of a car, travelling along a straight test track, varies with time. We interpret this graph in a descriptive way by noting that between O and A the distances travelled in equal intervals of time are progressively increasing: that is, the velocity is increasing as the car is accelerating. Between A and B the distances for equal time intervals are decreasing; the car is slowing down. Finally, there is no change in position, even though time passes, so the car must be at rest. We can use Figure 2.5 to deduce the details of the way in which the car's instantaneous velocity v varies with time. To do this, we draw tangents to the curve in Figure 2.5 at regular intervals of time, and measure the slope of each tangent to obtain values of v. The plot of v against t gives the graph in Figure 2.6. This confirms our descriptive interpretation: the velocity increases from zero to a maximum value, and then decreases to zero again. We will look at this example in more detail later on, where we shall see that the area under the velocity–time graph in Figure 2.6 gives the displacement x.

Acceleration

We have used the word *accelerating* in describing the increase in velocity of the car in the previous section. Acceleration is a measure of the rate at which the velocity of the particle is changing. **Average acceleration** is defined by the word equation

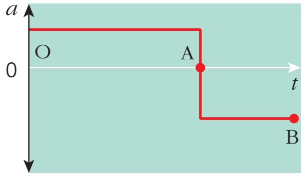
$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

The unit of acceleration is the unit of velocity (the metre per second) divided by the unit of time (the second), giving the metre per (second)^2 which is represented as $\mathrm{m\,s^{-2}}$.
In symbols, this equation is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where v_1 and v_2 are the velocities at time t_1 and t_2 respectively. To obtain the **instantaneous acceleration**, we take extremely small time intervals, just as we did when defining instantaneous velocity. Because it involves a change in velocity (a vector quantity), acceleration is also a vector quantity: we need to specify both its magnitude and its direction.

A particle moving with uniform (constant) velocity has zero acceleration. This means that the magnitude (speed) of the particle and its direction are not changing with time.



▲ **Figure 2.7**

We can deduce the acceleration of a particle from its velocity–time graph by drawing a tangent to the curve and finding the slope of the tangent. Figure 2.7 shows the result of doing this for the car's motion described by Figure 2.5 (the displacement–time graph) and Figure 2.6 (the velocity–time graph). The car accelerates at a constant rate between O and A, and then decelerates (that is, slows down) uniformly between A and B.

An acceleration with a very familiar value is the acceleration of free fall near the Earth's surface (discussed further below): this is $9.81\,\mathrm{m\,s^{-2}}$, often approximated to $10\,\mathrm{m\,s^{-2}}$. To illustrate the range of values you may come across, some accelerations are summarised in Table 2.2.

	acceleration/$\mathrm{m\,s^{-2}}$
due to circular motion of electron around nucleus	9×10^{26}
car crash	1×10^{3}
free fall on Earth	10
family car	2
free fall on Moon	2
at Equator, due to rotation of Earth	3×10^{-2}
due to circular motion of Earth around Sun	6×10^{-5}

▲ **Table 2.2** Examples of accelerations

WORKED EXAMPLE 2C

1 A sports car accelerates along a straight test track from rest to $70\,\mathrm{km\,h^{-1}}$ in 6.3 s. What is its average acceleration?

2 A railway train, travelling along a straight track, takes 1.5 minutes to come to rest from a speed of $115\,\mathrm{km\,h^{-1}}$. What is its average acceleration while braking?

Answers

1 First convert the data into consistent units. 70 km (7.0×10^4 m) in 1 hour (3.6×10^3 s) is $19\,\mathrm{m\,s^{-1}}$.
Since average acceleration is (change of velocity)/(time taken), the acceleration is 19/6.3 = $\mathbf{3.0\,m\,s^{-2}}$.

2 $115\,\mathrm{km\,h^{-1}}$ is $31.9\,\mathrm{m\,s^{-1}}$, and 1.5 minutes is 90 s. The average acceleration is (change of velocity)/(time taken) = −31.9/90 = $\mathbf{-0.35\,m\,s^{-2}}$.
Note that the acceleration is a negative quantity because the change of velocity is negative: the final velocity is less than the initial. A negative acceleration is often called a deceleration.

Questions

4 A sprinter, starting from the blocks, reaches his full speed of $9.0\,\text{m s}^{-1}$ in 1.5 s. What is his average acceleration?

5 A car is travelling at a speed of $25\,\text{m s}^{-1}$. At this speed, it is capable of accelerating at $1.8\,\text{m s}^{-2}$. How long would it take to accelerate from $25\,\text{m s}^{-1}$ to the speed limit of $31\,\text{m s}^{-1}$?

6 At an average speed of $24\,\text{km h}^{-1}$, how many kilometres will a cyclist travel in 75 minutes?

7 An aircraft travels 1600 km in 2.5 hours. What is its average speed, in m s^{-1}?

8 Does a car speedometer register speed or velocity? Explain.

9 An aircraft travels 1400 km at a speed of $700\,\text{km h}^{-1}$, and then runs into a headwind that reduces its speed over the ground to $500\,\text{km h}^{-1}$ for the next 800 km. What is the total time for the flight? What is the average speed of the aircraft?

10 A sports car can stop in 6.1 s from a speed of $110\,\text{km h}^{-1}$. What is its acceleration?

11 Can the velocity of a particle change if its speed is constant? Can the speed of a particle change if its velocity is constant? If the answer to either question is 'yes', give examples.

Uniformly accelerated motion

Having defined displacement, velocity and acceleration, we shall use the definitions to derive a series of equations, called the *kinematic equations*, which can be used to give a complete description of the motion of a particle in a straight line. The mathematics will be simplified if we deal with situations in which the acceleration does not vary with time; that is, the acceleration is uniform (or constant). This approximation applies for many practical cases. However, there are two important types of motion for which the kinematic equations do not apply: circular motion and the oscillatory motion called simple harmonic motion. We shall deal with these separately in Topic 12 and Topic 17.

Think about a particle moving along a straight line with constant acceleration a. Suppose that its initial velocity, at time $t = 0$, is u. After a further time t its velocity has increased to v. From the definition of acceleration as (change in velocity)/(time taken), we have $a = (v - u)/t$ or, re-arranging,

$$v = u + at$$

From the definition of average velocity $\bar{v}$ (distance travelled)/(time taken), over the time t the distance travelled s will be given by the average velocity multiplied by the time taken, or

$$s = \bar{v}t$$

The average velocity $\bar{v}$ is written in terms of the initial velocity u and final velocity v as

$$\bar{v} = \frac{u + v}{2}$$

and, using the previous equation for v,

$$\bar{v} = \frac{(u + u + at)}{2} = u + \frac{at}{2}$$

Substituting this we have

$$s = ut + \frac{1}{2}at^2$$

The right-hand side of this equation is the sum of two terms. The ut term is the distance the particle would have travelled in time t if it had been travelling with a constant speed u, and the $\frac{1}{2}at^2$ term is the additional distance travelled as a result of the acceleration.

The equation relating the final velocity v, the initial velocity u, the acceleration a and the distance travelled s is

$$v^2 = u^2 + 2as$$

If you wish to see how this is obtained from previous equations, see the Maths Note below.

MATHS NOTE

From $v = u + at$,

$t = (v - u)/a$

Substitute this in $s = ut + \frac{1}{2}at^2$

$s = u(v - u)/a + \frac{1}{2}a(v - u)^2/a^2$

Multiplying both sides by $2a$ and expanding the terms,

$2as = 2uv - 2u^2 + v^2 - 2uv + u^2$

or $v^2 = u^2 + 2as$

The five equations relating the various quantities which define the motion of the particle in a straight line in uniformly accelerated motion are

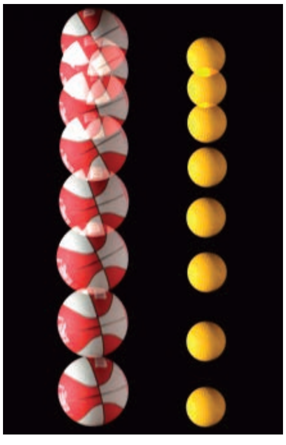
$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \frac{(u + v)t}{2}$$

In these equations u is the initial velocity, v is the final velocity, a is the acceleration, s is the distance travelled, and t is the time taken. The average velocity, $\bar{v}$ is given by $(u + v)/2$.

In solving problems involving kinematics, it is important to understand the situation before you try to substitute numerical values into an equation. Identify the quantity you want to know, and then make a list of the quantities you know already. This should make it obvious which equation is to be used.

Free fall acceleration

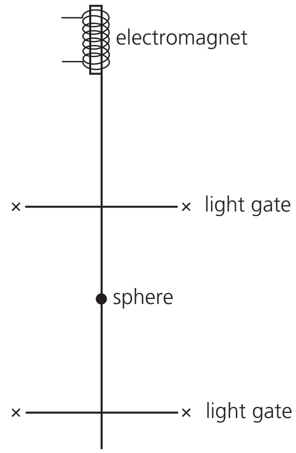
A very common example of uniformly accelerated motion is when an object falls freely near the Earth's surface. Because of the **gravitational field** of the Earth, the Earth exerts a force on all objects dropped near its surface. The gravitational field near the surface of the Earth is taken to be uniform, so all objects fall with the same uniform acceleration. A Level Topic 13 will describe gravitational fields in more detail. This acceleration is called the **acceleration of free fall**, and is represented by the symbol g. It has a value of $9.81\,\text{m}\,\text{s}^{-2}$, and is directed downwards. For completeness, we ought to qualify this statement by saying that the fall must be in the absence of **air resistance**, but in most situations this can be assumed to be true.



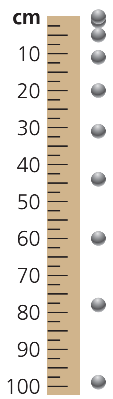
▲ **Figure 2.8** Strobe-flash photograph of objects in free fall

Determination of the acceleration of free fall

The acceleration of free fall may be determined in several ways. The most direct method involves timing the fall of an object from rest through a measured height. Note that, because the time of fall is likely to be only a few tenths of a second, a timing device that can measure to one-hundredth of a second is required.



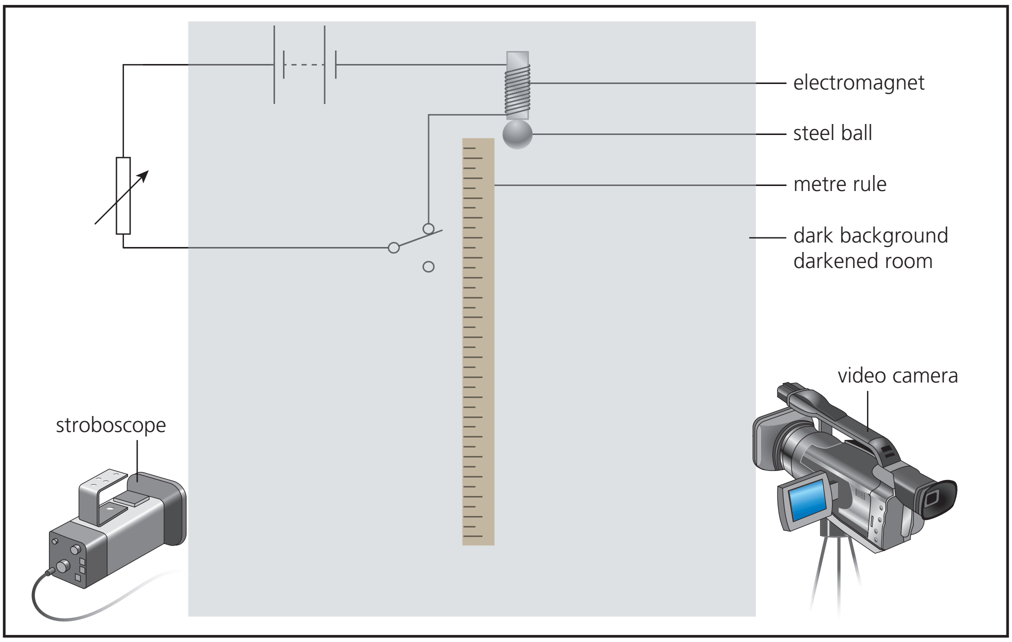
▲ **Figure 2.9** Determination of the acceleration of free fall using a ball falling between two light gates



▲ **Figure 2.11** Strobe-flash image of a ball in free fall

For example, a steel sphere is released from an electromagnet and falls under gravity. As it falls, it passes through two light gates which switch an electronic timer on and then off (Figure 2.9). The acceleration of free fall can be determined from the values of the time interval and distance between the two light gates.

In one example of an experiment to determine the acceleration of free fall, the fall of a steel ball is recorded using strobe-flash photography. A steel ball is released from an electromagnet and falls under gravity (Figure 2.10). A video camera is used to produce a film of the ball's fall. A stroboscope is used to flash a light at a selected frequency. The film shows the position of the ball at regular intervals of time against the scale on a measuring tape or metre rule as the ball falls vertically (Figure 2.11).



▲ **Figure 2.10** An experiment to determine the acceleration due to free fall

If the object falls from rest, we can use the second of the equations for uniformly accelerated motion in the form

$$s = \frac{1}{2}at^2$$

to calculate the value of g. The frequency of the stroboscope gives the time interval to one-hundredth of a second between flashes of light and hence the images of the ball on the film. Table 2.3 shows a typical set of results. A frequency of 20 Hz was used for the stroboscope and the time interval obtained from $T = 1/f$. The time of zero is taken at the first clear image of the ball. A graph of the displacement s against t^2 should give a straight line of gradient $\frac{1}{2}a$ from which g can be calculated.

position/m	time/s
0.012	0.05
0.049	0.10
0.110	0.15
0.196	0.20
0.306	0.25
0.441	0.30

▲ **Table 2.3** The position and time for a free falling steel ball, using the stroboscope method with a stroboscope frequency of 20 Hz

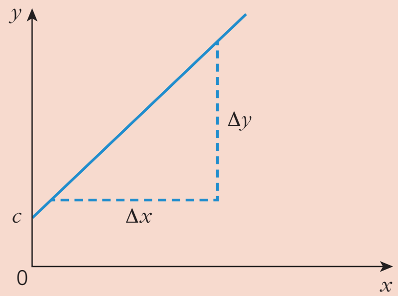
MATHS NOTE

Straight-line graphs

The representation of data in a graphical form is a very important means by which relationships between variables can be determined.

The plotting of data points provides an averaging which may well be superior to an arithmetical mean. Where an arithmetical mean is calculated, each set of data has an equal weighting. When using a best-fit line on a graph, the average is weighted towards those data points close to the line. A wayward point (anomalous point) can be detected and allowance made – perhaps taking a new set of measurements.

An important type of graph which is used frequently in Cambridge International AS & A Level Physics is the straight-line graph, as illustrated in Figure 2.12.



▲ **Figure 2.12**

The equation representing this graph is

$$y = mx + c$$

where m and c are constants.

The constant m is the gradient of the graph, $m = \Delta y/\Delta x$.

The constant c is the intercept on the y-axis.

If a variable y is thought to vary linearly with x, then plotting this graph will enable the following:

- the straight line with an intercept of c verifies a linear relationship between y and x
- determination of the values of the gradient m and the intercept c enables the exact form of the relationship to be established.

If the intercept is zero, the straight line passes through the origin. The relationship is $y = mx$. This special case with $c = 0$ means that y is proportional to x.

Relationships may also be based on powers. For example,

$$R = a + bS^n$$

where a, b and n are constants. A graph of R against S^n gives a straight-line graph with gradient b and an intercept on the R axis of a.

WORKED EXAMPLE 2D

Use Table 2.3 to calculate the average velocity:

a between the first two positions in the table and

b the last two positions in the table.

Hence show the average acceleration of the ball is $9.8\,\text{m}\,\text{s}^{-2}$.

Answers

a average velocity = (0.49 – 0.012)/0.05 = $\mathbf{0.74\,m\,s^{-1}}$

b average velocity = (0.441 – 0.306)/0.05 = $\mathbf{2.7\,m\,s^{-1}}$

The average acceleration = (2.7 – 0.74)/0.2 = $\mathbf{9.8\,m\,s^{-2}}$

Question

12 Use the data given in Table 2.3 to plot a graph of displacement against time squared. Determine the acceleration of free fall from the gradient of your graph.

Acceleration of free fall

Until the sixteenth century, the idea of the acceleration of a falling object was not fully appreciated. It was commonly thought that heavier bodies fell faster than light ones. This idea was a consequence of observing the effect of air resistance on light objects with a large surface area, such as feathers. However, Galileo Galilei (1564–1642) suggested that, in the absence of resistance, all bodies would fall with the same constant acceleration. He showed mathematically that, for an object falling from rest, the displacement travelled is proportional to the square of the time. Galileo tested the relation experimentally by timing the fall of objects from various levels of the Leaning Tower of Pisa (Figure 2.13).

This is the relation we have derived as $s = ut + \frac{1}{2}at^2$. For an object starting from rest, $u = 0$ and $s = \frac{1}{2}at^2$. That is, the displacement is proportional to time squared.

▲ **Figure 2.13** Leaning Tower of Pisa

▲ **Figure 2.14** Galileo in his study

WORKED EXAMPLE 2E

1 A car increases its speed from $25\,\text{m}\,\text{s}^{-1}$ to $31\,\text{m}\,\text{s}^{-1}$ with a uniform acceleration of $1.8\,\text{m}\,\text{s}^{-2}$. How far does it travel while accelerating?

2 The average acceleration of a sprinter from the time of leaving the blocks to reaching her maximum speed of $9.0\,\text{m}\,\text{s}^{-1}$ is $6.0\,\text{m}\,\text{s}^{-2}$. For how long does she accelerate? What distance does she cover in this time?

3 A cricketer throws a ball vertically upward into the air with an initial velocity of $18.0\,\text{m}\,\text{s}^{-1}$. How high does the ball go? How long is it before it returns to the cricketer's hands?

Answers

1 In this problem we want to know the distance s. We know the initial speed $u = 25\,\text{m}\,\text{s}^{-1}$, the final speed $v = 31\,\text{m}\,\text{s}^{-1}$, and the acceleration $a = 1.8\,\text{m}\,\text{s}^{-2}$. We need an equation linking s with u, v and a. This is

$$v^2 = u^2 + 2as$$

Substituting the values gives $31^2 = 25^2 + 2 \times 1.8s$. Re-arranging, $s = (31^2 - 25^2)/(2 \times 1.8) =$ **93 m**.

2 In the first part of this problem, we want to know the time t. We know the initial speed $u = 0$, the final speed $v = 9.0\,\text{m}\,\text{s}^{-1}$, and the acceleration $a = 6.0\,\text{m}\,\text{s}^{-2}$. We need an equation linking t with u, v and a. This is

$$v = u + at$$

Substituting the values, we have $9.0 = 0 + 6.0t$. Re-arranging, $t = 9.0/6.0 =$ **1.5 s**.

For the second part of the problem, we want to know the distance s. We know the initial speed $u = 0$, the final speed $v = 9.0\,\text{m}\,\text{s}^{-1}$, and the acceleration $a = 6.0\,\text{m}\,\text{s}^{-2}$; we have also just found the time $t = 1.5\,\text{s}$. There is a choice of equations linking s with u, v, a and t. We can use

$$s = ut + \frac{1}{2}at^2$$

Substituting the values, $s = 0 + \frac{1}{2} \times 6.0 \times (1.5)^2 =$ **6.8 m**.

Another relevant equation is $\bar{v} = \Delta x/\Delta t$. Here the average velocity $\bar{v}$ is given by $\bar{v} = (u + v)/2 = 4.5\,\text{m}\,\text{s}^{-1}$.

$\Delta x/\Delta t$ is the same as s/t, so $4.5 = s/1.5$, and $s = 4.5 \times 1.5 =$ **6.8 m** as before.

3 In the first part of the problem, we want to know the distance s. We know the initial velocity $u = 18.0\,\text{m s}^{-1}$ upwards and the acceleration $a = g = 9.81\,\text{m s}^{-2}$ downwards. At the highest point the ball is momentarily at rest, so the final velocity $v = 0$. The equation linking s with u, v and a is

$$v^2 = u^2 + 2as$$

Substituting the values, $0 = (18.0)^2 + 2(-9.81)s$. Thus $s = -(18.0)^2/2(-9.81) =$ **16.5 m**. Note that here the ball has an upward velocity but a downward acceleration, and that at the highest point the velocity is zero but the acceleration is not zero.

In the second part we want to know the time t for the ball's up-and-down flight. We know u and a, and also the overall displacement $s = 0$, as the ball returns to the same point at which it was thrown. The equation to use is

$$s = ut + \frac{1}{2}at^2$$

Substituting the values, $0 = 18.0t + (\frac{1}{2} - 9.81)t^2$. Doing some algebra, $t(36.0 - 9.81t) = 0$. There are two solutions, $t = 0$ and $t = 36.0/9.81 = 3.7\,\text{s}$. The $t = 0$ value corresponds to the time when the displacement was zero when the ball was on the point of leaving the cricketer's hands. The answer required here is **3.7 s**.

Questions

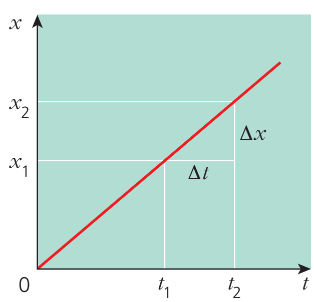
13 An airliner must reach a speed of $110\,\text{m s}^{-1}$ to take off. If the available length of the runway is 2.4 km and the aircraft accelerates uniformly from rest at one end, what minimum acceleration must be available if it is to take off?

14 A speeding motorist passes a traffic police officer on a stationary motorcycle. The police officer immediately gives chase: his uniform acceleration is $4.0\,\text{m s}^{-2}$, and by the time he draws level with the motorist he is travelling at $30\,\text{m s}^{-1}$. How long does it take for the police officer to catch the car? If the car continues to travel at a steady speed during the chase, what is that speed?

15 A cricket ball is thrown vertically upwards with a speed of $15.0\,\text{m s}^{-1}$. What is its velocity when it first passes through a point 8.0 m above the cricketer's hands?

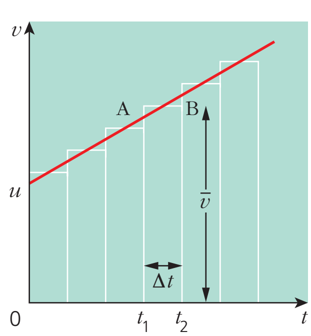
Graphs of the kinematic equations

It is often useful to represent the motion of a particle graphically, instead of by means of a series of equations. In this section we bring together the graphs which correspond to the equations we have already derived. We shall see that there are some important links between the graphs.



▲ Figure 2.15

First, think about a particle moving in a straight line with constant velocity. Constant velocity means that the particle covers equal distances in equal intervals of time. A graph of displacement x against time t is thus a straight line, as in Figure 2.15. Here the particle has started at $x = 0$ and at time $t = 0$. The slope of the graph is equal to the magnitude of the velocity, since, from the definition of average velocity, $\bar{v} = (x_2 - x_1)/(t_2 - t_1) = \Delta x/\Delta t$. Because this graph is a straight line, the average velocity and the instantaneous velocity are the same. The equation describing the graph is $x = vt$.



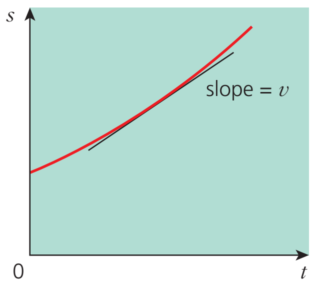
▲ Figure 2.16

Now think about a particle moving in a straight line with constant acceleration. The particle's velocity will change by equal amounts in equal intervals of time. A graph of the magnitude v of the velocity against time t will be a straight line, as in Figure 2.16.

Here the particle has started with velocity u at time $t = 0$. The slope of the graph is equal to the magnitude of the acceleration. The graph is a straight line showing that the acceleration is a constant. The equation describing the graph is $v = u + at$.

An important feature of the velocity–time graph is that we can deduce the displacement of the particle by calculating the area between the graph and the t-axis, between appropriate limits of time. Suppose we want to obtain the displacement of the particle between times t_1 and t_2 in Figure 2.16. Between these times the average $\bar{v}$ velocity is represented by the width of the horizontal line AB. The area between the graph and the t-axis is equal to the area of the rectangle whose top edge is AB. This area is $\bar{v}\Delta t$. But, by the definition of average velocity ($\bar{v} = \Delta x/\Delta t$), $\bar{v}\Delta t$ is equal to the displacement Δx during the time interval Δt.

We can deduce the graph of displacement s against time t from the velocity–time graph by calculating the area between the graph and the t-axis for a succession of values of t. As shown in Figure 2.16, we can split the area up into a number of rectangles. The displacement at a certain time is then just the sum of the areas of the rectangles up to that time. Since the graph is linear, the area under the graph is the area of a trapezium, which can also be found as the sum of the area of a rectangle and a triangle.



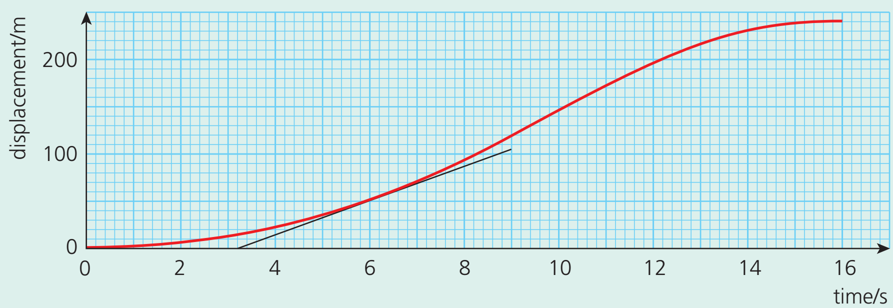
▲ **Figure 2.17**

Figure 2.17 shows the result of plotting the displacement s determined in this way against time t. It is a curve with a slope which increases the higher the value of t, indicating that the particle is accelerating. The slope at a particular time gives the magnitude of the instantaneous velocity. The equation describing Figure 2.17 is

$$s = ut + \frac{1}{2}at^2$$

WORKED EXAMPLE 2F

The displacement–time graph for a car on a straight test track is shown in Figure 2.18. Use this graph to draw velocity–time and acceleration–time graphs for the test run.



▲ **Figure 2.18 Displacement–time graph**

Answer

We have already met this graph when we discussed the concepts of velocity and acceleration (Figure 2.5). In Figure 2.18 it has been re-drawn to scale, and figures have been put on the displacement and time axes. We find the magnitude of the velocity by measuring the gradient of the displacement–time graph. As an example, a tangent to the graph has been drawn at $t = 6.0$ s. The slope of this tangent is $18\,\mathrm{m\,s^{-1}}$. If the process is repeated at different times, the following velocities are determined.

t/s	2	4	6	8	10	12	14	16
v/m s^{-1}	6	12	18	24	30	20	10	0

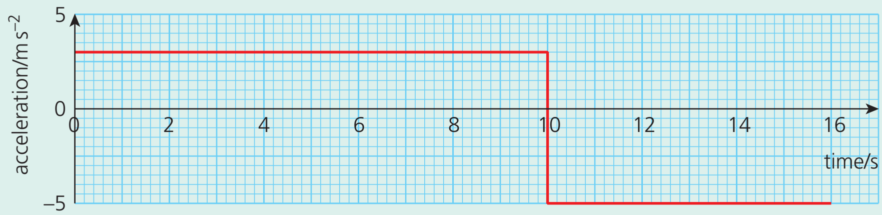
▲ **Table 2.4**

These values are plotted on the velocity–time graph of Figure 2.19. Check some of the values by drawing tangents yourself. *Hint*: When drawing tangents, use a mirror or a transparent ruler.

Figure 2.19 shows two straight-line portions. Initially, from $t = 0$ to $t = 10$ s, the car is accelerating uniformly, and from $t = 10$ s to $t = 16$ s it is decelerating. The acceleration is given by $a = \Delta v/\Delta t = 30/10 = 3\,\mathrm{m\,s^{-2}}$ up to $t = 10$ s. Beyond $t = 10$ s the acceleration is $-30/6 = -5\,\mathrm{m\,s^{-2}}$. (The minus sign shows that the car is decelerating.)

▲ **Figure 2.19** Velocity–time graph

The acceleration–time graph is plotted in Figure 2.20.



▲ **Figure 2.20** Acceleration–time graph

Finally, we can confirm that the area under a velocity–time graph gives the displacement.

The area under the line in Figure 2.19 is the sum of the area of two triangles:

$$\left(\frac{1}{2} \times 10 \times 30\right) + \left(\frac{1}{2} \times 6 \times 30\right) = 240\,\text{m}$$

the value of s at t = 16 s on Figure 2.18.

Displacement for non-uniform acceleration

We have shown that for constant acceleration, we can deduce the displacement of the particle by calculating the area between the velocity–time graph and the *t*-axis. For the simple case for constant acceleration, the velocity–time graph is linear and the area under the graph is the area of a triangle (or, for where the graph line doesn't start from the origin), the area of a trapezium. The unit for the displacement calculated by the area under the line is given by the product of the units on the axes for the velocity and time. For example, if the velocity is in $\text{m}\,\text{s}^{-1}$ and the time in seconds the displacement is given in m ($\text{m}\,\text{s}^{-1} \times \text{s}$). Similarly if velocity is in $\text{km}\,\text{h}^{-1}$ and the time in hours the displacement is in km ($\text{km}\,\text{h}^{-1} \times \text{h}$).

The analysis we did for Figure 2.16 also applies to cases where the acceleration is variable (non-uniform) and the velocity-time graph is curved. In this case the area under the curve can be estimated by counting the squares and working out from the scales on the axes what displacement the area of each square represents.

The area under the velocity–time graph represents the displacement.

Question

16 In a test of a sports car on a straight track, the following readings of velocity v were obtained at the times t stated.

t/s	0	5	10	15	20	25	30	35
v/m s^{-1}	0	15	23	28	32	35	37	38

▲ **Table 2.5**

a On graph paper, draw a velocity–time graph and use it to determine the acceleration of the car at time t = 5 s.

b Find also the total distance travelled between t = 0 and t = 30 s.

Note: These figures refer to a case of non-uniform acceleration, which is more realistic than the previous example. However, the same rules apply: the acceleration is given by the slope of the velocity–time graph at the relevant time, and the distance travelled can be found from the area under the graph.

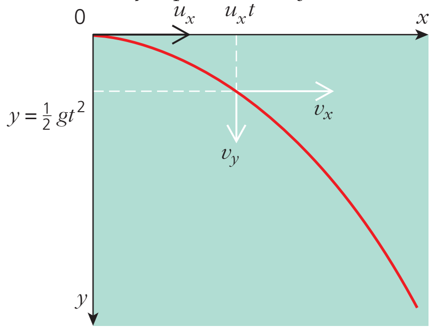
Two-dimensional motion under a constant force

So far we have been dealing with motion along a straight line; that is, one-dimensional motion. We will now think about the motion of particles moving in paths in two dimensions. We shall need to make use of ideas we have already learnt regarding vectors in Topic 1. The particular example we shall take is where a particle moves in a plane under the action of a constant force. An example is the motion of a ball thrown at an angle to the vertical (Figure 2.21), or an electron moving at an angle to an electric field. In the case of the ball, the constant force acting on it is its weight. For the electron, the constant force is the electric force provided by the electric field (Topic 18).

▲ **Figure 2.21** Cricketer bowling the ball

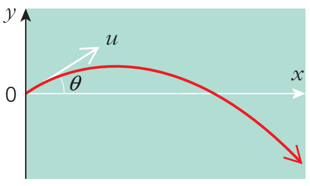
This topic is often called **projectile motion**. Galileo first gave an accurate analysis of this motion. He did so by splitting the motion up into its vertical and horizontal components, and considering these separately. The key is that the two components can be considered independently.

As an example, think about a particle sent off in a horizontal direction and subject to a vertical gravitational force (its weight). As before, air resistance will be neglected. We will analyse the motion in terms of the horizontal and vertical components of velocity. The particle is projected at time t = 0 at the origin of a system of x, y co-ordinates (Figure 2.22) with velocity u_x in the x-direction. Think first about the particle's vertical motion (in the y-direction). Throughout the motion, it has an acceleration of g (the acceleration of free fall) in the y-direction. The initial value of the vertical component of velocity is $u_y = 0$. The vertical component increases continuously under the uniform acceleration g. Using $v = u + at$, its value v_y at time t is given by $v_y = gt$. Also at time t, the vertical displacement y downwards is given by $y = \frac{1}{2}gt^2$. Now for the horizontal motion (in the x-direction): here the acceleration is zero, so the horizontal component of velocity remains constant at u_x. At time t the horizontal displacement x is given by $x = u_xt$. To find the velocity of the particle at any time t, the two components v_x and v_y must be added vectorially. The direction of the resultant vector is the direction of motion of the particle. The curve traced out by a particle subject to a constant force in one direction is a **parabola**.

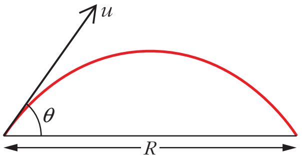


▲ **Figure 2.22**

▲ **Figure 2.23** Water jets from a garden sprinkler showing a parabola-shaped spray



▲ **Figure 2.24**



▲ **Figure 2.25**

If the particle had been sent off with velocity u at an angle θ to the horizontal, as in Figure 2.24, the only difference to the analysis of the motion is that the initial y-component of velocity is $u \sin\theta$. In the example illustrated in Figure 2.24, this is upwards. Because of the downwards acceleration g, the y-component of velocity decreases to zero, at which time the particle is at the crest of its path, and then increases in magnitude again but this time in the opposite direction. The path is again a parabola.

For the particular case of a particle projected with velocity u at an angle θ to the horizontal from a point on level ground (Figure 2.25), the range R is defined as the distance from the point of projection to the point at which the particle reaches the ground again. We can show that R is given by

$$R = \frac{(u^2 \sin 2\theta)}{g}$$

For details, see the Maths Note below.

MATHS NOTE

Suppose that the particle is projected from the origin ($x = 0$, $y = 0$). We can interpret the range R as being the horizontal distance x travelled at the time t when the value of y is again zero. The equation which links displacement, initial speed, acceleration and time is $s = ut + \frac{1}{2}at^2$. Adapting this for the vertical component of the motion, we have

$$0 = (u \sin\theta)t - \frac{1}{2}gt^2$$

The two solutions of this equation are $t = 0$ and $t = (2u \sin\theta)/g$. The $t = 0$ case is when the particle was projected; the second is when it returns to the ground at $y = 0$. We use this second value of t with the horizontal component of velocity $\cos\theta$ to find the distance x travelled (the range R). This is

$$x = R = (u \cos\theta)t = (2u^2 \sin\theta \cos\theta)/g$$

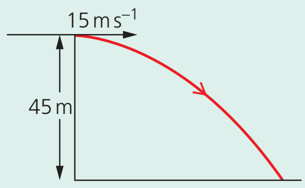
There is a trigonometric relationship $\sin 2\theta = 2 \sin\theta \cos\theta$, use of which puts the range expression in the required form

$$R = (u^2 \sin 2\theta)/g$$

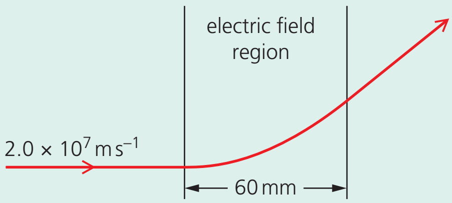
We can see that R will have its maximum value for a given speed of projection u when $\sin 2\theta = 1$, or $2\theta = 90°$, or $\theta = 45°$. The value of this maximum range is $R_{max} = u^2/g$.

WORKED EXAMPLE 2G

1 A stone is thrown from the top of a vertical cliff, 45 m high above level ground, with an initial velocity of $15\,\text{m s}^{-1}$ in a horizontal direction (Figure 2.26). How long does it take to reach the ground? How far from the base of the cliff is it when it reaches the ground?



▲ **Figure 2.26**



▲ **Figure 2.27**

2 An electron, travelling with a velocity of $2.0 \times 10^7\,\text{m s}^{-1}$ in a horizontal direction, enters a uniform electric field. This field gives the electron a constant acceleration of $5.0 \times 10^{15}\,\text{m s}^{-2}$ in a direction perpendicular to its original velocity (Figure 2.27). The field extends for a horizontal distance of 60 mm. What is the magnitude and direction of the velocity of the electron when it leaves the field?

Answers

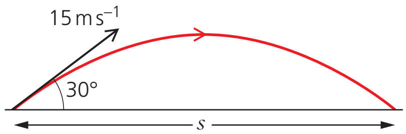
1 To find the time t for which the stone is in the air, work with the vertical component of the motion, for which we know that the initial component of velocity is zero, the displacement $y = 45$ m, and the acceleration a is $9.81\,\text{m s}^{-2}$. The equation linking these is $y = \frac{1}{2}gt^2$. Substituting the values, we have $45 = \frac{1}{2} \times 9.81t^2$. This gives $t = \sqrt{(2 \times 45/9.81)} =$ **3.0 s**.
For the second part of the question, we need to find the horizontal distance x travelled in the time t. Because the horizontal component of the motion is not accelerating, x is given simply by $x = u_x t$. Substituting the values, we have $x = 15 \times 3.0 =$ **45 m**.

2 The horizontal motion of the electron is not accelerated. The time t spent by the electron in the field is given by $t = x/u_x = 60 \times 10^{-3}/2.0 \times 10^7 = 3.0 \times 10^{-9}$ s. When the electron enters the field, its vertical component of velocity is zero; in time t, it has been accelerated to $v_y = at = 5.0 \times 10^{15} \times 3.0 \times 10^{-9} = 1.5 \times 10^7\,\text{m s}^{-1}$. When the electron leaves the field, it has a horizontal component of velocity $v_x = 2.0 \times 10^7\,\text{m s}^{-1}$, unchanged from the initial value u_x. The vertical component is $v_y = 1.5 \times 10^7\,\text{m s}^{-1}$. The resultant velocity v is given by $v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{[(2.0 \times 10^7)^2 + (1.5 \times 10^7)^2]} = 2.5 \times 10^7\,\text{m s}^{-1}$. The direction of this resultant velocity makes an angle θ to the horizontal, where θ is given by $\tan\theta = v_y/v_x = 1.5 \times 10^7/2.0 \times 10^7$. The angle θ is **37°**.

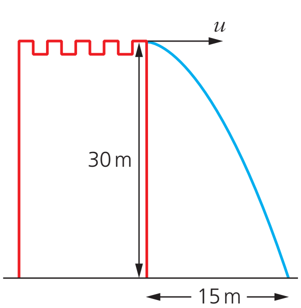
Questions

17 A ball is thrown horizontally from the top of a tower 30 m high and lands 15 m from its base (Figure 2.28). What is the ball's initial speed?

18 A football is kicked on level ground at a velocity of $15\,\text{m s}^{-1}$ at an angle of 30° to the horizontal (Figure 2.29). How far away is the first bounce?



▲ **Figure 2.29**



▲ **Figure 2.28**

19 A car accelerates from 5.0 m s^{-1} to 20 m s^{-1} in 6.0 s. Assuming uniform acceleration, how far does it travel in this time?

20 If a raindrop were to fall from a height of 1 km, with what velocity would it hit the ground if there were no air resistance?

21 Traffic police can estimate the speed of vehicles involved in accidents by the length of the marks made by skidding tyres on the road surface. It is known that the maximum deceleration that a car can attain when braking on a normal road surface is about 9 m s^{-2}. In one accident, the tyre-marks were found to be 125 m long. Estimate the speed of the vehicle before braking.

22 On a theme park ride, a cage is travelling upwards at constant speed. As it passes a platform alongside, a passenger drops coin A through the cage floor. At exactly the same time, a person standing on the platform drops coin B from the platform.

a Which coin, A or B (if either), reaches the ground first?

b Which (if either) has the greater speed on impact?

23 William Tell was faced with the agonising task of shooting an apple from his son Jemmy's head.

Assume that William is placed 25 m from Jemmy; his crossbow fires a bolt with an initial speed of 45 m s^{-1}. The crossbow and apple are on the same horizontal line. At what angle to the horizontal should William aim so that the bolt hits the apple?

24 The position of a sports car on a straight test track is monitored by taking a series of photographs at fixed time intervals. The following record of position *x* was obtained at the stated times *t*.

t/s	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
x/m	0	0.4	1.8	4.2	7.7	12.4	18.3	25.5	33.9	43.5	54.3

▲ **Table 2.6**

On graph paper, draw a graph of *x* against *t*. Use your graph to obtain values for the velocity *v* of the car at a number of values of *t*. Draw a second graph of *v* against *t*. From this graph, what can you deduce about the acceleration of the car?

SUMMARY

- Distance is the length along the actual path travelled and is a scalar quantity. Displacement is the distance travelled in a straight line in a specified direction and is a vector quantity.
- Speed is a scalar quantity and is described by magnitude only. Velocity is a vector quantity and requires magnitude and direction.
- Average speed is defined by: (actual distance moved)/(time taken).
- Average velocity is defined by: (displacement)/(time taken) or $\Delta x/\Delta t$.
- The instantaneous velocity is the average velocity measured over an infinitesimally short time interval.
- Average acceleration is defined by: (change in velocity)/(time taken) or $\Delta v/\Delta t$.
- Acceleration is a vector. Instantaneous acceleration is the average acceleration measured over an infinitesimally short time interval.
- The gradient of a displacement-time graph gives the velocity.
- The gradient of a velocity–time graph gives the acceleration.
- The area between a velocity–time graph and the time axis gives the displacement.
- The equations for an object moving in a straight line with uniform acceleration are:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \frac{(u + v)t}{2}$$

- Objects falling freely near the surface of the Earth in the absence of air resistance, experience the same acceleration, the acceleration of free fall g, which has the value g = 9.81 m s^{-2}.
- The acceleration of free fall for a falling object can be determined by measuring the position of the object at different times after being released.
- The motion of projectiles is analysed in terms of two independent motions at right angles. The horizontal component of the motion is at a constant velocity, while the vertical motion is subject to a constant acceleration g.

END OF TOPIC QUESTIONS

1 In a driving manual, it is suggested that, when driving at $13\,\text{m s}^{-1}$ (about 45 km per hour), a driver should always keep a minimum of two car-lengths between the driver's car and the one in front.

   a Suggest a scientific justification for this safety tip, making reasonable assumptions about the magnitudes of any quantities you need.

   b How would you expect the length of this 'exclusion zone' to depend on speed for speeds higher than $13\,\text{m s}^{-1}$?

2 A student, standing on the platform at a railway station, notices that the first two carriages of an arriving train pass her in 2.0 s, and the next two in 2.4 s. The train is decelerating uniformly. Each carriage is 20 m long. When the train stops, the student is opposite the last carriage. How many carriages are there in the train?

3 A ball is to be kicked so that, at the highest point of its path, it just clears a horizontal cross-bar on a pair of goal-posts. The ground is level and the cross-bar is 2.5 m high. The ball is kicked from ground level with an initial velocity of $8.0\,\text{m s}^{-1}$.

   a Calculate the angle of projection of the ball and the distance of the point where the ball was kicked from the goal-line.

   b Also calculate the horizontal velocity of the ball as it passes over the cross-bar.

   c For how long is the ball in the air before it reaches the ground on the far side of the cross-bar?

4 An athlete competing in the long jump leaves the ground at an angle of 28° and makes a jump of 7.40 m.

   a Calculate the speed at which the athlete took off.

   b If the athlete had been able to increase this speed by 5%, what percentage difference would this have made to the length of the jump?

5 A hunter, armed with a bow and arrow, takes direct aim at a monkey hanging from the branch of a tree. At the instant that the hunter releases the arrow, the monkey takes avoiding action by releasing its hold on the branch. By setting up the relevant equations for the motion of the monkey and the motion of the arrow, show that the monkey was mistaken in its strategy.

6 A car travels due north at a constant speed of $30\,\text{m s}^{-1}$ for 2.0 minutes. The car then travels due east at a constant speed of $20\,\text{m s}^{-1}$ for four minutes. What is the total distance travelled by the car?

   A 140 m   B 8400 m   C 9000 m   D 9100 m

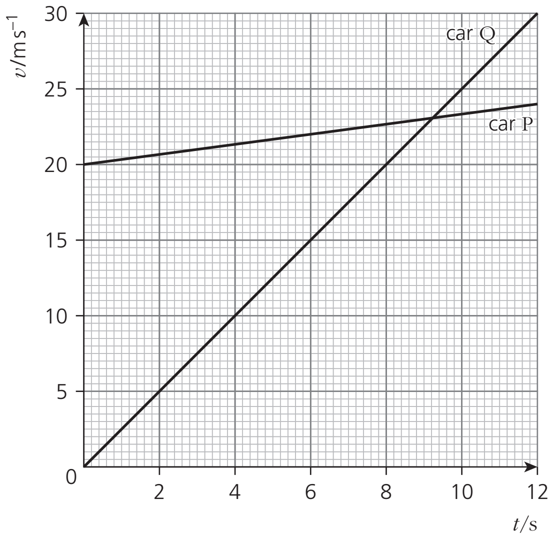
7 A ball is thrown vertically upwards with an initial speed of $20\,\text{m s}^{-1}$ from a platform 15 m above the ground. The ball travels to a maximum height and then returns to ground level. How long does the ball take to travel from the platform to its maximum height and then to the ground?

   A 2.7 s   B 4.1 s   C 4.7 s   D 5.3 s

8 A ball is thrown at an angle to the horizontal of 30° with an initial speed of $45\,\text{m s}^{-1}$. What time does the ball take to return to the same horizontal level as its release point?

   A 2.3 s   B 4.0 s   C 4.6 s   D 8.0 s

9 The variation with time t of the velocity v of two cars P and Q is shown in Fig. 2.30.



▲ **Figure 2.30**

The cars travel in the same direction along a straight road. Car P passes car Q at time $t = 0$.

a The speed limit for cars on the road is 100 km h^{-1}. State and explain whether car Q exceeds the speed limit. **[1]**

b Calculate the acceleration of car P. **[2]**

c Determine the distance between the two cars at time $t = 12$ s. **[3]**

d From time $t = 12$ s, the velocity of each car remains constant at its value at $t = 12$ s. Determine the time t at which car Q passes car P. **[2]**

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10 a Define *speed* and *velocity* and use these definitions to explain why one of these quantities is a scalar and the other is a vector. **[2]**

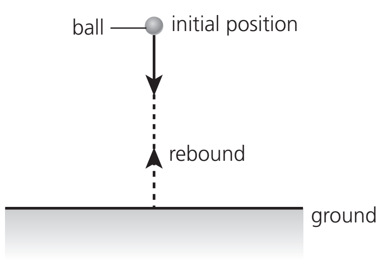
b A ball is released from rest and falls vertically. The ball hits the ground and rebounds vertically, as shown in Fig. 2.31.

The variation with time t of the velocity v of the ball is shown in Fig. 2.32.

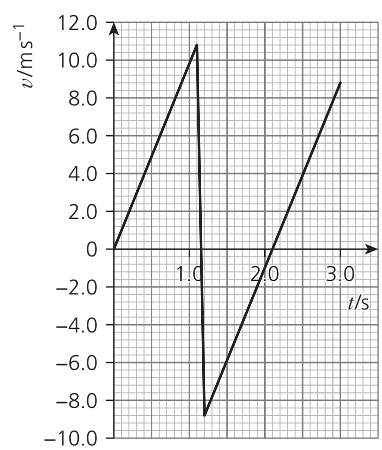
Air resistance is negligible.

i Without calculation, use Fig. 2.32 to describe the variation with time t of the velocity of the ball from $t = 0$ to $t = 2.1$ s. **[3]**

ii Calculate the acceleration of the ball after it rebounds from the ground. Show your working. **[3]**



▲ **Figure 2.31**



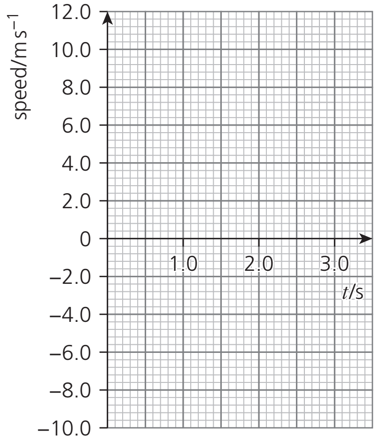
▲ **Figure 2.32**

iii Calculate, for the ball, from $t = 0$ to $t = 2.1$ s:

1 the distance moved, [3]

2 the displacement from the initial position. [2]

iv On a copy of Fig. 2.33, sketch the variation with t of the speed of the ball. [2]



▲ **Figure 2.33**

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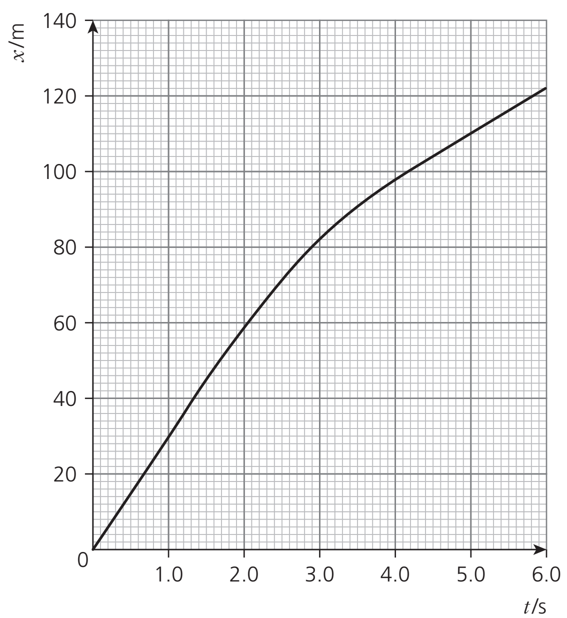
11 a Define:

i *velocity*, [1]

ii *acceleration*. [1]

b A car of mass 1500 kg travels along a straight horizontal road.

The variation with time t of the displacement x of the car is shown in Fig. 2.34.



▲ **Figure 2.34**

i Use Fig. 2.34 to describe qualitatively the velocity of the car during the first six seconds of the motion shown.

Give reasons for your answers. [3]

ii Calculate the average velocity during the time interval $t = 0$ to $t = 1.5$ s. [1]

iii Show that the average acceleration between $t = 1.5$ s and $t = 4.0$ s is $-7.2\,\text{m}\,\text{s}^{-2}$. [2]

iv Calculate the average force acting on the car between $t = 1.5$ s and $t = 4.0$ s. [2]

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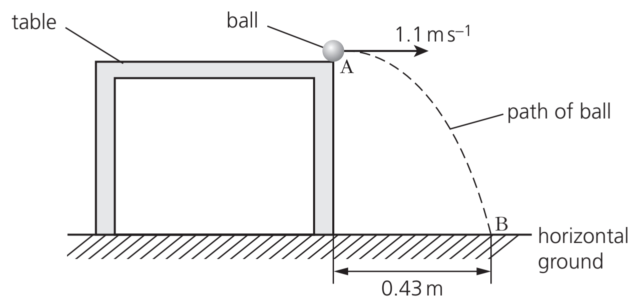
12 a Copy and complete Fig. 2.35 to indicate whether each of the quantities is a vector or a scalar. **[2]**

quantity	vector or scalar
acceleration	
speed	
power	

▲ **Figure 2.35**

b A ball is projected with a horizontal velocity of 1.1 m s^{-1} from point A at the edge of a table, as shown in Fig. 2.36.



▲ **Figure 2.36**

The ball lands on horizontal ground at point B which is a distance of 0.43 m from the base of the table.

Air resistance is negligible.

i Calculate the time taken for the ball to fall from A to B. **[1]**

ii Use your answer in **b i** to determine the height of the table. **[2]**

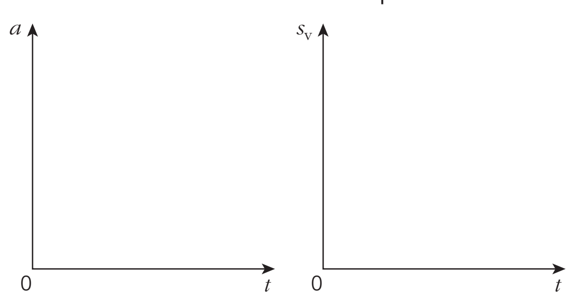
iii The ball leaves the table at time $t = 0$.

For the motion of the ball between A and B, sketch graphs on copies of Fig. 2.37 to show the variation with time t of

1 the acceleration a of the ball,

2 the vertical component s_v of the displacement of the ball from A.

Numerical values are not required. **[2]**



▲ **Figure 2.37**

c A ball of greater mass is projected from the table with the same velocity as the ball in **b**. Air resistance is still negligible.

State and explain the effect, if any, of the increased mass on the time taken for the ball to fall to the ground. **[1]**

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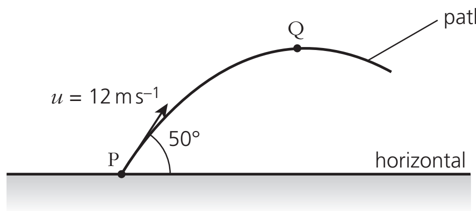
13 A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal, as illustrated in Fig. 2.38.



▲ **Figure 2.38**

The ball reaches maximum height at Q. Air resistance is negligible.

a Calculate:

i the horizontal component of u, [1]

ii the vertical component of u. [1]

b Show that the maximum height reached by the ball is 4.3 m. [2]

c Determine the magnitude of the displacement PQ. [4]

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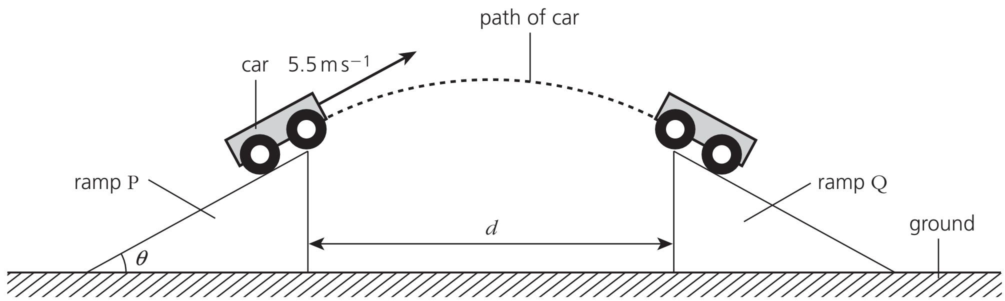
14 a Define:

i *displacement*, [1]

ii *acceleration*. [1]

b A remote-controlled toy car moves up a ramp and travels across a gap to land on another ramp, as illustrated in Fig. 2.39.



▲ **Figure 2.39**

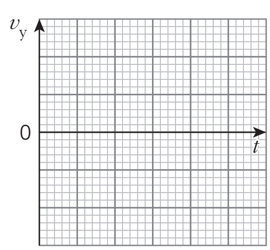
The car leaves ramp P with a velocity of 5.5 m s^{-1} at an angle θ to the horizontal. The horizontal component of the car's velocity as it leaves the ramp is 4.6 m s^{-1}. The car lands at the top of ramp Q. The tops of both ramps are at the same height and are distance d apart. Air resistance is negligible.

i Show that the car leaves ramp P with a vertical component of velocity of 3.0 m s^{-1}. [1]

ii Determine the time taken for the car to travel between the ramps. [2]

iii Calculate the horizontal distance d between the tops of the ramps. [1]

c Ramp Q is removed. The car again leaves ramp P as in **b** and now lands directly on the ground. The car leaves ramp P at time t = 0 and lands on the ground at time t = T. On a copy of Fig. 2.40, sketch the variation with time t of the vertical component v_y of the car's velocity from t = 0 to t = T. Numerical values of v_y and t are not required. [2]



▲ **Figure 2.40**

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3 Dynamics

Learning outcomes

By the end of this topic, you will be able to:

3.1 Momentum and Newton's laws of motion

1. understand that mass is the property of an object that resists change in motion
2. recall the relationship $F = ma$, and solve problems using it, understanding that acceleration and force are always in the same direction
3. define and use linear momentum as the product of mass and velocity
4. define and use force as the rate of change of momentum
5. state and apply each of Newton's laws of motion
6. describe and use the concept of weight as the effect of a gravitational field on a mass and recall that the weight of an object is equal to the product of its mass and the acceleration of free fall

3.2 Non-uniform motion

1. show a qualitative understanding of frictional forces and viscous forces/drag including air resistance (no treatment of the coefficients of friction and viscosity is required, and a simple model of drag force increasing as speed increases is sufficient)
2. describe and explain qualitatively the motion of objects in a uniform gravitational field with air resistance
3. understand that objects moving against a resistive force may reach a terminal (constant) velocity

3.3 Linear momentum and its conservation

1. state the principle of conservation of momentum
2. apply the principle of conservation of momentum to solve simple problems including elastic and inelastic interactions between objects in both one and two dimensions
3. recall that, for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation
4. understand that, while momentum of a system is always conserved in interactions between objects, some change in kinetic energy may take place

Starting points

- ★ Motion of an object can be described in terms of displacement, velocity and acceleration.
- ★ A force is required to make an object accelerate.
- ★ Kinetic energy is the energy stored in an object due to its motion.
- ★ Energy cannot be created or destroyed. It can only be converted from one form to another.
- ★ A single vector may be resolved into two separate components at right angles to each other.

3.1 Momentum and Newton's laws of motion

Relationships involving force and mass

When you push a trolley in a supermarket or pull a case behind you at an airport, you are exerting a force. When you hammer in a nail, a force is being exerted. When you drop a book and it falls to the floor, the book is falling because of the force of gravity. When you lean against a wall or sit on a chair, you are exerting a force. Forces can change the shape or dimensions of objects. You can crush a drinks can by squeezing it and applying a force; you can stretch a rubber band by pulling it. In everyday life, we have a good understanding of what is meant by force and the situations in which forces are involved. In physics the idea of force is used to add detail to the descriptions of moving objects.

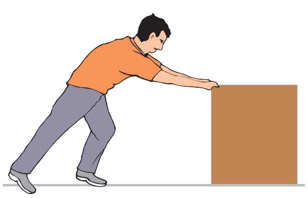
As with all physical quantities, a method of measuring force must be established. One way of doing this is to make use of the fact that forces can change the dimensions of objects in a reproducible way. It takes the same force to stretch a spring by the same change in length (provided the spring is not overstretched by applying a very large force). This principle is used in the spring balance. A scale shows how much the spring has been extended, and the scale can be calibrated in terms of force. Laboratory spring balances are often called newton balances, because the newton is the SI unit of force.

Forces are vector quantities: they have magnitude as well as direction. A number of forces acting on an object are often shown by means of a force diagram drawn to scale, in which the forces are represented by lines of length proportional to the magnitude of the force, and in the appropriate direction (see Topic 1.4). The combined effect of several forces acting on an object is known as the **resultant** force.

Force and motion

The Greek philosopher Aristotle believed that the natural state of an object was a state of rest, and that a force was necessary to make it move and to keep it moving. This argument requires that the greater the force, the greater the speed of the object.

Nearly two thousand years later, Galileo questioned this idea. He suggested that motion at a constant speed could be just as natural a state as the state of rest. He introduced an understanding of the effect of **friction** on motion.



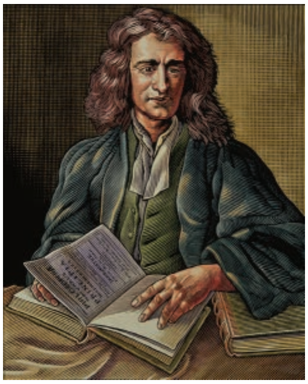
▲ **Figure 3.1**

Imagine a heavy box being pushed along a rough floor at constant speed (Figure 3.1). This may take a considerable force. The force required can be reduced if the floor is made smooth and polished, and reduced even more if a lubricant, for example grease, is applied between the box and the floor. We can imagine a situation where, when friction is reduced to a vanishingly small value, the force required to push the box at constant speed is also vanishingly small.

Galileo realised that the force of friction was a force that opposed the pushing force. When the box is moving at constant speed, the pushing force is exactly equal to the frictional force, but in the opposite direction, so that there is a net force of zero acting on the box. In the situation of vanishingly small friction, the box will continue to move with constant speed, because there is no force to slow it down.

Newton's laws of motion

Isaac Newton (1642–1727) used Galileo's ideas to produce a theory of motion, expressed in his three laws of motion. The **first law of motion** or **Newton's first law** re-states Galileo's deduction about the natural state of an object.



▲ **Figure 3.2** Isaac Newton

> Every object continues in its state of rest, or with uniform velocity, unless acted on by a resultant force.

This law tells us one of the effects of a force: it disturbs the state of rest or velocity of an object. The property of an object to stay in a state of rest or uniform velocity is called **inertia**.

Newton's second law tells us what happens if a force is exerted on an object. It causes the velocity to change. A force exerted on an object at rest makes it move – it gives it a velocity. A force exerted on a moving object may make its speed increase or decrease, or change its direction of motion. A change in speed or velocity is acceleration.

Newton's second law relates the magnitude of this acceleration to the force applied. It also introduces the idea of the mass of an object. Mass is a measure of the inertia of an object to change in velocity. The bigger the mass, the more difficult it is to change its state of rest or velocity. A simplified form of Newton's second law is

For an object of constant mass, its acceleration is directly proportional to the resultant force applied to it.

The direction of the acceleration is in the direction of the resultant force. In a word equation the relation between force and acceleration is

force = mass × acceleration

and in symbols

$$F = ma$$

where F is the **resultant** force, m is the mass and a is the acceleration. Here we have made the constant of proportionality equal to unity (that is, we use an equals sign rather than a proportionality sign) by choosing quantities with units which will give us this simple relation. In SI units, the force F is in newtons, the mass m in kilograms and the acceleration a in metre (second)$^{-2}$.

▲ **Figure 3.3**

One newton is defined as the force which will give a 1 kg mass an acceleration of $1\,m\,s^{-2}$ in the direction of the force.

When you push a supermarket trolley, the trolley experiences a force (Figure 3.3). The trolley applies an equal and opposite force on another object – you. Newton understood that the object on which the force is exerted applies another force back on the object which is applying the force. When object A applies a force on object B then object B applies an equal and opposite force on object A. **Newton's third law** relates these two forces.

Whenever one object exerts a force on another, the second object exerts an equal and opposite force on the first.

▲ **Figure 3.4** Space rocket launch

This law highlights the very important point that the two forces act on *different objects*. To take the example of the supermarket trolley, the force exerted by you on the trolley is equal and opposite to the force exerted by the trolley on you.

Newton's third law has applications in every branch of everyday life. We walk because of this law. When you take a step forward, your foot presses against the ground. The ground then exerts an equal and opposite force on you. This is the force, on you, which propels you in your path. Space rockets work because of the law (Figure 3.4).

To expel the exhaust gases from the rocket, the rocket exerts a force on the gases. The gases exert an equal and opposite force on the rocket, accelerating it forward.

WORKED EXAMPLE 3A

1 An object of mass 1.5 kg is to be accelerated at $2.2\,\mathrm{m\,s^{-2}}$. What force is required?
2 A car of mass 1.5 tonnes (1.5×10^3 kg), travelling at $80\,\mathrm{km\,h^{-1}}$, is to be stopped in 11 s. What force is required?

Answers

1 From Newton's second law, $F = ma = 1.5 \times 2.2 =$ **3.3 N**.
2 The acceleration of the car can be obtained from $v = u + at$ (see Topic 2).
The initial speed u is $80\,\mathrm{km\,h^{-1}}$, or $22\,\mathrm{m\,s^{-1}}$. The final speed v is 0.
Then $a = -22/11 = -2.0\,\mathrm{m\,s^{-2}}$.
This is negative because the car is decelerating.
By Newton's second law, $F = ma = 1.5 \times 10^3 \times 2.0 = \mathbf{3.0 \times 10^3\,N}$.

Questions

1 A force of 5.0 N is applied to an object of mass 3.0 kg. What is the acceleration of the object?
2 A stone of mass 50 g is accelerated from a catapult to a speed of $8.0\,\mathrm{m\,s^{-1}}$ from rest over a distance of 30 cm. What average force is applied by the rubber of the catapult?

Momentum

We shall now introduce a quantity called **momentum**, and see how Newton's laws are related to it.

The momentum of a particle is defined as the product of its mass and its velocity.

In words

momentum = mass × velocity

and in symbols

$$p = mv$$

The unit of momentum is the unit of mass times the unit of velocity; that is, $\mathrm{kg\,m\,s^{-1}}$. An alternative unit is the newton second (N s). Momentum, like velocity, is a vector quantity. Its complete name is **linear momentum**, to distinguish it from angular momentum, which does not concern us here.

As stated above, **Newton's first law** states that every object continues in a state of rest, or with uniform velocity, unless acted on by a resultant force. We can express this law in terms of momentum. If an object maintains its uniform velocity, its momentum is unchanged. If an object remains at rest, again its momentum (zero) does not change. Thus, an alternative statement of the first law is that the momentum of a particle remains constant unless an external resultant force acts on the particle. As an equation

p = constant (provided the external resultant force is zero)

This is a special case, for a single particle, of a very important conservation law: the principle of conservation of momentum. This word 'conservation' here means that the quantity remains constant.

We already have Newton's second law in a form which relates the force acting on an object to the product of the mass and the acceleration of the object. Newton's second law can also be expressed in terms of momentum. Remember that the acceleration of an object is the rate of change of its velocity. The product of mass and acceleration then is just the mass times the rate of change of velocity. For an object of constant mass, this is just the same as the rate of change of (mass × velocity). But (mass × velocity) is momentum, so the product of mass and acceleration is identical to the rate of change of momentum. Thus, Newton's second law is stated as

The resultant force acting on an object is proportional to the rate of change of its momentum.

The constant of proportionality is made equal to unity as described earlier, by the definition of the newton. Hence the second law used in problem solving is

The resultant force acting on an object is equal to the rate of change of momentum.

Expressed in terms of symbols

$$F = \Delta p/\Delta t$$

for constant mass m

$$F = \Delta(mv)/\Delta t = m(\Delta v/\Delta t) = ma$$

Note that F represents the **resultant force** acting on the object.

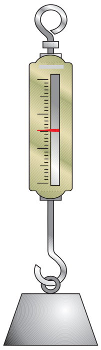
Continuing with the idea of force being equal to rate of change of momentum, the third law becomes: the rate of change of momentum due to the force on one object is equal and opposite to the rate of change of momentum due to the force on the other object. The two forces act on each object for the same time (Δt). Hence $F\Delta t$ is equal for each object. Therefore, when two objects exert forces on each other, their changes of momentum are equal and opposite.

Weight

We saw in Topic 2 that all objects released near the surface of the Earth fall with the same, constant acceleration (the acceleration of free fall) if air resistance can be neglected. The force causing this acceleration is the gravitational attraction of the Earth on the object, or the force of gravity. The force of gravity which acts on an object is called the **weight** of the object. We can apply Newton's second law to the weight. For an object of mass m falling with the acceleration of free fall g, the weight W is given by

$$W = mg$$

The SI unit of force is the newton (N). This is also the unit of weight. The weight of an object is obtained by multiplying its mass in kilograms by the acceleration of free fall, 9.81 $\mathrm{m\,s^{-2}}$. Thus a mass of one kilogram has a weight of 9.81 N. Because weight is a force and force is a vector, we ought to be aware of the direction of the weight of an object. It is towards the centre of the Earth. Because weight always has this direction, we do not need to specify direction every time we give the magnitude of the weight of objects.

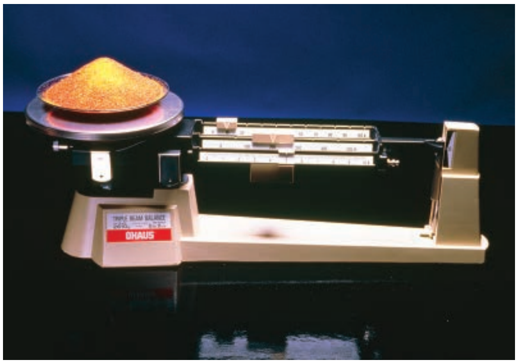


▲ **Figure 3.5** Newton balance

How do we measure mass and weight? If you hang an object from a newton balance, you are measuring its weight (Figure 3.5). The unknown weight of the object is balanced by a force provided by the spring in the balance. From a previous calibration, this force is related to the extension of the spring. There is the possibility of confusion here. Laboratory newton balances may, indeed, be calibrated in newtons. But all commercial spring balances – for example, the balances at fruit and vegetable counters in supermarkets – are calibrated in kilograms. Such balances are really measuring the weight of the fruit and vegetables, but the scale reading is in mass units, because there is no distinction between mass and weight in everyday life. The average shopper thinks of 5 kg of mangoes as having a weight of 5 kg. In fact, the mass of 5 kg has a weight of 49 N.

▲ **Figure 3.6** Top-pan balance

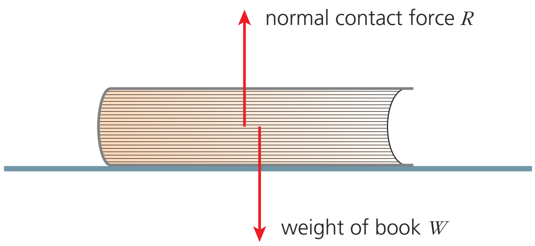
The word 'balance' in the spring balance and in the laboratory top-pan balance (see Figure 3.6) relates to the balance of forces. In each case, the unknown force (the weight) is equalled by a force which is known through the previous calibration of the balance.



▲ **Figure 3.7** Lever balance

A way of comparing masses is to use a beam balance, or lever balance (see Figure 3.7). Here the weight of the object is balanced against the weight of some masses, which have previously been calibrated in mass units. The word 'balance' here refers to the equilibrium of the beam: when the beam is horizontal, the moment of the weight on one side of the pivot is equal and opposite to the moment on the other side of the pivot. Because weight is given by the product of mass and the acceleration of free fall, the equality of the weights means that the masses are also equal.

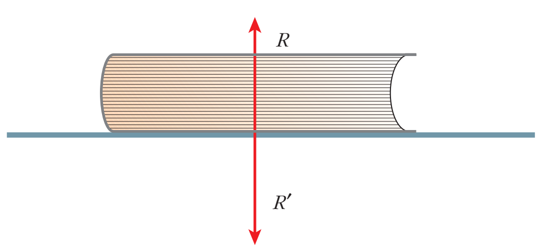
We have introduced the idea of weight by thinking about an object in free fall. But objects at rest also have weight: the gravitational attraction on a book is the same whether it is falling or whether it is resting on a table. The fact that the book is at rest tells us, by Newton's first law, that the resultant force acting on it is zero. So there must be another force acting on the book which exactly balances its weight. In Figure 3.8 the table exerts an upwards force on the book. This force is equal in magnitude to the weight but opposite in direction. It is a **normal contact force**: 'contact' because it occurs due to the contact between book and table, and 'normal' because it acts perpendicularly to the plane of contact.



▲ **Figure 3.8 A book resting on a table: forces on the book. (The forces act in the same vertical line, but are separated slightly for clarity.)**

The book remains at rest on the table because the weight *W* of the book downwards is exactly balanced by the normal contact force *R* exerted by the table on the book. The vector sum of these forces is zero, so the book is in equilibrium. A very common mistake is to state that 'By Newton's third law, *W* is equal to *R*'. But these two forces are both acting on the book, and cannot be related by the third law. Third-law forces always act on *different* bodies.

To see the application of the third law, think about the normal contact force *R*. This is an upwards force exerted by the *table* on the book. There is a downwards force *R*′ exerted by the *book* on the table. By Newton's third law, these forces are equal and opposite. This situation is illustrated in Figure 3.9.

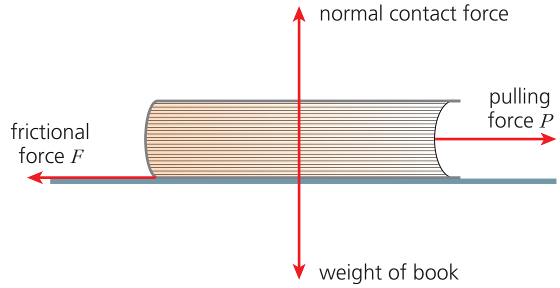


▲ **Figure 3.9 A book resting on a table: *R* and *R*′ act on different objects and are equal and opposite.**

Having considered the forces acting between book and table, we should consider the force that is equal and opposite to the weight of the book, even when the book is not on the table. This is not easy, because there does not seem to be an obvious second force. But remember that the weight is due to the gravitational attraction of the Earth on the book. If the Earth attracts the book, the book also attracts the Earth. This gravitational force of the book on the Earth is the second force. We can test whether the two forces do, indeed, act on different objects. The weight of the book acts on the book and the second force (the attraction of the Earth to the book) acts on the Earth. Thus, the condition that the two forces act on different objects is satisfied.

3.2 Non-uniform motion

As noted at the start of Topic 3.1, frictional forces are important in considering the motion of an object. A friction force acts along the common surface of contact between two objects. A frictional force always acts in the opposite direction to the relative motion of the objects. The frictional force is larger for rough surfaces and zero for smooth surfaces. In Figure 3.10 the pulling force *P* and the frictional force *F* are in opposite directions. If *P* is greater than *F* then the book accelerates. If *P* equals *F* the book travels at constant velocity.

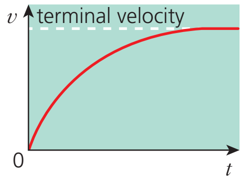


▲ **Figure 3.10** A book being pulled on a table

We use the term **viscous force** (or **drag force**) to describe the frictional force in a fluid (a liquid or a gas). The property of the fluid determining this force is the viscosity of the fluid. The frictional force is low for fluids with low viscosity such as water. The frictional force is large for fluids with high viscosity such as glue. An example of such a force is air resistance.

We mentioned in Topic 2 that, in most situations, air resistance for objects in free fall can be neglected. In fact, there are some applications in which the size of this force becomes important.

▲ **Figure 3.11** A parachutist about to land



▲ **Figure 3.12** Motion of an object falling in air

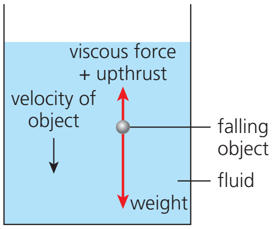
The velocity of an object moving through a resistive fluid (a liquid or a gas) does not increase indefinitely, but eventually reaches a maximum velocity, called the **terminal velocity**. The drag force due to air resistance is zero when an object's velocity is zero and increases with speed.

For an object falling in a viscous fluid the resultant downward force (weight – viscous force) decreases as the viscous force increases. When the resistive force has reached a value equal and opposite to the weight of the falling object the resultant force downwards is zero so the object no longer accelerates but continues at uniform velocity. This is a case of motion with non-uniform acceleration. The acceleration is initially equal to g, but decreases to zero at the time when the terminal velocity is achieved. Thus, raindrops and parachutists are normally travelling at a constant speed by the time they approach the ground (Figure 3.11).

An object released from rest at a considerable height above the Earth's surface at first increases in velocity as it accelerates due to gravity, but soon reaches a terminal (constant) velocity. Figure 3.12 shows the variation of the velocity v with time t for the object until its terminal velocity is reached.

The examples of objects falling through air in the Earth's gravitational field are assumed to fall through a uniform gravitational field where the value of g is constant. The weight of the object remains constant in these situations.

When an object is immersed in a fluid, it experiences an upward force due to the pressure of the fluid on it (see Topic 4.3). We call this an upthrust or buoyancy force.as we shall see in Topic 4, the upthrust force depends on the density of the fluid. The density of air is very small and, therefore, the effect of the upthrust force on an object in air is considered to be negligible.



▲ **Figure 3.13** Forces on an object falling in a fluid

In fluids such as oil and glycerine the upthrust force is not negligible. An object falling through such a fluid experiences the forces shown in Figure 3.13. The viscous force increases with the velocity of the object. The resultant downward force equals weight – (upthrust + viscous force).

The object reaches terminal velocity when the upthrust and the viscous force equals its weight.

weight = upthrust + viscous force

and causes the object to accelerate until the upthrust and the viscous force equals its weight. The object then continues to fall at its terminal velocity.

Problem solving

In dealing with problems involving Newton's laws, start by drawing a general sketch of the situation. Then consider each object in your sketch. Show all the forces acting on that object, both known forces and unknown forces you may be trying to find. Here it is a real help to try to draw the arrows which represent the forces in approximately the correct direction and approximately to scale. Label each force with its magnitude or with a symbol if you do not know the magnitude. For each force, you must know the cause of the force (gravity, friction, and so on), and you must also know *on* what object that force acts and *by* what object it is exerted. This labelled diagram is referred to as a **free-body diagram**, because it detaches the object from the others in the situation. Having established all the forces acting on the object, you can use Newton's second law to find unknown quantities. This procedure is illustrated in the example below.

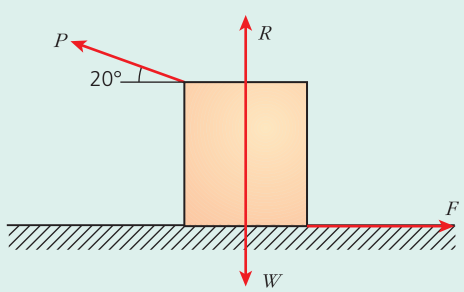
Newton's second law equates the resultant force acting on an object to the product of its mass and its acceleration. In some problems, the system of objects is in equilibrium. They are at rest, or are moving in a straight line with uniform speed. In this case, the acceleration is zero, so the resultant force is also zero. In other cases, the resultant force is not zero and the objects in the system are accelerating.

Whichever case applies, you should remember that forces are vectors. You will probably have to resolve the forces into two components at right angles (see Topic 1.4), and then apply the second law to each set of components separately. Problems can often be simplified by making a good choice of directions for resolution. You will end up with a set of equations, based on the application of Newton's second law, which must be solved to determine the unknown quantity.

WORKED EXAMPLE 3B

1 A box of mass 5.0 kg is pulled along a horizontal floor by a force P of 25 N, applied at an angle of 20° to the horizontal (Figure 3.14). A frictional force F of 20 N acts parallel to the floor.

Calculate the acceleration of the box.



▲ **Figure 3.14**

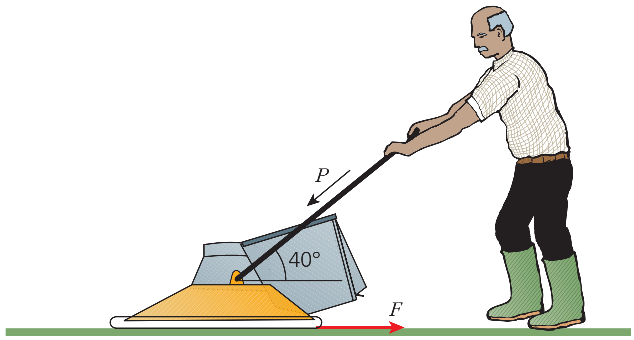
2 What is the magnitude of the momentum of an α-particle of mass 6.6×10^{-27} kg travelling with a speed of $2.0 \times 10^{7}\,\mathrm{m\,s^{-1}}$?

Answers

1 The free-body diagram is shown in Figure 3.14. Resolving the forces parallel to the floor, the component of the pulling force, acting to the left, is 25 cos 20° = 23.5 N.
The frictional force, acting to the right, is 20 N.
The resultant force to the left is thus 23.5 − 20.0 = 3.5 N.
From Newton's second law, $a = F/m = 3.5/5.0 =$ $\mathbf{0.70\,m\,s^{-2}}$.

2 $p = mv = 6.6 \times 10^{-27} \times 2.0 \times 10^{7} =$ $\mathbf{1.3 \times 10^{-19}\,kg\,m\,s^{-1}}$

Questions



▲ **Figure 3.15**

3 A person gardening pushes a lawnmower of mass 18 kg at constant speed. To do this requires a force *P* of 80 N directed along the handle, which is at an angle of 40° to the horizontal (Figure 3.15).

   a Calculate the horizontal frictional force *F* on the mower.

   b If this frictional force were constant, what force, applied along the handle, would accelerate the mower from rest to $1.2\,m\,s^{-1}$ in 2.0 s?

4 What is the magnitude of the momentum of an electron of mass 9.1×10^{-31} kg travelling with a speed of $7.5 \times 10^{6}\,m\,s^{-1}$?

5 Explain the changes in the resultant force acting on an object as it moves through a fluid

   a as the object falls vertically downwards and

   b moves vertically upwards in a fluid.

3.3 Linear momentum and its conservation

▲ **Figure 3.16** System of two particles

We have already seen that Newton's first law states that the momentum of a single particle is constant, if no external resultant force acts on the particle. Now think about a system of two particles (Figure 3.16). We allow these particles to exert some sort of force on each other: it could be gravitational attraction or, if the particles were charged, it could be electrostatic attraction or repulsion.

These two particles are isolated from the rest of the Universe, and experience no outside forces at all. If the first particle exerts a force *F* on the second, Newton's third law tells us that the second exerts a force –*F* on the first. The minus sign indicates that the forces are in opposite directions. As we saw in the last section, we can express this law in terms of change of momentum. The change of momentum of the second particle as a result of the force exerted on it by the first is equal and opposite to the change of momentum of the first particle as a result of the force exerted on it by the second. Thus, the changes of momentum of the individual particles cancel out, and the momentum of the system of two particles remains constant. The particles have merely exchanged some momentum.

The situation is expressed by the equation

$$p = p_1 + p_2 = \text{constant}$$

where p is the total momentum, and p_1 and p_2 are the individual momenta.

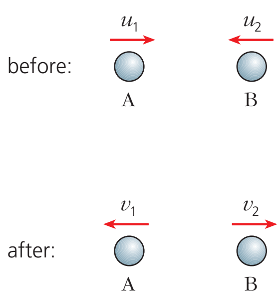
We could extend this idea to a system of three, four or indeed any number n of particles.

If no external resultant force acts on a system, the total momentum of the system remains constant, or is conserved.

A system on which no external resultant force acts is often called an isolated system. The fact that the total momentum of an isolated system is constant is the **principle of conservation of momentum**. It is a direct consequence of Newton's third law of motion.

Collisions

We now use the principle of conservation of momentum to analyse a system consisting of two colliding particles. (If you want a real example to think about, try snooker balls.)



▲ **Figure 3.17** Collision between two particles

Consider two particles A and B making a direct, head-on collision. Particle A has mass m_1 and is moving with velocity u_1 in the direction from left to right; B has mass m_2 and has velocity u_2 in the direction from right to left (Figure 3.17). As velocity is a vector quantity, this is the same as saying that the velocity is $-u_2$ from left to right. The particles collide. After the collision they have velocities $-v_1$ and v_2 respectively in the direction from left to right. That is, both particles are moving back along their directions of approach.

According to the principle of conservation of momentum, the total momentum of this isolated system remains constant, whatever happens as a result of the interaction of the particles. Thus, the total momentum before the collision must be equal to the total momentum after the collision. The momentum before the collision is

$$m_1u_1 - m_2u_2$$

and the momentum after is

$$-m_1v_1 + m_2v_2$$

Because total momentum is conserved

$$m_1u_1 - m_2u_2 = -m_1v_1 + m_2v_2$$

Knowing the masses of the particles and the velocities before collision, this equation would allow us to calculate the relation between the velocities after the collision.

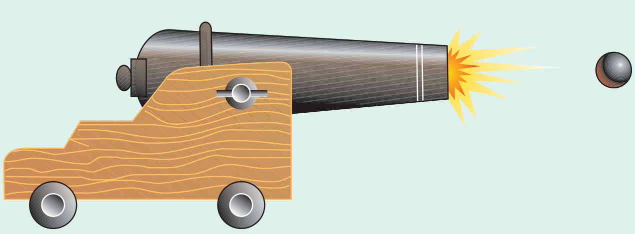
The way to approach collision problems is as follows.

- Draw a labelled diagram showing the colliding particles before collision. Draw a separate diagram showing the situation after the collision. Take care to define the directions of all the velocities.
- Obtain an expression for the total momentum before the collision, remembering that momentum is a vector quantity. Similarly, find the total momentum after the collision, taking the same reference direction.
- Then equate the momentum before the collision to the momentum afterwards.

WORKED EXAMPLE 3C

A cannon of mass 1.5 tonnes (1.5×10^3 kg) fires a cannon-ball of mass 5.0 kg (Figure 3.18).

The speed with which the ball leaves the cannon is $70\,\text{m}\,\text{s}^{-1}$ relative to the Earth. What is the initial speed of recoil of the cannon?



▲ **Figure 3.18**

Answer

The system under consideration is the cannon and the cannon-ball. The total momentum of the system before firing is zero. Because the total momentum of an isolated system is constant, the total momentum after firing must also be zero. That is, the momentum of the cannon-ball, which is $5.0 \times 70 = 350\,\text{kg}\,\text{m}\,\text{s}^{-1}$ to the right, must be exactly balanced by the momentum of the cannon. If the initial speed of recoil is v, the momentum of the cannon is $1500v$ to the left.

Thus, $1500v = 350$ and $v =$ **$0.23\,\text{m}\,\text{s}^{-1}$**.

Question

6 An ice-skater of mass 80 kg, initially at rest, pushes his partner, of mass 65 kg, away from him so that she moves with an initial speed of 1.5 m s^{-1}. What is the initial speed of the first skater after this manoeuvre?

Momentum and impulse

It is now useful to introduce a quantity called **impulse** and relate it to a change in momentum.

If a constant force F acts on an object for a time Δt, the impulse of the force is given by $F\Delta t$.

The unit of impulse is given by the unit of force, the newton, multiplied by the unit of time, the second: it is the newton second (N s).

We know from Newton's second law that the force acting on an object is equal to the rate of change of momentum of the object. We have already expressed this as the equation

$$F = \Delta p/\Delta t$$

If we multiply both sides of this equation by Δt, we obtain

$$F\Delta t = \Delta p$$

We have already defined $F\Delta t$ as the impulse of the force. The right-hand side of the equation (Δp) is the change in the momentum of the object. So, Newton's second law tells us that the **impulse of a force is equal to the change in momentum**.

It is useful for dealing with forces that act over a short interval of time, as in a collision. The forces between colliding objects are seldom constant throughout the collision, but the equation can be applied to obtain information about the average force acting.

Note that the idea of impulse explains why there is an alternative unit for momentum. In the section on momentum we introduced the kg m s^{-1} and the N s as possible units for momentum. The kg m s^{-1} is the logical unit, the one you arrive at if you take momentum as being the product of mass and velocity. The N s comes from the impulse–momentum equation: it is the unit of impulse, and because impulse is equal to change of momentum, it is also a unit for momentum.

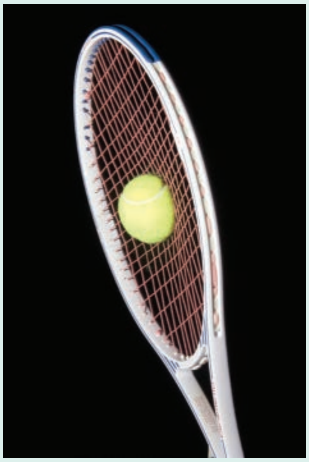
WORKED EXAMPLE 3D

Some tennis players can serve the ball at a speed of 55 m s^{-1}. The tennis ball has a mass of 60 g. In an experiment, it is determined that the ball is in contact with the racket for 25 ms during the serve (Figure 3.19). Calculate the average force exerted by the racket on the ball.

Answer

The change in momentum of the ball as a result of the serve is 0.060 × 55 = 3.3 kg m s^{-1}. By the impulse–momentum equation, the change in momentum is equal to the impulse of the force. Since impulse is the product of force and time, Ft = 3.3 N s.

Here t is 0.025 s; thus F = 3.3/0.025 = **130 N**.



▲ Figure 3.19

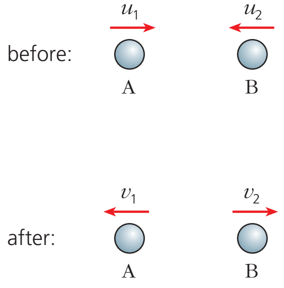
Question

7 A golfer hits a ball of mass 45 g at a speed of $40\,m\,s^{-1}$ (Figure 3.20). The golf club is in contact with the ball for 3.0 ms. Calculate the average force exerted by the club on the ball.

▲ **Figure 3.20**

Elastic and inelastic collisions

In some collisions, kinetic energy is conserved as well as momentum. By the **conservation of kinetic energy**, we mean that the total kinetic energy of the colliding particles before collision is the same as the total kinetic energy afterwards. This means that no energy is lost in the permanent deformation of the colliding particles, or as heat and sound. There is a transformation of energy during the collision: while the colliding particles are in contact, some of the kinetic energy is transformed into elastic potential energy, but as the particles separate, it is transformed into kinetic energy again.



▲ **Figure 3.21** Collision between two particles

Using the same notation for the masses and speeds of the colliding particles as in the section on Collisions (see Figure 3.21), the total kinetic energy of the particles before collision is

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2$$

The total kinetic energy afterwards is

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

If the collision is **elastic**, the total kinetic energy before collision is equal to the total kinetic energy after collision.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

Note that because energy is a scalar, the directions of motion of the particles are not indicated by the signs of the various terms.

In solving problems about elastic collisions, this equation is useful because it gives another relation between masses and velocities, in addition to that obtained from the principle of conservation of momentum.

When the velocity directions are as defined in Figure 3.21, application of the two conservation conditions shows that

$$u_1 + u_2 = v_1 + v_2$$

for a perfectly elastic collision.

That is, the relative speed of approach ($u_1 + u_2$) is equal to the relative speed of separation ($v_1 + v_2$). Note that this useful relation applies *only for a perfectly elastic collision*.

Elastic collisions occur in the collisions of atoms and molecules. We shall see in Topic 15.3 that one of the most important assumptions in the kinetic theory of gases is that the collisions of the gas molecules with the walls of the container are perfectly elastic. However, in larger-scale collisions, such as those of snooker balls, collisions cannot be perfectly elastic. (The 'click' of snooker balls on impact indicates that a very small fraction of the total energy of the system has been transformed into sound.)

Nevertheless, we often make the assumption that such a collision is perfectly elastic.

Collisions in which the total kinetic energy is not the same before and after the event are called **inelastic**.

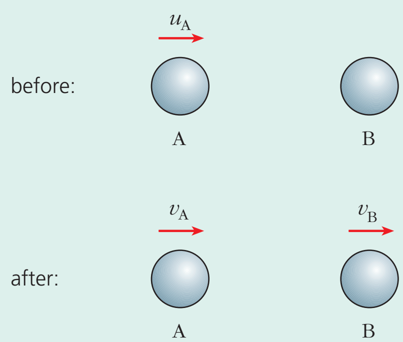
Total energy must, of course, be conserved. But in an inelastic collision the kinetic energy that does not re-appear in the same form is transformed into heat, sound and other forms of energy. In an extreme case, all the kinetic energy may be lost. A lump of modelling clay dropped on to the floor does not bounce. All the kinetic energy it possessed just before hitting the floor has been transformed into the work done in flattening the lump, and (a much smaller amount) into the sound emitted as a 'squelch'.

Although kinetic energy may or may not be conserved in a collision, momentum is always conserved, and so is total energy.

The truth of this statement may not be entirely obvious, especially when considering examples such as the lump of modelling clay which was dropped on to the floor. Surely the clay had momentum just before the collision with the floor, but had no momentum afterwards? True! But for the system of the lump of clay alone, external forces (the attraction of the Earth on the clay, and the force exerted by the floor on the clay on impact) were acting. When external forces act, the conservation principle does not apply. We need to consider a system in which no external forces act. Such a system is the lump of modelling clay and the Earth. While the clay falls towards the floor, gravitational attraction will also pull the Earth towards the clay. Conservation of momentum can be applied in that the total momentum of clay and Earth remains constant throughout the process: before the collision, and after it. The effects of the transfer of the clay's momentum to the Earth are not noticeable due to the difference in mass of the two objects.

WORKED EXAMPLE 3E

1 A snooker ball A moves with speed u_A directly towards a similar ball B which is at rest (Figure 3.22). The collision is elastic. What are the speeds v_A and v_B after the collision?



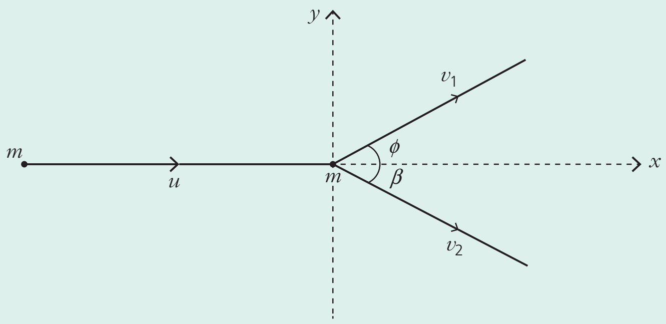
▲ **Figure 3.22**

2 A particle of mass m makes a glancing collision with a similar particle, also of mass m, which is at rest (Figure 3.23). The collision is elastic. After the collision the particles move off at angles ϕ and β. State the equations that relate:

a the x components of the momentum of the particles

b the y components of the momentum of the particles

c the kinetic energy of the particles.



▲ **Figure 3.23**

Answers

1 It is convenient to take the direction from left to right as the direction of positive momentum. If the mass of a billiard ball is m, the total momentum of the system before the collision is mu_A. By the principle of conservation of momentum, the total momentum after collision is the same as that before, or

$mu_A = mv_A + mv_B$

The collision is perfectly elastic, so the total kinetic energy before the collision is the same as that afterwards, or

$\frac{1}{2}mu_A{}^2 = \frac{1}{2}mv_A{}^2 + \frac{1}{2}mv_B{}^2$

Solving these equations gives $v_A = 0$ and $v_B = u_A$. That is, ball A comes to a complete standstill, and ball B moves off with the same speed as that with which ball A struck it. (Another solution is possible algebraically: $v_A = u_A$ and $v_B = 0$. This corresponds to a non-collision. Ball A is still moving with its initial speed, and ball B is still at rest. In cases where algebra gives us two possible solutions, we need to decide which one is physically appropriate.)

2 From the conservation of momentum:

a $mu = mv_1 \cos\phi + mv_2 \cos\beta$

b $0 = mv_1 \sin\phi - mv_2 \sin\beta$

c total kinetic energy is constant as the collision is elastic.
Hence, $\frac{1}{2}mu^2 = \frac{1}{2}mv_1{}^2 + \frac{1}{2}mv_2{}^2$

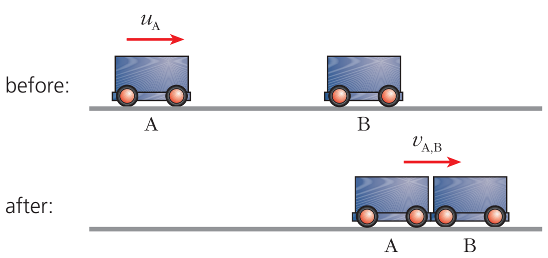
Question

8 A trolley A moves with speed u_A towards a trolley B of equal mass which is at rest (Figure 3.24).

The trolleys stick together and move off as one with speed $v_{A,B}$.

a Determine $v_{A,B}$.

b What fraction of the initial kinetic energy of trolley A is converted into other forms in this inelastic collision?



▲ **Figure 3.24**

SUMMARY

- The mass of an object is the property of an object that resists change in motion.
- The linear momentum p of an object is defined as the product of its mass m and its velocity v. In symbols: $p = mv$. Momentum has units $kg\,m\,s^{-1}$ or N s. It is a vector quantity.
- Newton's laws of motion are:
  - First law: Every object continues in its state of rest, or with uniform velocity, unless acted upon by a resultant force.
  - Second law: The resultant force acting on an object is proportional to the rate of change of its momentum (this is used to define force). In symbols: $F \propto \Delta p/\Delta t$. If SI units are used $F = \Delta p/\Delta t$.
  - Third law: When one object exerts a force on another object, the second object exerts an equal and opposite force on the first object.
- Newton's first and third laws of motion can also be stated in terms of momentum:
  - First law: The momentum of an object remains constant unless a resultant external force acts on the object: p = constant
  - Third law: When two objects collide, their changes in momentum are equal and opposite.
- If the mass is constant, the resultant force is equal to mass × acceleration or $F = ma$, where force F is in newtons, mass m is in kilograms and acceleration a is in $m\,s^{-2}$.
- The acceleration of free fall g provides the link between the mass m and the weight W of an object: $W = mg$
- A frictional force always acts in the opposite direction to the relative motion of the objects.
- The term viscous force is used to describe the frictional force in a fluid (a liquid or a gas).
- As an object falls in a uniform gravitational field the air resistance increases with the speed of the object until the resultant force is zero. The object then moves with terminal (constant) velocity.
- The principle of conservation of momentum states that the total momentum of an isolated system is constant. An isolated system is one on which no external resultant force acts.
- In collisions between objects, application of the principle of conservation of momentum shows that the total momentum of the system before the collision is equal to the total momentum after the collision.
- An elastic collision is one in which the total kinetic energy remains constant. In this situation, the relative speed of approach is equal to the relative speed of separation.
- An inelastic collision is one in which the total kinetic energy is not the same before and after the event.
- Although kinetic energy may or may not be conserved in a collision, momentum is always conserved, and so is total energy.
- The impulse of a force F is the product of the force and the time Δt for which it acts:
  impulse = $F\Delta t$
  The impulse of a force acting on an object is equal to the change of momentum of the object: $F\Delta t = \Delta p$.
  The unit of impulse is N s.

END OF TOPIC QUESTIONS

1 A net force of 95 N accelerates an object at $1.9\,m\,s^{-2}$.
Calculate the mass of the object.

2 A parachute trainee jumps from a platform 3.0 m high. When he reaches the ground, he bends his knees to cushion the fall. His torso decelerates over a distance of 0.65 m. Calculate:

- **a** the speed of the trainee just before he reaches the ground,
- **b** the deceleration of his torso,
- **c** the average force exerted on his torso (of mass 45 kg) by his legs during the deceleration.

3 If the acceleration of an object is zero, does this mean that no forces act on it?

4 A railway engine pulls two carriages of equal mass with uniform acceleration. The tension in the coupling between the engine and the first carriage is T. Deduce the tension in the coupling between the first and second carriages.

5 Calculate the magnitude of the momentum of a car of mass 1.5 tonnes (1.5×10^3 kg) travelling at a speed of $22\,m\,s^{-1}$.

6 When a certain space rocket is taking off, the propellant gases are expelled at a rate of 900 kg s^{-1} and speed of 40 km s^{-1}. Calculate the thrust on the rocket.

7 An insect of mass 4.5 mg, flying with a speed of 0.12 m s^{-1}, encounters a spider's web, which brings it to rest in 2.0 ms. Calculate the average force exerted by the insect on the web.

8 What is your mass? What is your weight?

9 An atomic nucleus at rest emits an α-particle of mass 4 u. The speed of the α-particle is found to be 5.6×10^6 m s^{-1}.

Calculate the speed with which the daughter nucleus, of mass 218 u, recoils.

10 A heavy particle of mass m_1, moving with speed u, makes a head-on collision with a light particle of mass m_2, which is initially at rest. The collision is perfectly elastic, and m_2 is very much less than m_1. Describe the motion of the particles after the collision.

11 A light object and a heavy object have the same momentum.

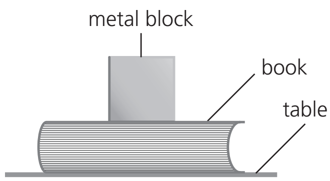
Which has the greater kinetic energy?

12 A resultant force 30 N acts on an initially stationary mass of 10 kg for 4.0 s. How far does the mass move?

A 6.0 m **B** 12 m **C** 24 m **D** 48 m

13 A mass of 0.50 kg falls vertically and collides with the ground at a speed of 5.0 m s^{-1}. The mass rebounds vertically with a speed of 3.0 m s^{-1}. The mass is in contact with the ground for 20 ms. What is the magnitude of the force acting on the mass as it makes contact with the ground?

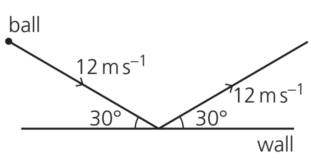
A 50 N **B** 75 N **C** 130 N **D** 200 N



▲ **Figure 3.25**

14 A book rests on a horizontal table. A metal block rests on the top of the book as shown in Fig. 3.25. Newton's third law describes how forces exist in pairs. One such pair of forces is the weight of the book and another force Y. What is the name of the body on which force Y acts?

A book **B** Earth **C** ground **D** metal block



▲ **Figure 3.26**

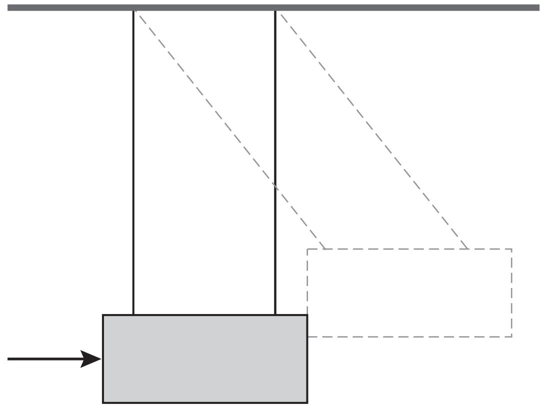
15 A 45 g ball with speed of 12 m s^{-1} hits a wall at an angle of 30° (see Fig. 3.26). The ball rebounds with the same speed and angle. The contact time of the ball with the wall is 15 ms. Calculate:

a the change in momentum of the ball,

b the impulse of the ball,

c the force exerted on the ball by the wall.

16 A bullet of mass 12 g is fired horizontally from a gun with a velocity of 180 m s^{-1}. It hits, and becomes embedded in, a block of wood of mass 2000 g, which is freely suspended by long strings, as shown in Fig. 3.27.


▲ **Figure 3.27**

Calculate:

a **i** the magnitude of the momentum of the bullet just before it enters the block,

ii the magnitude of the initial velocity of the block and bullet after impact,

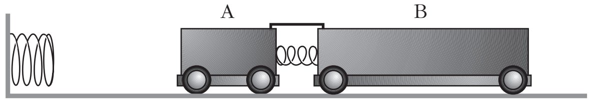
iii the kinetic energy of the block and embedded bullet immediately after the impact.

b Deduce the maximum height above the equilibrium position to which the block and embedded bullet rise after impact.

17 A nucleus A of mass 222 u is moving at a speed of 350 m s^{-1}. While moving, it emits an α-particle of mass 4 u. After the emission, it is determined that the daughter nucleus, of mass 218 u, is moving with speed 300 m s^{-1} in the original direction of the parent nucleus. Calculate the speed of the α-particle.

18 A safety feature of modern cars is the air-bag, which, in the event of a collision, inflates and is intended to decrease the risk of serious injury. Use the concept of impulse to explain why an air-bag might have this effect.

19 Two frictionless trolleys A and B, of mass m and $3m$ respectively, are on a horizontal track (Fig. 3.28). Initially they are clipped together by a device which incorporates a spring, compressed between the trolleys. At time $t = 0$ the clip is released. The velocity of trolley B is u to the right.



▲ **Figure 3.28**

a Calculate the velocity of trolley A as the trolleys move apart.

b At time $t = t_1$, trolley A collides elastically with a fixed spring and rebounds. (Compression and expansion of the spring take a negligibly short time.) Trolley A catches up with trolley B at time $t = t_2$.

i Calculate the velocity of trolley A between $t = t_1$ and $t = t_2$.

ii Find an expression for t_2 in terms of t_1.

c When trolley A catches up with trolley B at time t_2 the clip operates so as to link them again, this time without the spring between them, so that they move together with velocity v.

Calculate the common velocity v in terms of u.

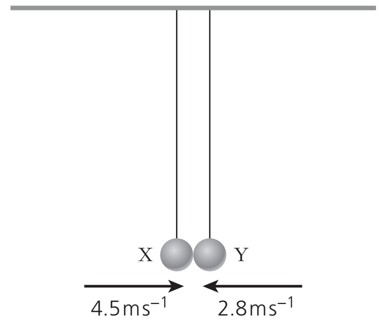
d Initially the trolleys were at rest and the total momentum of the system was zero. However, the answer to **c** shows that the total momentum after $t = t_2$ is not zero. Discuss this result with reference to the principle of conservation of momentum.

20 A ball of mass m makes a perfectly elastic head-on collision with a second ball, of mass M, initially at rest. The second ball moves off with half the original speed of the first ball.

a Express M in terms of m.

b Determine the fraction of the original kinetic energy retained by the ball of mass m after the collision.

21 Two balls X and Y are supported by long strings, as shown in Fig. 3.29.



▲ **Figure 3.29**

The balls are each pulled back and pushed towards each other. When the balls collide at the position shown in Fig. 3.29, the strings are vertical. The balls rebound in opposite directions. Fig. 3.30 shows data for X and Y during this collision.

ball	mass	velocity just before collision/m s^{-1}	velocity just after collision/m s^{-1}
X	50 g	+4.5	−1.8
Y	M	−2.8	+1.4

▲ **Figure 3.30**

The positive direction is horizontal and to the right.

a Use the conversation of linear momentum to determine the mass M of Y. **[3]**

b State and explain whether the collision is elastic. **[1]**

c Use Newton's second and third laws to explain why the magnitude of the change in momentum of each ball is the same. **[3]**

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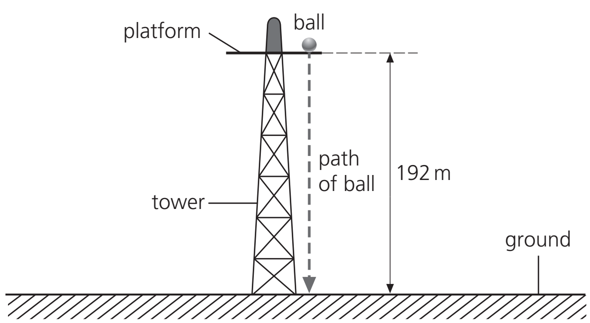
22 A steel ball falls from a platform on a tower to the ground below, as shown in Fig. 3.31.



▲ **Figure 3.31**

The ball falls from rest through a vertical distance of 192 m. The mass of the ball is 270 g.

a Assume air resistance is negligible.

i Calculate:

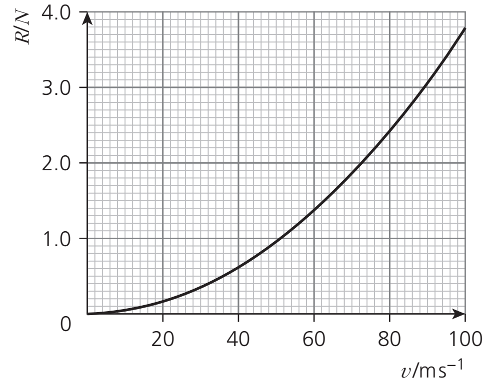
1 the time taken for the ball to fall to the ground, **[2]**

2 the maximum kinetic energy of the ball. **[2]**

ii State and explain the variation of the velocity of the ball with time as the ball falls to the ground. **[1]**

iii Show that the velocity of the ball on reaching the ground is approximately 60 m s^{-1}. **[1]**

b In practice, air resistance is not negligible. The variation of the air resistance R with the velocity v of the ball is shown in Fig. 3.32.



▲ **Figure 3.32**

i Use Fig. 3.32 to state and explain qualitatively the variation of the acceleration of the ball with the distance fallen by the ball. **[3]**

ii The speed of the ball reaches $40\,m\,s^{-1}$. Calculate its acceleration at this speed. **[2]**

iii Use information from **a iii** and Fig. 3.32 to state and explain whether the ball reaches terminal velocity. **[2]**

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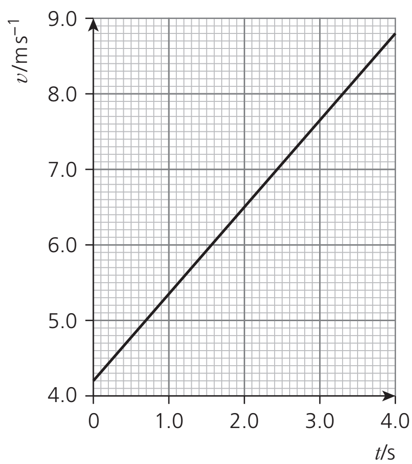
23 a State Newton's second law of motion. **[1]**

b A constant resultant force F acts on an object A. The variation with time t of the velocity v for the motion of A is shown in Fig. 3.33.



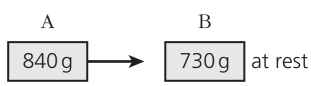
▲ **Figure 3.33**

The mass of A is 840 g. Calculate, for the time $t = 0$ to $t = 4.0\,s$:

i the change in momentum of A, **[2]**

ii the force F. **[1]**

c The force F is removed at $t = 4.0\,s$. Object A continues at constant velocity before colliding with an object B, as illustrated in Fig. 3.34.



▲ **Figure 3.34**

Object B is initially at rest. The mass of B is 730 g. The objects A and B join together and have a velocity of $4.7\,m\,s^{-1}$.

i By calculation, show that the changes in momentum of A and of B during the collision are equal and opposite. **[2]**

ii Explain how the answers obtained in **i** support Newton's third law. **[2]**

iii By reference to the speeds of A and B, explain whether the collision is elastic. **[1]**

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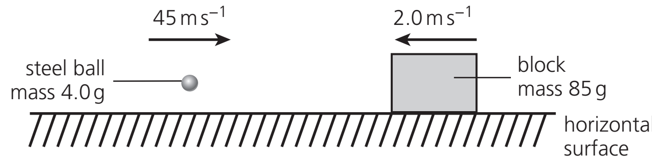
24 A wooden block moves along a horizontal frictionless surface, as shown in Fig. 3.35.



▲ **Figure 3.35**

The block has mass 85 g and moves to the left with a velocity of 2.0 m s^{-1}. A steel ball of mass 4.0 g is fired to the right. The steel ball, moving horizontally with a speed of 45 m s^{-1}, collides with the block and remains embedded in it. After the collision the block and steel ball both have speed v.

a Calculate v. **[2]**

b **i** For the block and ball, state:

1 the relative speed of approach before collision,

2 the relative speed of separation after collision. **[1]**

ii Use your answers in **i** to state and explain whether the collision is elastic or inelastic. **[1]**

c Use Newton's third law to explain the relationship between the rate of change of momentum of the ball and the rate of change of momentum of the block during the collision. **[2]**

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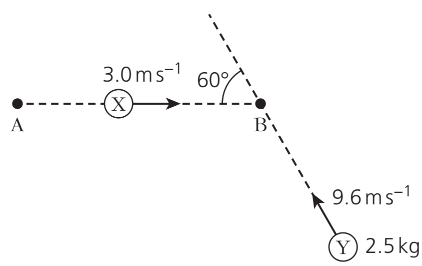
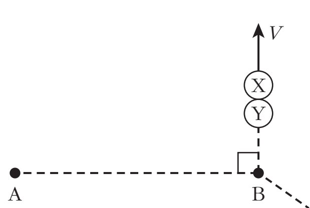
25 Two balls, X and Y, move along a horizontal frictionless surface, as illustrated in Fig. 3.36.



▲ **Figure 3.36**

Ball X has an initial velocity of 3.0 m s^{-1} in a direction along line AB. Ball Y has a mass of 2.5 kg and an initial velocity of 9.6 m s^{-1} in a direction at an angle of 60° to line AB. The two balls collide at point B. The balls stick together and then travel along the horizontal surface in a direction at right angles to the line AB, as shown in Fig. 3.37.



▲ **Figure 3.37**

a By considering the components of momentum in the direction from A to B, show that ball X has a mass of 4.0 kg. **[2]**

b Calculate the common speed V of the two balls after the collision. **[2]**

c Determine the difference between the initial kinetic energy of ball X and the initial kinetic energy of ball Y. **[2]**

Cambridge International AS and A Level Physics (9702) Paper 22 Q3 March 2019

4 Forces, density and pressure

Learning outcomes

By the end of this topic, you will be able to:

4.1 Turning effects of forces

1. understand that the weight of an object may be taken as acting at a single point known as the centre of gravity
2. define and apply the moment of a force
3. understand that a couple is a pair of forces that acts to produce rotation only
4. define and apply the torque of a couple

4.2 Equilibrium of forces

1. state and apply the principle of moments
2. understand that, when there is no resultant force and no resultant torque, a system is in equilibrium
3. use a vector triangle to represent coplanar forces in equilibrium

4.3 Density and pressure

1. define and use density
2. define and use pressure
3. derive, from the definitions of pressure and density, the equation for hydrostatic pressure $\Delta p = \rho g \Delta h$
4. use the equation $\Delta p = \rho g \Delta h$
5. understand that the upthrust acting on an object in a fluid is due to a difference in hydrostatic pressure
6. calculate the upthrust acting on an object in a fluid using the equation $F = \rho g V$ (Archimedes' principle)

Starting points

- ★ Understand the concept of weight as the effect of a gravitational field.
- ★ The use of vector triangles to add vectors.
- ★ For zero resultant force, the velocity of an object does not change (Newton's first law).

4.1 Turning effects of forces

Centre of gravity

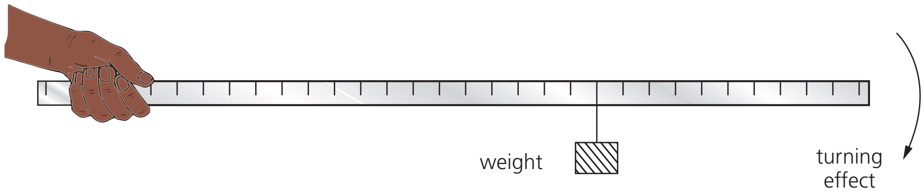
An object may be made to balance at a particular point. When it is balanced at this point, the object does not turn and so all the weight on one side of the pivot is balanced by the weight on the other side. Supporting the object at the pivot means that the only force which has to be applied at the pivot is one to stop the object falling – that is, a force equal to the weight of the object. Although all parts of the object have weight, the whole weight of the object appears to act at this balance point. This point is called the **centre of gravity** (C.G.) of the object.

The centre of gravity of an object is the point at which the whole weight of the object may be considered to act.

The weight of an object can be shown as a force acting vertically downwards at the centre of gravity. For a uniform object such as a ruler, the centre of gravity is at the geometrical centre.

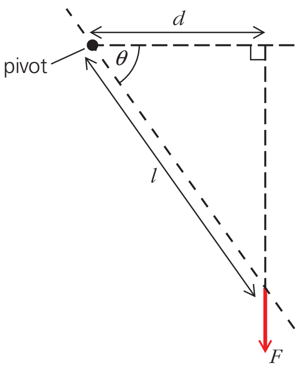
Moment of a force

When a force acts on an object, the force may cause the object to move in a straight line. It could also cause the object to turn or spin (rotate).



▲ **Figure 4.1** Turning effect on a metre ruler

Think about a metre rule held in the hand at one end so that the rule is horizontal (Figure 4.1). If a weight is hung from the ruler we can feel a turning effect on the ruler. The turning effect increases if the weight is increased or it is moved further from the hand along the ruler. The turning effect acts at the hand where the metre rule is pivoted. Keeping the weight and its distance along the rule constant, the turning effect can be changed by holding the ruler at an angle to the horizontal. The turning effect becomes smaller as the rule approaches the vertical position.



▲ **Figure 4.2** Finding the moment of a force

The turning effect of a force is called the **moment** of the force.

The moment of a force depends on the magnitude of the force and also on the distance of the force from the pivot or fulcrum. This distance must be defined precisely. In the simple experiment above, we saw that the moment of the force depended on the angle of the ruler to the horizontal. Varying this angle means that the line of action of the force from the pivot varies (see Figure 4.2). The distance required when finding the moment of a force is the perpendicular distance d of the line of action of the force from the pivot.

The moment of a force is defined as the product of the force and the perpendicular distance of the line of action of the force from the pivot.

Referring to Figure 4.2, the force has magnitude F and acts at a point distance l from the pivot. Then, when the ruler is at angle θ to the horizontal,

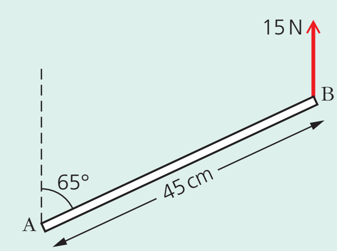
$$\text{moment of force} = F \times d$$

$$= F \times l \cos\theta$$

Since force is measured in newtons and distance is measured in metres, the unit of the moment of a force is newton-metre (N m).

WORKED EXAMPLE 4A

In Figure 4.3, a light rod AB of length 45 cm is held at A so that the rod makes an angle of 65° to the vertical. A vertical force of 15 N acts on the rod at B. Calculate the moment of the force about the end A.



▲ **Figure 4.3**

Answer

moment of force = force × perpendicular distance from pivot

$= 15 \times 0.45 \sin 65°$

(Remember that the distance must be in metres.)

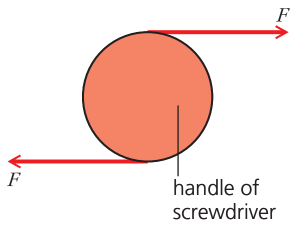
$= \mathbf{6.1\ N\,m}$

Question

1 Referring to Figure 4.3, calculate the moment of the force about A for a vertical force of 25 N with the rod at an angle of 30° to the vertical.

Couples

When a screwdriver is used, we apply a turning effect to the handle. We do not apply one force to the handle because this would mean the screwdriver would move sideways. Rather, we apply two forces of equal size but opposite direction on opposite sides of the handle (see Figure 4.4).

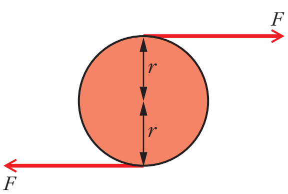


▲ **Figure 4.4** Two forces acting as a couple

A **couple** consists of two forces, equal in magnitude but opposite in direction whose lines of action do not coincide.

Consider the two parallel forces, each of magnitude F acting as shown in Figure 4.5 on opposite ends of a diameter of a disc of radius r. Each force produces a moment about the centre of the disc of magnitude Fr in a clockwise direction. The total moment about the centre is

$F \times 2r$ or $F \times$ perpendicular distance between the forces



▲ **Figure 4.5** Torque of a couple

Although a turning effect is produced, this turning effect is not called a moment because it is produced by two forces, not one. Instead, this turning effect is referred to as a **torque**. The unit of torque is the same as that of the moment of a force, i.e. newton-metre.

The torque of a couple is the product of one of the forces and the perpendicular distance between the forces.

It is interesting to note that, in engineering, the tightness of nuts and bolts is often stated as the maximum torque to be used when screwing up the nut on the bolt. Spanners used for this purpose are called torque wrenches because they have a scale on them to indicate the torque that is being applied.

▲ **Figure 4.6** Tightening a wheel nut requires the application of a torque.

WORKED EXAMPLE 4B

Calculate the torque produced by two forces, each of magnitude 30 N, acting in opposite directions with their lines of action separated by a distance of 25 cm.

Answer

torque = force × separation of forces

= 30 × 0.25 (distance in metres)

= **7.5 N m**

Question

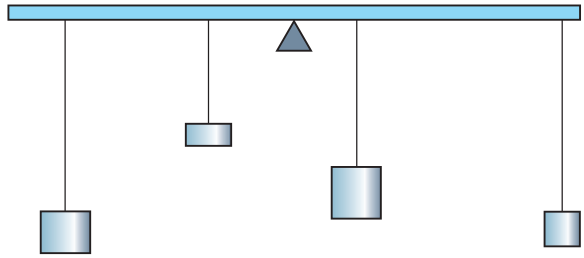
2 The torque produced by a person using a screwdriver is 0.18 N m. This torque is applied to the handle of diameter 4.0 cm. Calculate the force applied to the handle.

4.2 Equilibrium of forces

The principle of moments

A metre rule may be balanced on a pivot so that the rule is horizontal. Hanging a weight on the rule will make the rule rotate about the pivot. Moving the weight to the other side of the pivot will make the rule rotate in the opposite direction. Hanging weights on both sides of the pivot as shown in Figure 4.7 (overleaf) means that the ruler may rotate clockwise, or anticlockwise, or it may remain horizontal. In this horizontal position, there is no resultant turning effect and so the total turning effect of the forces in the clockwise direction equals the total turning effect in the anticlockwise direction.

You can check this very easily with the apparatus of Figure 4.7.



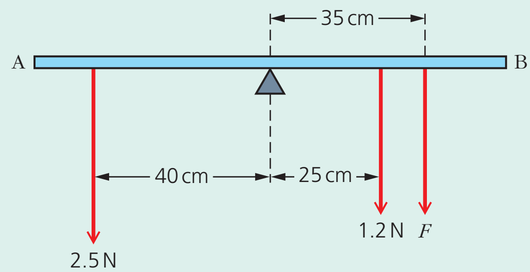
▲ **Figure 4.7**

When an object has no tendency to change its speed of rotation, it is said to be in rotational equilibrium.

The **principle of moments** states that, for an object to be in rotational equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about that same point.

WORKED EXAMPLE 4C

Some weights are hung from a light rod AB as shown in Figure 4.8. The rod is pivoted. Calculate the magnitude of the force F required to balance the rod horizontally.



▲ **Figure 4.8**

Answer

Sum of clockwise moments = $(0.25 \times 1.2) + 0.35F$

Sum of anticlockwise moments = 0.40×2.5

By the principle of moments

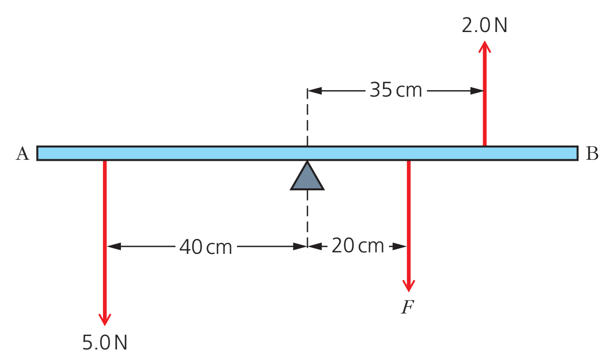
$(0.25 \times 1.2) + 0.35F = 0.40 \times 2.5$

$0.35F = 1.0 - 0.3$

$F = \mathbf{2.0\,N}$

Question

3 Some weights are hung from a light rod AB as shown in Figure 4.9. The rod is pivoted. Calculate the magnitude of the force F required to balance the rod horizontally.



▲ **Figure 4.9**

Equilibrium

The principle of moments gives the condition necessary for an object to be in rotational equilibrium. However, the object could still have a resultant force acting on it which would cause it to accelerate linearly. Thus, for complete **equilibrium**, there cannot be any resultant force in any direction.

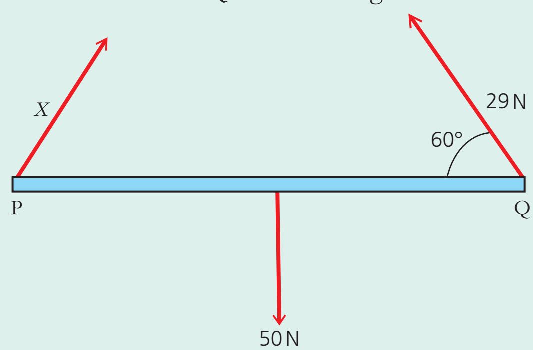
In Topic 1.4 we added forces (vectors) using a **vector triangle**. When three forces act on an object the condition for equilibrium is that the vector diagram for these forces forms a closed triangle. When four or more forces act on an object the same principles apply.

For equilibrium, the closed vector triangle then becomes a closed vector polygon.

For an object to be **in equilibrium**:
1 the sum of the forces in any direction must be zero
2 the sum of the moments of the forces about any point must be zero.

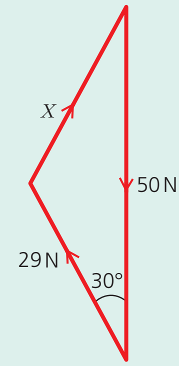
WORKED EXAMPLE 4D

The uniform rod PQ shown in Figure 4.10 is horizontal and in equilibrium.



▲ **Figure 4.10**

The weight of the rod is 50 N. A force of 29 N that acts at end Q is 60° to the horizontal. The force at end P is labelled *X*. Draw a vector triangle to represent the forces acting on the rod and determine the magnitude and direction of force *X*.



▲ **Figure 4.11**

Answer

The forces keep the rod in equilibrium and hence form a closed triangle as shown in Figure 4.11.

A scale diagram can be drawn to show that *X* is 29 N and acts at 60° to the horizontal.

Question

4 The same uniform rod PQ is in equilibrium, as in the above example.
  a i Show that the upward forces equal the downward forces.
    ii Show that the horizontal force to the left equals the horizontal force to the right.
  b The length of the rod in Figure 4.10 is 100 cm. Determine the force *X* by taking moments about Q.

4.3 Density and pressure

In this section we will bring together density and pressure to show an important link between them.

Density

The density of a substance is defined as its mass per unit volume.

$$\rho = m/V$$

The symbol for density is ρ (Greek rho) and its SI unit is $kg\,m^{-3}$.

WORKED EXAMPLE 4E

An iron sphere of radius 0.18 m has mass 190 kg. Calculate the density of iron.

Answer

First calculate the volume of the sphere from $V = \frac{4}{3}\pi r^3$. This works out at $0.024\,m^3$.

Application of the formula for density gives ρ = **7800 kg m^{-3}**.

Questions

5 The mass of a metal cylinder is 200 g. The length of the cylinder is 6.0 cm and its diameter is 2.0 cm. Calculate the density of the metal in $kg\,m^{-3}$.

6 A spherical metal ball has a radius of 2.5 cm. The density of the metal is $2700\,kg\,m^{-3}$. Calculate the mass of the ball in kg.

Pressure

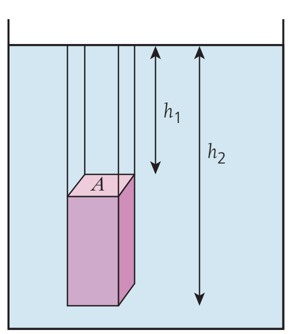
Pressure is defined as force per unit area, where the force F acts perpendicularly to the area A.

$$p = F/A$$

The symbol for pressure is p and its SI unit is the pascal (Pa), which is equal to one newton per square metre ($N\,m^{-2}$).

Pressure in a liquid

The link between pressure and density comes when we deal with liquids or with fluids in general. Consider a point at a depth h_1 below the surface of a liquid in a container. What is the pressure due to the liquid? Very simply, the pressure is caused by the weight of the column of liquid above a small area at that depth, as shown in Figure 4.12. The weight of the column is $W = mg = \rho Ah_1g$, and the pressure ρ_1 is $W/A = \rho gh_1$. The pressure at a depth of h_2 is due the column of liquid above this depth and is given by $p_2 = \rho gh_2$.



▲ **Figure 4.12** Column of liquid above the area A

$$p = \rho gh$$

The difference in pressure due to the difference in water depth is

$$\Delta p\ (= p_2 - p_1) = \rho g(h_2 - h_1) = \rho g\Delta h$$

The pressure in a liquid increases with depth. The change in pressure in a fluid, Δp due to the change in depth Δh is given by:

$$\Delta p = \rho g\Delta h$$

A fluid that is at rest has all points in the fluid in equilibrium. Hence the pressure at any point in a fluid must act in **all** directions at that point. The forces due to the pressure of a fluid acting on an object immersed in the fluid will act in all directions on that object.

The pressure is proportional to the depth below the surface of the liquid. If an external pressure, such as atmospheric pressure, acts on the surface of the liquid, this must be taken into account in calculating the absolute pressure. The absolute pressure is the sum of the external pressure and the pressure due to the depth below the surface of the liquid.

WORKED EXAMPLE 4F

Calculate the excess pressure over atmospheric at a point 1.2 m below the surface of the water in a swimming pool. The density of water is $1.0 \times 10^3\,\text{kg}\,\text{m}^{-3}$.

Answer

This is a straightforward calculation from $p = \rho gh$.

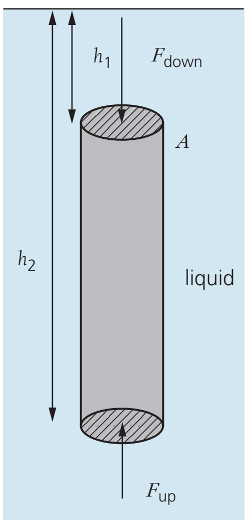
Substituting, $p = 1.0 \times 10^3 \times 9.81 \times 1.2 = \mathbf{1.2 \times 10^4\,Pa}$.

If the total pressure had been required, this value would be added to atmospheric pressure p_A. Taking p_A to be 1.01×10^5 Pa, the total pressure is 1.13×10^5 Pa.

Question

7 Calculate the difference in blood pressure between the top of the head and the soles of the feet of a student 1.3 m tall, standing upright.
Take the density of blood as $1.1 \times 10^3\,\text{kg}\,\text{m}^{-3}$.

Upthrust



▲ **Figure 4.13** Origin of the buoyancy force (upthrust)

When an object is immersed in a fluid (a liquid or a gas), it appears to weigh less than when in a vacuum. Large stones under water are easier to lift than when they are out of the water. The reason for this is that immersion in the fluid provides an **upthrust** or buoyancy force.

We can see the reason for the upthrust when we think about an object, such as the cylinder in Figure 4.13, submerged in water. Remember that the pressure in a liquid increases with depth. Thus, the pressure at the bottom of the cylinder is greater than the pressure at the top of the cylinder. The pressure difference Δp is given by

$$\Delta p = \rho g h_2 - \rho g h_1$$

This difference in pressure means that there is a bigger force acting upwards on the base of the cylinder, than there is acting downwards on the top. The difference in these forces is the upthrust or buoyancy force F_b. Looking at Figure 4.13, we can see that

$$F_b = F_{up} - F_{down}$$

and, since

$$p = F/A$$

$$F_b = \rho g A(h_2 - h_1) = \rho g A l$$
$$= \rho g V$$

where ρ is the density of the liquid, l is the length of the cylinder, A is the cross-sectional area of the cylinder, and V is its volume and $V = Al$. Since the column occupies a volume equal to the volume of the liquid it displaces, the liquid displaced by the column has mass ρV, and weight ρgV. Hence the upthrust is equal to the **weight of the liquid displaced** by the immersed object. This relation has been derived for a cylinder, but it will also apply to objects of any shape.

The rule that the upthrust acting on an object immersed in a fluid is equal to the weight of the fluid displaced is known as **Archimedes' principle**.

WORKED EXAMPLE 4G

Calculate:

a the force needed to lift a metal cylinder when in air

b the force needed to lift the cylinder when immersed in water.

The density of the metal is 7800 kg m^{-3} and the density of water is 1000 kg m^{-3}. The volume of the cylinder is 0.50 m^3.

Answers

a Force needed in air = weight of cylinder = 0.50 × 7800 × 9.81 = **3.8 × 10^4 N**

b Force needed in water
= weight of cylinder – upthrust
= 0.50 × 7800 × 9.81 – 0.50 × 1000 × 9.81
= **3.3 × 10^4 N**

The difference in the values in **a** and **b** is the upthrust on the metal cylinder when immersed in water. i.e. the weight of water displaced by the cylinder.

[The upthrust of the cylinder in air was neglected as the density of air is very much less than that of the metal.]

Questions

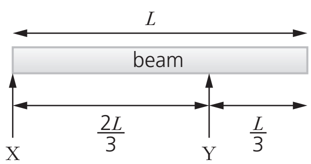
8 Explain why a boat made of metal is in equilibrium when stationary and floating on water.

9 A sphere of radius 4.5 cm is immersed in a liquid of density 800 kg m^{-3}. Calculate the upthrust on the sphere.

SUMMARY

- The centre of gravity of an object is the point at which the whole weight of the object may be considered to act.
- The moment of a force is the product of the force and the perpendicular distance of the line of action of the force from the pivot.
- A couple consists of two equal forces acting in opposite directions whose lines of action do not coincide.
- The torque of a couple is a measure of the turning effect of the couple.
- The torque of a couple is the product of one of the forces and the perpendicular distance between the lines of action of the forces.
- The principle of moments states that for an object in rotational equilibrium the sum of the clockwise moments about a point is equal to the sum of the anticlockwise moments about the point.
- For an object to be in equilibrium:
  - the sum of the forces in any direction must be zero
  - the sum of the moments of the forces about any point must be zero.
- Density ρ is defined by the equation $\rho = m/V$, where m is the mass of an object and V is its volume.
- Pressure p is defined by the equation $p = F/A$, where F is the force acting perpendicularly to an area A.
- Pressure increases with depth in a fluid. The difference in pressure is proportional to the difference in depth between two points in the fluid, and the pressure difference is given by $\Delta p = \rho g \Delta h$.
- The total pressure p at a point at a depth h below the surface of a fluid of density ρ is $p = p_A + \rho gh$, p_A being the atmospheric pressure; the difference in pressure between the surface and a point at a depth h is ρgh.
- The upthrust F on an object immersed in a fluid is equal to the weight of the fluid displaced ($F = \rho gV$).

END OF TOPIC QUESTIONS

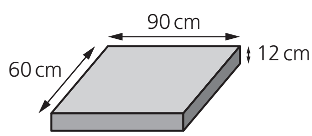


▲ **Figure 4.14**

1 A uniform beam of length L is supported by two forces X and Y so that it is in equilibrium. The position of the forces is shown in Fig. 4.14.

What is the ratio of the forces X : Y?

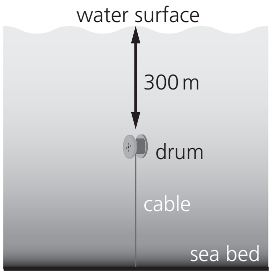
A 1 : 2 **B** 1 : 3 **C** 1 : 1 **D** 2 : 1



▲ **Figure 4.15**

2 A rectangular block of mass 150 kg has sides of 60 cm, 90 cm and 12 cm (Fig. 4.15). What is the minimum pressure that the block exerts on the ground when it is resting on one of its sides?

A 0.27 kPa **B** 0.28 kPa **C** 1.4 kPa **D** 2.7 kPa



▲ **Figure 4.16**

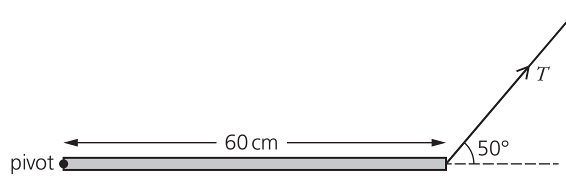
3 A metal drum is held 300 m beneath the surface of the sea by a vertical cable as shown in Fig. 4.16. The drum has a volume of 0.500 m^3 and weight 800 N. The density of the sea water is 1030 kg m^{-3}. What is the tension in the cable?

A 0.80 kN **B** 4.3 kN **C** 5.1 kN **D** 5.9 kN

4 A uniform rod of length 60 cm has a weight of 14 N. It is pivoted at one end and held in a horizontal position by a thread tied to its other end, as shown in Fig. 4.17. The thread makes an angle of 50° with the horizontal. Calculate:

a the moment of the weight of the rod about the pivot,

b the tension T in the thread required to hold the rod horizontally.

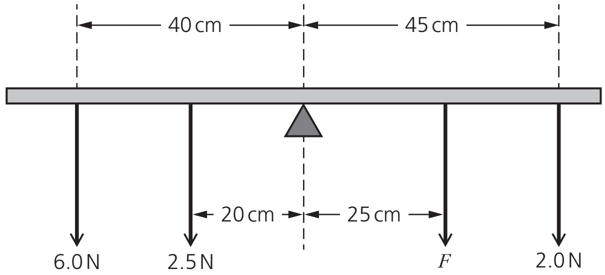


▲ **Figure 4.17**

5 A ruler is pivoted at its centre of gravity and weights are hung from the ruler as shown in Fig. 4.18. Calculate:

a the total anticlockwise moment about the pivot,

b the magnitude of the force F.

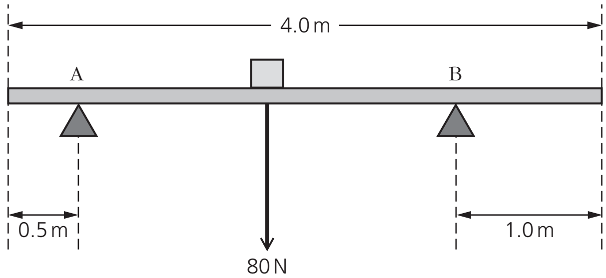


▲ **Figure 4.18**

6 A uniform plank of weight 120 N rests on two stools as shown in Fig. 4.19. A weight of 80 N is placed on the plank, midway between the stools. Calculate:

a the force acting on the stool at A,

b the force acting on the stool at B.

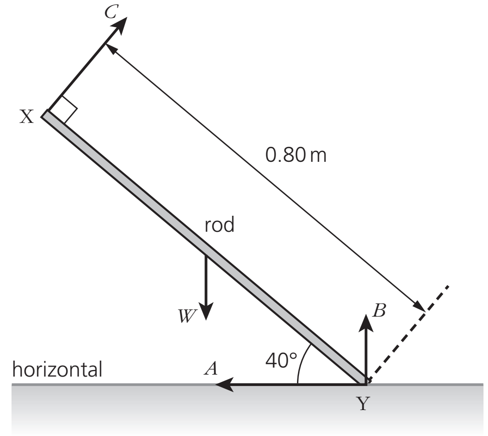


▲ **Figure 4.19**

7 A nut is to be tightened to a torque of 16 N m. Calculate the force which must be applied to the end of a spanner of length 24 cm in order to produce this torque.

8 **a** State the conditions required for an object to be in equilibrium.

b A uniform rod XY of length 0.80 m is at an angle of 40° to a horizontal surface, as shown in Fig. 4.20.



▲ **Figure 4.20**

Four forces A, B, C and W act on the rod to maintain equilibrium. Force C acts at right angles to the rod at end X and is 2.0 N. A vertical force B and a horizontal force A act on the rod at end Y. W is the weight of the rod.

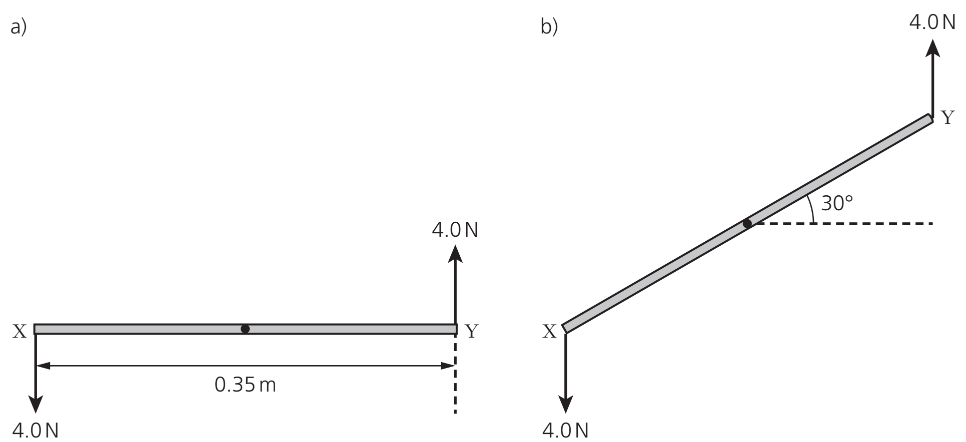
i Show that the weight of the rod is 5.2 N by taking moments about end Y.

ii State the name of the force B and A.

iii Determine the force A.

c The cross-sectional area of the rod is $8.5 \times 10^{-5}\,\text{m}^2$. Calculate the density of the rod.

9 Figs. 4.21a and b show a rod XY of length 0.35 m.



▲ **Figure 4.21**

Forces of 4.0 N act at X and Y.

Calculate the torque on rod XY in:

a Fig. 4.21a,

b Fig. 4.21b.

10 A solid cylinder has a radius of 2.0 cm and length 45 cm. The weight of the cylinder is 49 N. The cylinder is completely immersed in oil of density 920 kg m^{-3}. The apparent weight of the cylinder when immersed in the oil is W_A.

Calculate:

a the density of the cylinder,

b **i** the upthrust on the cylinder when immersed in the oil,

ii W_A,

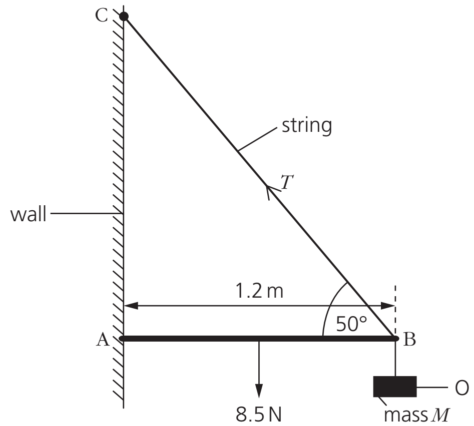
c the pressure difference between the top and bottom of the cylinder when immersed in the oil.

11 The water in a storage tank is 15 m above a water tap in the kitchen of a house. Calculate the pressure of the water leaving the tap.
Density of water = 1.0 × 10^3 kg m^{-3}.

12 Show that the pressure p due to a liquid of density ρ is proportional to the depth h below the surface of the liquid.

13 **a** Define *centre of gravity*. **[2]**

b A uniform rod AB is attached to a vertical wall at A. The rod is held horizontally by a string attached at B and to point C, as shown in Fig. 4.22.



▲ **Figure 4.22**

The angle between the rod and the string at B is 50°. The rod has length 1.2 m and weight 8.5 N. An object O of mass M is hung from the rod at B. The tension T in the string is 30 N.

i Use the resolution of forces to calculate the vertical component of T. **[1]**

ii State the *principle of moments*. **[1]**

iii Use the principle of moments and take moments about A to show that the weight of the object O is 19 N. **[3]**

iv Hence determine the mass M of the object O. **[1]**

c Use the concept of equilibrium to explain why a force must act on the rod at A. **[2]**

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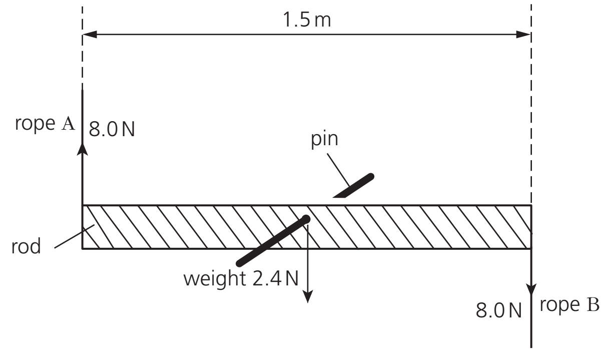
14 a Define the *torque* of a couple. **[2]**

b A uniform rod of length 1.5 m and weight 2.4 N is shown in Fig. 4.23.



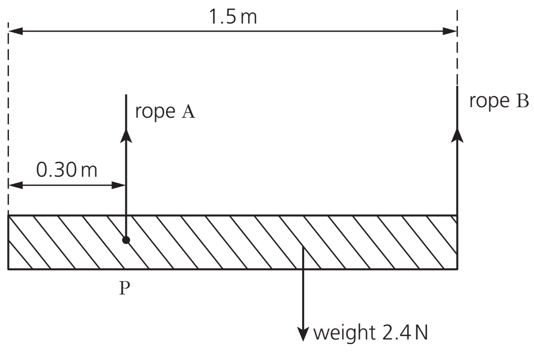
▲ **Figure 4.23**

The rod is supported on a pin passing through a hole in its centre. Ropes A and B provide equal and opposite forces of 8.0 N.

i Calculate the torque on the rod produced by ropes A and B. **[1]**

ii Discuss, briefly, whether the rod is in equilibrium. **[2]**

c The rod in **b** is removed from the pin and supported by ropes A and B, as shown in Fig. 4.24.



▲ **Figure 4.24**

Rope A is now at point P 0.30 m from one end of the rod and rope B is at the other end.

i Calculate the tension in rope B. **[2]**

ii Calculate the tension in rope A. **[1]**

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15 a Define *density*. **[1]**

b A paving slab has a mass of 68 kg and dimensions 50 mm × 600 mm × 900 mm.

i Calculate the density, in kg m^{-3}, of the material from which the paving slab is made. **[2]**

ii Calculate the maximum pressure a slab could exert on the ground when resting on one of its surfaces. **[3]**

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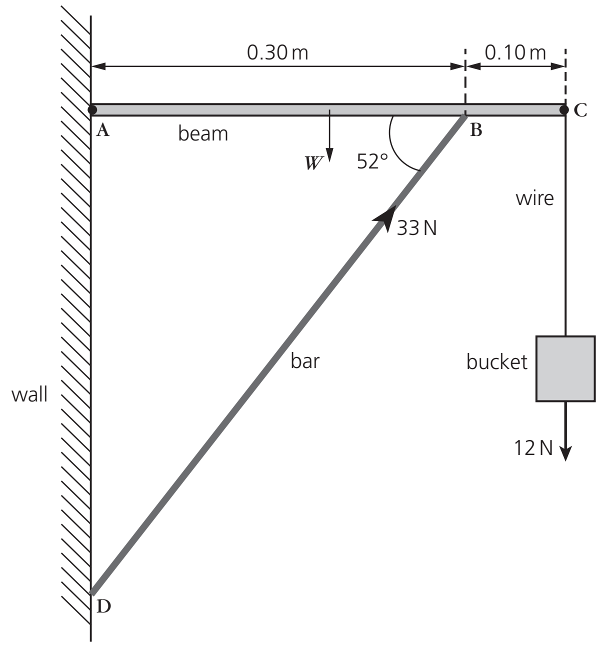
16 a State the two conditions for an object to be in equilibrium. **[2]**

b A uniform beam AC is attached to a vertical wall at end A. The beam held horizontal by a rigid bar BD, as shown in Fig. 4.25.



▲ **Figure 4.25**

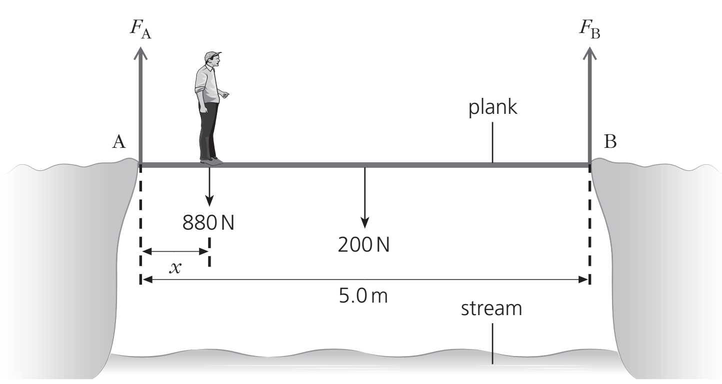
The beam is of length 0.40 m and weight W. An empty bucket of weight 12 N is suspended by a light metal wire from end C. The bar exerts a force on the beam of 33 N at 52° to the horizontal. The beam is in equilibrium.

i Calculate the vertical component of the force exerted by the bar on the beam. **[1]**

ii By taking moments about A, calculate the weight W of the beam. **[3]**

Cambridge International AS and A Level Physics (9702) Paper 22 Q3 parts a bi and ii Oct/Nov 2016

17 A uniform plank AB of length 5.0 m and weight 200 N is placed across a stream, as shown in Fig. 4.26.



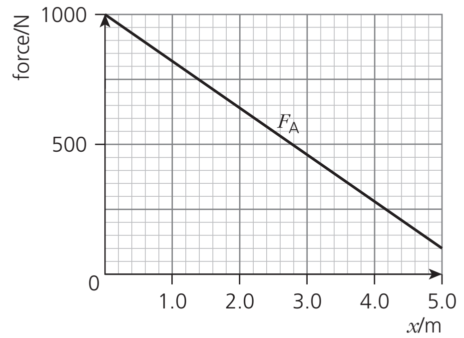
▲ **Figure 4.26**

A man of weight 880 N stands a distance x from end A. The ground exerts a vertical force F_A on the plank at end A and a vertical force F_B on the plank at end B. As the man moves along the plank, the plank is always in equilibrium.

a **i** Explain why the sum of the forces F_A and F_B is constant no matter where the man stands on the plank. **[2]**

ii The man stands a distance $x = 0.50$ m from end A. Use the principle of moments to calculate the magnitude of F_B. **[4]**

b The variation with distance x of force F_A is shown in Fig. 4.27.



▲ **Figure 4.27**

On a copy of Fig. 4.27, sketch a graph to show the variation with x of force F_B. **[3]**

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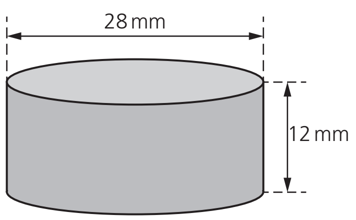
18 A cylindrical disc is shown in Fig. 4.28.



▲ **Figure 4.28**

The disc has diameter 28 mm and thickness 12 mm. The material of the disc has density 6.8×10^3 kg m^{-3}. Calculate, to two significant figures, the weight of the disc. **[4]**

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19 **a** Define *pressure*.

b A solid sphere of diameter 30.0 cm is fully immersed near the surface of the sea. The sphere is released from rest and moves vertically downwards through the seawater. The weight of the sphere is 1100 N. An upthrust U acts on the sphere. The upthrust remains constant as the sphere moves downwards.

The density of the seawater is 1030 kg m^{-3}.

i Calculate the density of the material of the sphere.

ii Briefly explain the origin of the upthrust acting on the sphere.

iii Show that the upthrust U is 140 N.

iv Calculate the initial acceleration of the sphere.

v The viscous (drag) force D acting on the sphere is given by

$D = \frac{1}{2} C\rho\pi r^2 v^2$

where r is the radius of the sphere and v is its speed. ρ is the density of the seawater.

The constant C has no units and is equal to 0.50.

Determine the constant (terminal) speed reached by the sphere.

5 Work, energy and power

Learning outcomes

By the end of this topic, you will be able to:

5.1 Energy conservation

1. understand the concept of work, and recall and use *work done = force × displacement* in the direction of the force
2. recall and apply the principle of conservation of energy
3. recall and understand that the efficiency of a system is the ratio of useful energy output from the system to the total energy input
4. use the concept of efficiency to solve problems
5. define power as work done per unit time
6. solve problems using the relationships $P = W/t$
7. derive $P = Fv$ and use it to solve problems

5.2 Gravitational potential energy and kinetic energy

1. derive, using $W = Fs$, the formula $\Delta E_p = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field
2. recall and use the formula $\Delta E_p = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field
3. derive, using the equations of motion, the formula for kinetic energy $E_k = \frac{1}{2}mv^2$
4. recall and use $E_k = \frac{1}{2}mv^2$

Starting points

★ Know that there are various forms of energy.

★ Understand that energy can be converted from one form to another.

★ Machines enable us to do useful work by converting energy from one form to another.

5.1 Energy conservation

Work

I'm going to work today.'

'Where do you work?'

'I've done some work in the garden.'

'Lots of work was done lifting the box.'

'I've done my homework.'

The words 'work', 'energy' and 'power' are in use in everyday English language but they have a variety of meanings. In physics, they have very precise meanings. The word **work** has a definite interpretation. The vagueness of the term 'work' in everyday speech causes problems for some students when they come to give a precise scientific definition of work.

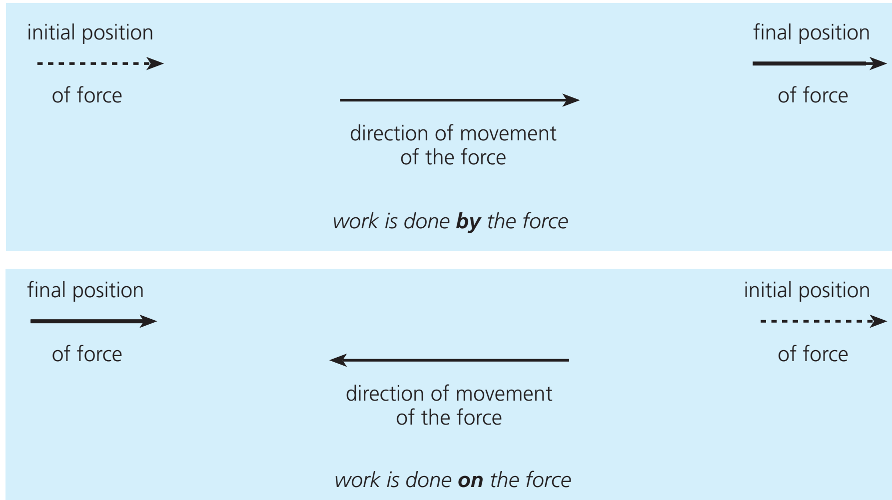
Work is done when a force moves the point at which it acts (the point of application) in the direction of the force.

work done = force × displacement in the direction of the force

▲ **Figure 5.1** The weight-lifter uses a lot of energy to lift the weights but they can be rolled along the ground with little effort.

It is very important to include direction in the definition of work done. A car can be pushed horizontally quite easily but, if the car is to be lifted off its wheels, much more work has to be done and a machine, such as a car-jack, is used.

When a force moves its point of application in the direction of the force, the force does work and the work done by the force is said to be *positive*. Conversely, if the direction of the force is *opposite* to the direction of movement, work is done on the force. This work done is then said to be *negative*. This is illustrated in Figure 5.2.



▲ **Figure 5.2**

▲ **Figure 5.3** The useful work done by the small tug-boat is found using the component of the tension in the rope along the direction of motion of the ship.

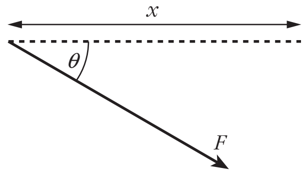
The term **displacement** represents the distance moved in a particular direction.

Displacement is a vector quantity, as is force. However, work done has no direction, only magnitude (size), and is a scalar quantity. It is measured in joules (J).

When a force of one newton moves its point of application by one metre in the direction of the force, one joule of work is done.

work done in joules
= force in newtons × distance moved in metres in the direction of the force

It follows that a joule (J) may be said to be a newton-metre (N m). If the force and the displacement are not both in the same direction, then the component of the force in the direction of the displacement must be found by resolving (see Topic 1.4).



▲ **Figure 5.4**

Consider a force F acting along a line at an angle θ to the displacement, as shown in Figure 5.4. The component of the force along the direction of the displacement is $F\cos\theta$.

$$\text{work done for displacement } x = F\cos\theta \times x = Fx\cos\theta$$

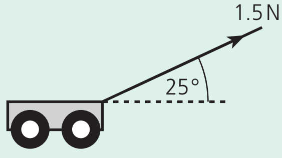
Note that the component $F\sin\theta$ of the force is at right angles to the displacement. Since there is no displacement in the direction of this component, no work is done in that direction.

WORKED EXAMPLE 5A

A child tows a toy by means of a string as shown in Figure 5.5.

The tension in the string is 1.5 N and the string makes an angle of 25° with the horizontal.

Calculate the work done in moving the toy horizontally through a distance of 265 cm.



▲ **Figure 5.5**

Answer

work done = horizontal component of tension × distance moved

$= 1.5\cos 25° \times 265 \times 10^{-2}$

$= \mathbf{3.6\,J}$

Questions

1 A box weighs 45 N. Calculate the work done in lifting the box through a vertical height of:

a 4.0 m

b 67 cm.

2 A force of 36 N acts at an angle of 55° to the vertical. The force moves its point of application by 64 cm in the direction of the force. Calculate the work done by:

a the horizontal component of the force

b the vertical component of the force.

▲ **Figure 5.6** The spring stores energy as it is stretched, releasing the energy as it returns to its original shape.

Energy

In order to wind up a spring, work has to be done because a force must be moved through a distance. When the spring is released, it can do work; for example, making a child's toy move. When the spring is wound, it stores the ability to do work. Anything that is able to do work is said to have energy.

An object that can do work must have **energy**.

An object with no energy is unable to do work. Energy and work are both scalars. Since work done is measured in joules (J), energy is also measured in joules. Table 5.1 lists some typical values of energy rounded to the nearest order of magnitude.

	energy/J
radioactive decay of a nucleus	10^{-13}
sound of speech on ear for 1 second	10^{-8}
moonlight on face for 1 second	10^{-3}
beat of the heart	1
burning a match	10^{3}
large cream cake	10^{6}
energy released from 100 kg of coal	10^{10}
earthquake	10^{19}
energy received on Earth from the Sun in one year	10^{25}
rotational energy of the Milky Way galaxy	10^{50}
estimated energy of formation of the Universe	10^{70}

▲ **Table 5.1** Typical energy values

Energy conversion and conservation

Newspapers sometimes refer to a 'global energy crisis'. In the near future, there may well be a shortage of fossil fuels. Fossil fuels are sources of chemical energy. It would be more accurate to refer to a 'fuel crisis'. When chemical energy is used, the energy is transformed into other forms of energy, some of which are useful and some of which are not. Eventually, all the chemical energy is likely to end up as energy that is no longer useful to us. For example, when petrol is burned in a car engine, some of the chemical energy is converted into the kinetic energy of the car and some is wasted as heat (thermal) energy. When the car stops, its kinetic energy is converted into internal energy in the brakes. The temperature of the brakes increases and thermal energy is released. The outcome is that the chemical energy has been converted into thermal energy which dissipates in the atmosphere and is of no further use. However, the total energy present in the Universe has remained constant. All energy changes are governed by the **law of conservation of energy**. This law states that

Energy cannot be created or destroyed. It can only be converted from one form to another.

There are many different forms of energy and you will meet a number of these during your Cambridge International AS & A Level Physics studies. Some of the more common forms are listed in Table 5.2.

energy	notes
gravitational potential energy	energy due to position of a mass in a gravitational field
kinetic energy	energy due to motion
elastic potential energy	energy stored due to stretching or compressing an object
electric potential energy	energy due to the position of a charge in an electric field
electromagnetic radiation	energy associated with waves in the electromagnetic spectrum
solar energy	electromagnetic radiation from the Sun
internal energy	random kinetic and potential energy of the molecules in an object
chemical energy	energy released during chemical reactions
thermal energy	energy transferred due to temperature difference (sometimes called heat energy)

▲ **Table 5.2** Forms of energy

WORKED EXAMPLE 5B

Map out the energy changes taking place when a battery is connected to a lamp.

Answer

Chemical energy in battery → energy transferred by current in wires → light energy and internal energy of the lamp

Question

3 Map out the following energy changes:

a a child swinging on a swing
b an aerosol can producing hairspray
c a lump of clay thrown into the air which subsequently hits the ground.

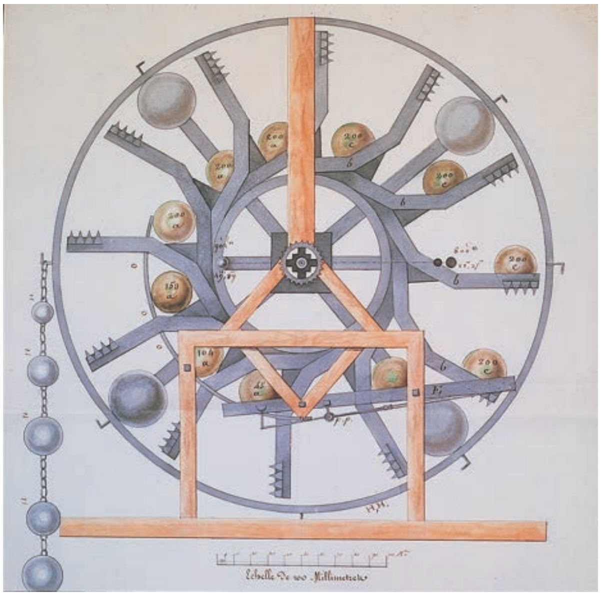
Efficiency

Machines are used to change energy from one form into some other more useful form. In most energy changes some energy is 'lost' as heat (thermal) energy. For example, when a ball rolls down a slope, the total change in gravitational potential energy is not equal to the gain in kinetic energy because heat (thermal) energy has been produced as a result of frictional forces.

Efficiency gives a measure of how much of the total energy may be considered useful and is not 'lost'.

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

Efficiency may be given either as a ratio or as a percentage. Since energy cannot be created, efficiency can never be greater than 100% and a 'perpetual motion' machine is not possible (Figure 5.7).



▲ **Figure 5.7** An attempt to design a machine to get something for nothing by breaking the law of conservation of energy

WORKED EXAMPLE 5C

A man lifts a weight of 480 N through a vertical distance of 3.5 m using a rope and some pulleys. The man pulls on the rope with a force of 200 N and a length of 10.5 m of rope passes through his hands. Calculate the efficiency of the pulley system.

Answer

work done by man = force × distance moved (in direction of the force)
= 200 × 10.5
= 2100 J

work done lifting load = 480 × 3.5
= 1680 J

Since energy is the ability to do work and from the definition of efficiency,

efficiency = work got out/work put in
= 1680/2100
= **0.80 or 80%**

Questions

4 An electric heater transfers energy from the mains supply into thermal energy. Suggest why this process may be 100% efficient.

5 The electric motor of an elevator (lift) uses 630 kJ of energy when raising the elevator and passengers, of total weight 12 500 N, through a vertical height of 29 m. Calculate the efficiency of the elevator.

Power

Machines such as wind turbines or engines do work for us when they change energy into a useful form. However, not only is the availability of useful forms of energy important, but also the rate at which it can be converted from one form to another. The rate of converting energy or using energy is known as **power**.

We have seen that energy is the ability to do work. Consider a family car and a Grand Prix racing car which both contain the same amount of fuel. They are capable of doing the same amount of work, but the racing car is able to travel much faster. This is because the engine of the racing car can convert the chemical energy of the fuel into useful energy at a much faster rate. The engine is said to be more powerful. Power is the rate of doing work. Power is given by the formula

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

The unit of power is the **watt** (symbol W) and is equal to a rate of working of 1 joule per second. This means that a light bulb of power 1 W will convert 1 J of energy to other forms of energy (e.g. light and heat) every second. Table 5.3 gives some values of power rounded to the nearest order of magnitude.

	power/W
power to operate a small calculator	10^{-6}
light power from a torch	10^{-3}
loudspeaker output	10
manual labourer working continuously	100
water buffalo working continuously	10^3
hair dryer	10^3
motor car engine	10^4
electric train	10^6
electricity generating station output	10^9

▲ **Table 5.3** Values of power

Power, like energy, is a scalar quantity.

Care must be taken when referring to power. It is common in everyday language to say that a strong person is 'powerful'. In physics, strength, or force, and power are *not* the same. Large forces may be exerted without any movement and thus no work is done and the power is zero! For example, a large rock resting on the ground is not moving, yet it is exerting a large force.

Consider a force F which moves a distance x at constant velocity v in the direction of the force, in time t. The work done W by the force is given by

$$W = Fx$$

Dividing both sides of this equation by time t gives

$$W/t = F(x/t)$$

Now, W/t is the rate of doing work, i.e. the power P and $x/t = v$. Hence,

$$P = Fv$$

power = force × velocity

WORKED EXAMPLE 5D

A small electric motor is used to lift a weight of 1.5 N through a vertical distance of 120 cm in 2.7 s. Calculate the useful power output of the motor.

Answer

work done = force × displacement in the direction of the force
= 1.5 × 1.2 (the displacement must be in metres)
= 1.8 J

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

= (1.8/2.7)

= **0.67 W**

Questions

6 Calculate the amount of energy converted into thermal energy when an electric fire, rated at 2.4 kW, is left switched on for a time of 3.0 minutes.

7 The output power of the electric motors of a train is 3.6 MW when the train is travelling at 30 m s^{-1}. Calculate the total force opposing the motion of the train.

8 A boy of mass 60 kg runs up a flight of steps in a time of 1.8 s. There are 22 steps and each one is of height 20 cm. Calculate the useful power developed in the boy's legs. (Take the acceleration of free fall as 9.81 m s^{-2}.)

5.2 Gravitational potential energy and kinetic energy

Gravitational potential energy

Potential energy is the ability of an object to do work as a result of its position or shape.

We have already seen that a wound-up spring stores energy. This energy is potential energy because the spring is strained. More specifically, the energy may be called **elastic** (or **strain**) **potential energy** (see Topic 6.2). Elastic potential energy is stored in objects which have had their shape changed elastically. Examples include stretched wires and twisted elastic bands.

▲ **Figure 5.8** The cars on the rollercoaster have stored gravitational potential energy. This energy is released as the cars fall.

Newton's law of gravitation (see Topic 13.2) tells us that all masses attract one another. We rely on the force of gravity to keep us on Earth! When two masses are pulled apart, work is done on them and so they gain **gravitational potential energy**. If the masses move closer together, they lose gravitational potential energy.

Gravitational potential energy is energy possessed by a mass due to its position in a gravitational field.

Changes in gravitational potential energy are of particular importance for an object near to the Earth's surface because we frequently do work raising masses and, conversely, the energy stored is released when the mass is lowered again. The gravitational field near the surface of the Earth is taken to be uniform and so the acceleration of free fall g has a constant value, 9.81 m s^{-2}. An object of mass m near the Earth's surface has weight mg (see Topic 3.1). This weight is the force with which the Earth attracts the mass (and the mass attracts the Earth). If the mass moves a *vertical* distance h,

$$\begin{aligned} \text{work done} &= Fs \\ &= \text{force} \times \text{displacement in the direction of the force} \\ &= mgh \end{aligned}$$

When the mass is raised, the work done is stored as gravitational potential energy and this energy can be recovered when the mass falls.

Change in gravitational potential energy $\Delta E_p = mg\Delta h$

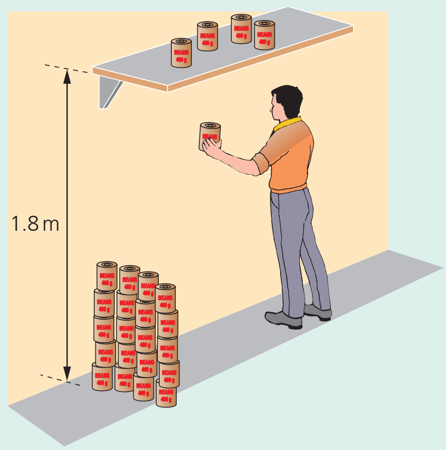
It is important to remember that, for the energy to be measured in joules, the mass m must be in kilograms, the acceleration of free fall g in metres (second)$^{-2}$ and the change in height Δh in metres.

Notice that a zero point of gravitational potential energy has not been stated. We are concerned with *changes* in potential energy when a mass rises or falls.

WORKED EXAMPLE 5E

1 Map out the energy changes taking place when an object moves from its lowest point to its highest point on the end of a vertical spring after the spring is stretched.

2 A shop assistant stacks a shelf with 25 tins of beans, each of mass 472 g (Figure 5.9). Each tin has to be raised through a distance of 1.8 m. Calculate the gravitational potential energy gained by the tins of beans, given that the acceleration of free fall is 9.81 m s^{-2}.



▲ **Figure 5.9**

Answers

1 (maximum) elastic potential energy in stretched spring → gravitational potential energy and kinetic energy and (reduced) elastic potential energy of object (as it moves up) → (maximum) gravitational potential energy (zero kinetic energy) and elastic potential energy in the compressed spring at its highest point

2 total mass raised = 25 × 472 = 11 800 g

= 11.8 kg

increase in potential energy = $m \times g \times h$

= 11.8 × 9.81 × 1.8

= **210 J (to 2 significant figures)**

Question

9 The acceleration of free fall is 9.81 m s^{-2}. Calculate the change in gravitational potential energy when:

a a person of mass 70 kg climbs a cliff of height 19 m

b a book of mass 940 g is raised vertically through a distance of 130 cm

c an aircraft of total mass 2.5×10^5 kg descends by 980 m.

Kinetic energy

As an object falls, it loses gravitational potential energy and, in so doing, it speeds up. Energy is associated with a moving object. In fact, we know that a moving object can be made to do work as it slows down. For example, a moving hammer hits a nail and, as it stops, does work to drive the nail into a piece of wood.

Kinetic energy is energy due to motion.

Consider an object of mass m moving with a constant acceleration a. In a distance s, the object accelerates from velocity u to velocity v. Then, by referring to the equations of motion (see Topic 2),

$$v^2 = u^2 + 2as$$

By Newton's law (see Topic 3), the force F giving rise to the acceleration a is given by

$$F = ma$$

Combining these two equations,

$$v^2 = u^2 + 2(F/m)s$$

Re-arranging,

$$mv^2 = mu^2 + 2Fs$$

$$2Fs = mv^2 - mu^2$$

$$Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

By definition, the term Fs is the work done by the force moving a distance s. Therefore, since Fs represents work done, then the other terms in the equation, $\tfrac{1}{2}mv^2$ and $\tfrac{1}{2}mu^2$, must also have the units of work done, or energy (see Topic 1). The magnitude of each of these terms depends on velocity squared and so $\tfrac{1}{2}mv^2$ and $\tfrac{1}{2}mu^2$ are terms representing energy which depends on velocity (or speed).

The kinetic energy E_k of an object of mass m moving with speed v is given by $E_k = \tfrac{1}{2}mv^2$.

For the kinetic energy to be in joules, mass must be in kilograms and speed in metres per second.

The full name for the term $E_k = \tfrac{1}{2}mv^2$ is *translational kinetic energy* because it is energy due to an object moving in a straight line. It should be remembered that rotating objects also have kinetic energy and this form of energy is known as *rotational kinetic energy*.

▲ **Figure 5.10** When the mass falls, it gains kinetic energy and drives the pile into the ground.

WORKED EXAMPLE 5F

Calculate the kinetic energy of a car of mass 900 kg moving at a speed of 20 m s^{-1}. State the form of energy from which the kinetic energy is derived.

Answer

$$\text{kinetic energy} = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2} \times 900 \times 20^2$$

$$= \mathbf{1.8 \times 10^5\,J}$$

This energy is derived from the chemical energy of the fuel.

Questions

10 Calculate the kinetic energy of a car of mass 800 kg moving at 100 kilometres per hour.

11 A cycle and cyclist have a combined mass of 80 kg and are moving at $5.0\,\mathrm{m\,s^{-1}}$. Calculate:

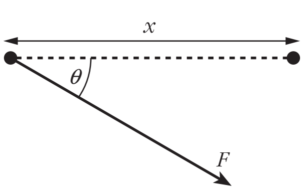
a the kinetic energy of the cycle and cyclist

b the increase in kinetic energy for an increase in speed of $5.0\,\mathrm{m\,s^{-1}}$.

SUMMARY

- When a force moves its point of application in the direction of the force, work is done.
- Work done = $Fx\cos\theta$, where θ is the angle between the direction of the force F and the displacement x.
- Energy is needed to do work; energy is the ability to do work.
- Energy cannot be created or destroyed. It can only be converted from one form to another.
- Efficiency = useful energy output/total energy input
- Power is defined as the rate of doing work or work done per unit time:
  power = work done/time taken, $P = W/t$.
- The unit of power is the watt (W).
  1 watt = 1 joule per second
- Power = force × velocity
- Potential energy is the energy stored in an object due to its position or shape; examples are elastic potential energy and gravitational potential energy.
- When an object of mass m moves vertically through a distance Δh in a uniform gravitational field, then the change in gravitational potential energy is given by: $\Delta E_p = mg\Delta h$ where g is the acceleration of free fall.
- Kinetic energy is the energy stored in an object due to its motion.
- For an object of mass m moving with speed v, the kinetic energy is given by: $E_k = \frac{1}{2}mv^2$.

END OF TOPIC QUESTIONS



▲ **Figure 5.11**

1 A force F moves its point of application by a distance x in a direction making an angle θ with the direction of the force, as shown in Fig. 5.11.

The force does an amount W of work. Copy and complete the following table.

F/N	x/m	θ/°	W/J
15	6.0	0	
15	6.0	90	
15	6.0	30	
46		23	6.4
2.4×10^3	1.6×10^2		3.1×10^5
	2.8	13	7.1×10^3

2 An elastic band is stretched so that its length increases by 2.4 cm. The force required to stretch the band increases linearly from 6.3 N to 9.5 N. Calculate:

a the average force required to stretch the elastic band,

b the work done in stretching the band.

3 Name each of the following types of energy:

a energy used in muscles,

b energy of water in a mountain lake,

c energy captured by a wind turbine,

d energy produced when a firework explodes,

e energy of a compressed gas.

4 A child of mass 35 kg moves down a sloping path on a skate board. The sloping path makes an angle of 4.5° with the horizontal. The constant speed of the child along the path is $6.5\,\mathrm{m\,s^{-1}}$. Calculate:

a the vertical distance through which the child moves in 1.0 s,

b the rate at which potential energy is being lost ($g = 9.81\,\mathrm{m\,s^{-2}}$).

5 A stone of mass 120 g is dropped down a well. The surface of the water in the well is 9.5 m below ground level. The acceleration of free fall of the stone is $9.81\,\text{m s}^{-2}$. Calculate, for the stone falling from ground level to the water surface:

a the loss of potential energy,

b its speed as it hits the water, assuming all the potential energy has been converted into kinetic energy.

6 An aircraft of mass 3.2×10^5 kg accelerates along a runway. Calculate the change in kinetic energy, in MJ, when the aircraft accelerates:

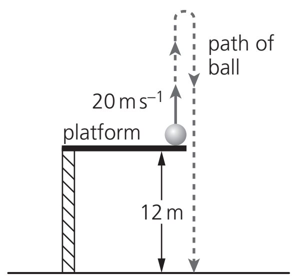
a from zero to $10\,\text{m s}^{-1}$,

b from $30\,\text{m s}^{-1}$ to $40\,\text{m s}^{-1}$,

c from $60\,\text{m s}^{-1}$ to $70\,\text{m s}^{-1}$.

7 In order to strengthen her legs, an athlete steps up on to a box and then down again 30 times per minute. The girl has mass 50 kg and the box is 35 cm high. The exercise lasts 4.0 minutes and as a result of the exercise, her leg muscles generate 120 kJ of heat energy. Calculate the efficiency of the leg muscles ($g = 9.81\,\text{m s}^{-2}$).

8 By accident, the door of a refrigerator is left open. Use the law of conservation of energy to explain whether the temperature of the room will rise, stay constant or fall after the refrigerator has been working for a few hours.



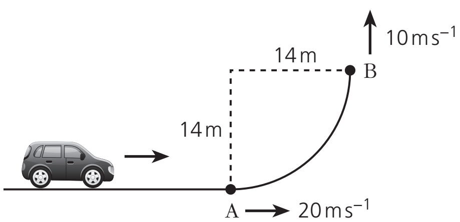
▲ **Figure 5.12**

9 A ball of mass 0.50 kg is thrown vertically upwards with a speed of $20\,\text{m s}^{-1}$. It is thrown from a platform 12 m above the ground reaches a maximum height before the ball falls to the ground, as shown in Fig. 5.12. What is the kinetic energy of the ball just as it hits the ground? Assume air resistance is negligible.

A 59 J **B** 100 J **C** 160 J **D** 260 J

10 A car moves along a track that is in a vertical plane and follows an arc of a circle of radius 14 m, as shown in Fig. 5.13. The car has a mass 500 kg moves past point A with a speed of $20\,\text{m s}^{-1}$. The car has a speed of $10\,\text{m s}^{-1}$ at B. What is the average resistive force acting on the car as it moves from A to B?

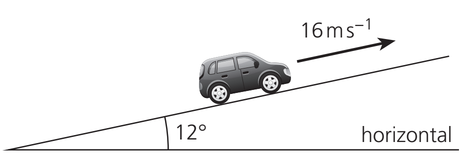
A 290 N **B** 450 N **C** 3400 N **D** 5400 N



▲ **Figure 5.13**

11 A car of mass 1500 kg moves up an incline at a constant speed, as shown in Fig. 5.14. The incline is at 12° to the horizontal. What is the power provided by the engine of the car for it to travel up the incline at $16\,\text{m s}^{-1}$?

A 5.0 kW **B** 23 kW **C** 49 kW **D** 230 kW



▲ **Figure 5.14**

12 A car travels in a straight line at speed v along a horizontal road. The car moves against a resistive force F given by the equation

$$F = 400 + kv^2$$

where F is in newtons, v in $m\,s^{-1}$ and k is a constant.
At speed v = 15 $m\,s^{-1}$, the resistive force F is 1100 N.

a Calculate, for this car:

i the power necessary to maintain the speed of 15 $m\,s^{-1}$,

ii the total resistive force at a speed of 30 $m\,s^{-1}$,

iii the power required to maintain the speed of 30 $m\,s^{-1}$.

b Determine the energy expended in travelling 1.2 km at a constant speed of:

i 15 $m\,s^{-1}$,

ii 30 $m\,s^{-1}$.

c Using your answers to part **b**, suggest why, during a fuel shortage, the maximum permitted speed of cars may be reduced.

13 a Distinguish between gravitational potential energy and elastic potential energy. **[2]**

b A ball of mass 65 g is thrown vertically upwards from ground level with a speed of 16 $m\,s^{-1}$. Air resistance is negligible.

i Calculate, for the ball:

1 the initial kinetic energy, **[2]**

2 the maximum height reached. **[2]**

ii The ball takes time t to reach maximum height. For time $t/2$ after the ball has been thrown, calculate the ratio:

$$\frac{\text{potential energy of ball}}{\text{kinetic energy of ball}}$$ **[3]**

iii State and explain the effect of air resistance on the time taken for the ball to reach maximum height. **[1]**

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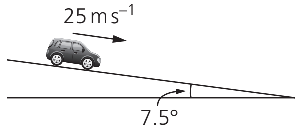

Figure 5.15

14 a Explain what is meant by *work done*. **[1]**

b A car is travelling along a road that has a uniform downhill gradient, as shown in Fig. 5.15.

The car has a total mass of 850 kg. The angle of the road to the horizontal is 7.5°. Calculate the component of the weight of the car down the slope. **[2]**

c The car in **b** is travelling at a constant speed of 25 $m\,s^{-1}$. The driver then applies the brakes to stop the car. The constant force resisting the motion of the car is 4600 N.

i Show that the deceleration of the car with the brakes applied is 4.1 $m\,s^{-2}$. **[2]**

ii Calculate the distance the car travels from when the brakes are applied until the car comes to rest. **[2]**

iii Calculate:

1 the loss of kinetic energy of the car, **[2]**

2 the work done by the resisting force of 4600 N. **[1]**

iv The quantities in **iii** part **1** and in **iii** part **2** are not equal. Explain why these two quantities are not equal. **[1]**

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15 A ball is thrown vertically down towards the ground with an initial velocity of 4.23 $m\,s^{-1}$. The ball falls for a time of 1.51 s before hitting the ground. Air resistance is negligible.

a i Show that the downwards velocity of the ball when it hits the ground is 19.0 $m\,s^{-1}$. **[2]**

ii Calculate, to three significant figures, the distance the ball falls to the ground. **[2]**

b The ball makes contact with the ground for 12.5 ms and rebounds with an upwards velocity of 18.6 m s^{-1}. The mass of the ball is 46.5 g.

i Calculate the average force acting on the ball on impact with the ground. [4]

ii Use conservation of energy to determine the maximum height the ball reaches after it hits the ground. [2]

c State and explain whether the collision the ball makes with the ground is elastic or inelastic. [1]

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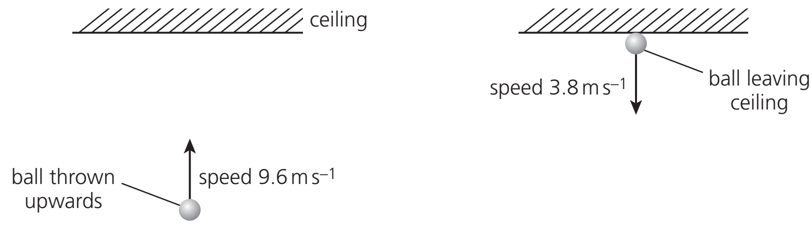
16 A ball is thrown vertically upwards towards a ceiling and then rebounds, as illustrated in Fig. 5.16.



▲ **Figure 5.16**

The ball is thrown with speed 9.6 m s^{-1} and takes a time of 0.37 s to reach the ceiling. The ball is then in contact with the ceiling for a further time of 0.085 s until leaving it with a speed of 3.8 m s^{-1}. The mass of the ball is 0.056 kg.
Assume that air resistance is negligible.

a Show that the ball reaches the ceiling with a speed of 6.0 m s^{-1}. [1]

b Calculate the height of the ceiling above the point from which the ball was thrown. [2]

c Calculate:

i the increase in gravitational potential energy of the ball for its movement from its initial position to the ceiling, [2]

ii the decrease in kinetic energy of the ball while it is in contact with the ceiling. [2]

d State how Newton's third law applies to the collision between the ball and the ceiling. [2]

e Calculate the change in momentum of the ball during the collision. [2]

f Determine the magnitude of the average force exerted by the ceiling on the ball during the collision. [2]

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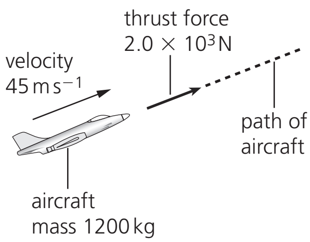

▲ **Figure 5.17**

17 a i Define *power*. [1]

ii State what is meant by *gravitational potential energy*. [1]

b An aircraft of mass 1200 kg climbs upwards with a constant velocity of 45 m s^{-1}, as shown in Fig. 5.17.
The aircraft's engine produces a thrust force of 2.0×10^3 N to move the aircraft through the air. The rate of increase in height of the aircraft is 3.3 m s^{-1}.

i Calculate the power produced by the thrust force. [2]

ii Determine, for a time interval of 3.0 minutes:

1 the work done by the thrust force to move the aircraft, [2]

2 the increase in gravitational potential energy of the aircraft, [2]

3 the work done against air resistance. [1]

iii Use your answer in **b ii** part **3** to calculate the force due to air resistance acting on the aircraft. [1]

iv With reference to the motion of the aircraft, state and explain whether the aircraft is in equilibrium. [2]

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6 Deformation of solids

Learning outcomes

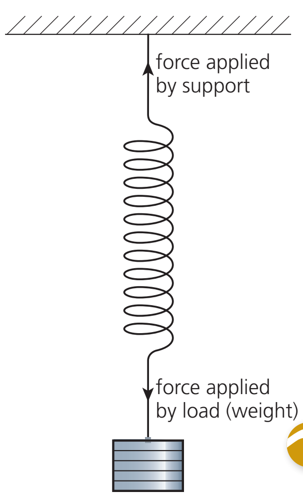
By the end of this topic, you will be able to:

6.1 Stress and strain

1. understand that deformation is caused by tensile or compressive forces (forces and deformations will be assumed to be in one dimension only)
2. understand and use the terms load, extension, compression and limit of proportionality
3. recall and use Hooke's law
4. recall and use the formula for the spring constant $k = F/x$
5. define and use the terms stress, strain and the Young modulus
6. describe an experiment to determine the Young modulus of a metal in the form of a wire

6.2 Elastic and plastic behaviour

1. understand and use the terms elastic deformation, plastic deformation and elastic limit
2. understand that the area under the force–extension graph represents the work done
3. determine the elastic potential energy of a material deformed within its limit of proportionality from the area under the force–extension graph
4. recall and use $E_p = \frac{1}{2}Fx = \frac{1}{2}kx^2$ for a material deformed within its limit of proportionality



▲ **Figure 6.1** A loaded helical spring

Starting points

- When forces are applied to a solid object, its shape or size may change.
- The change of shape or size is called **deformation**.
- The deformation is called a **tensile** deformation if an object is stretched or a **compressive** deformation if the object squeezed/compressed.
- The load or force that stretches a wire is called a tensile force.
- Work done = force × displacement in the direction of the force
- Potential energy is the energy stored in an object due to its position (relative to other bodies) or shape.

6.1 Stress and strain

Hooke's law

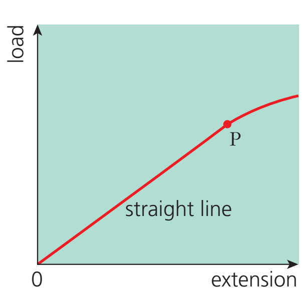
A helical spring, attached to a fixed point, hangs vertically and has weights attached to its lower end, as shown in Figure 6.1. As the magnitude of the weight is increased the spring becomes longer. The increase in length or deformation of the spring is called the **extension** of the spring and the weight attached to the spring is called the **load**.

The extension is equal to the extended length – natural/original length.

The load is the tensile force that causes the extension.

Note that a force acts on the spring at each end. An upwards force acts on the spring from the support at the fixed point as well as the load and the spring is in equilibrium.

Figure 6.2 shows a load against extension for the spring. The section of the line from the origin to the point P is straight. In this region, the extension of the spring is proportional to the load. The point P is referred to as the **limit of proportionality**.



▲ **Figure 6.2** Extension of a loaded spring

The fact that there is a proportional relationship between load and extension is expressed in **Hooke's law**. It should be appreciated that, although we have used a spring as an illustration, the law applies to any object, provided the limit of proportionality has not been exceeded, for example a wire.

Hooke's law states that, provided the limit of proportionality is not exceeded, the extension of an object is proportional to the applied load.

The law can be expressed in the form of an equation

force $F \propto$ extension e

Removing the proportionality sign gives

$F = ke$

where k is a constant, known as the **spring constant** (or force constant).

The spring constant is the force per unit extension, $k = F/x$.

The unit of the constant is newton per metre (N m^{-1}).

The spring constant is different for each spring or wire.

It should be noted that if a load (compressive force) causes the object to be compressed then Hooke's law still applies up to the limit of proportionality. The **compression** is then equal to the original length – reduced length.

WORKED EXAMPLE 6A

An elastic cord has an original length of 25 cm. When the cord is extended by applying a force at each end, the length of the cord becomes 40 cm for forces of 0.75 N. Calculate the force constant of the cord.

Answer

extension of cord = 15 cm, the force causing the extension (the load) is 0.75 N

force constant = 0.75/0.15 (extension in metres)

= **5.0 N m^{-1}**

Questions

1 Explain what is meant by the *limit of proportionality*.

2 Calculate the spring constant for a spring which extends by a distance of 3.5 cm when a load of 14 N is hung from its end.

3 A steel wire extends by 1.5 mm when it is under a tensile force of 45 N. Calculate:

a the spring constant of the wire

b the tensile force required to produce an extension of 1.8 mm, assuming that the limit of proportionality is not exceeded.

The Young modulus

The spring constant is different for each specimen of a material that has a different shape or size. The extension produced by a given force depends on other factors. For example, the extension of a wire depends on its length and diameter as well as the type of material. In order to compare materials a quantity is defined which enables the extensions to be calculated if the dimensions of a specimen of a material are known. This quantity is called the **Young modulus**.

When an object has its shape or size changed by forces acting on it, strain is produced in the object. The strain is a measure of the extent of the deformation. When a tensile force acts on an object such as a wire or spring the deformation is a change in length.

If an object of original length L_o is extended by an amount e, the tensile **strain** (ε) is defined as

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$

$\varepsilon = e/L_o$

Strain is the ratio of two lengths and does not have a unit.

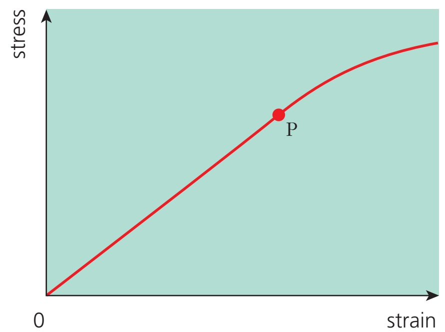
The strain produced within an object is caused by a **stress**. In our case, we are dealing with changes in length and so the stress is referred to as a **tensile stress**. When a tensile force F acts normally to an area A, the stress (σ) is given by

$$\text{stress} = \frac{\text{force}}{\text{area normal to the force}}$$

$\sigma = F/A$

The unit of tensile stress is newton per square metre (N m^{-2}). This unit is also the unit of pressure and so an alternative unit for stress is the pascal (Pa).

In Figure 6.2, we plotted a graph of load against extension. Since load is related to stress and extension is related to strain, a graph of stress plotted against strain would have the same basic shape, as shown in Figure 6.3. Once again, there is a straight line region between the origin and P, the limit of proportionality. In this region, changes of strain with stress are proportional.



▲ **Figure 6.3 Stress–strain graph**

In the region where the changes are proportional, it can be seen that

stress ∝ strain

or, removing the proportionality sign,

stress = E × strain

The constant E is known as the Young modulus of the material.

$$\text{Young modulus } E = \frac{\text{stress}}{\text{strain}}$$

provided the limit of proportionality is not exceeded.

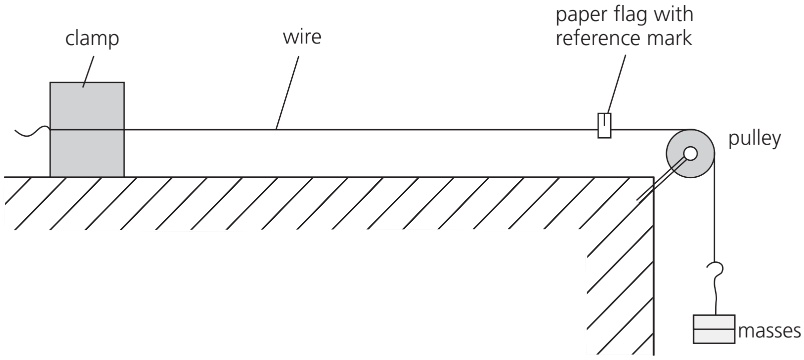
The unit of the Young modulus is the same as that for stress because strain is a ratio and has no unit. Hence the unit of E is the newton per square metre (N m^{-2}) or the pascal (Pa).

This definition for the Young modulus can be used to derive the expression

$E = (F/A) \times (L_o/e) = (FL_o)/(Ae)$

This expression is used to determine the Young modulus of a metal.

The Young modulus of a metal in the form of a wire may be measured by applying loads to a wire and measuring the extensions caused. The original length and the cross-sectional area must also be measured. A suitable laboratory arrangement is shown in Figure 6.4. A copper wire is often used. This is because, for wires of the same diameter under the same load, a copper wire will give larger, more measurable, extensions than a steel wire. (Why is this?) A paper flag with a reference mark on it is attached to the wire at a distance of approximately one metre from the clamped end. The original length L_o is measured from the clamped end to the reference mark, using a metre rule. The diameter d of the wire is measured using a micrometer screw gauge, and the cross-sectional area A calculated from $A = \frac{1}{4}\pi d^2$. Extensions e are measured as masses m are added to the mass-carrier. (Think of a suitable way of measuring these extensions.) The load F is calculated from $F = mg$. A graph of F (y-axis) against e (x-axis) has gradient EA/L_o, so the Young modulus E is equal to gradient × (L_o/A). This method is only applicable where Hooke's law is valid and the graph obtained is a straight line. Care should be taken not to exceed the limit of proportionality when extending the wire.



▲ **Figure 6.4** Simple experiment to measure the Young modulus of a wire

Some values of the Young modulus for different materials are shown in Table 6.1.

material	Young modulus E/Pa
aluminium	7.0×10^{10}
copper	1.1×10^{11}
steel	2.1×10^{11}
glass	4.1×10^{10}
rubber	5.0×10^{8}

▲ **Table 6.1** Young modulus for different materials

WORKED EXAMPLE 6B

A steel wire of diameter 1.0 mm and length 2.5 m is suspended from a fixed point and a mass of weight 45 N is suspended from its free end. The Young modulus of the material of the wire is 2.1×10^{11} Pa. Assuming that the limit of proportionality of the wire is not exceeded, calculate:

a the applied stress
b the strain
c the extension of the wire.

Answers

a area = $\pi \times (0.5 \times 10^{-3})^2$
= $7.9 \times 10^{-7}\,\text{m}^2$
stress = force/area
= $45/7.9 \times 10^{-7}$
= $\mathbf{5.7 \times 10^{7}\,Pa}$

b strain = stress/Young modulus
= $5.7 \times 10^{7}/2.1 \times 10^{11}$
= $\mathbf{2.7 \times 10^{-4}}$

c extension = strain × length
= $2.7 \times 10^{-4} \times 2.5$
= $6.8 \times 10^{-4}\,\text{m}$
= **0.68 mm**

Questions

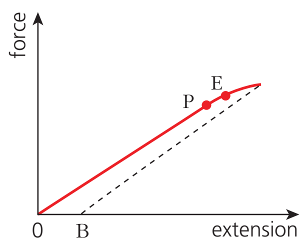
4 A copper wire of diameter 1.78 mm and length 1.40 m is suspended from a fixed point and a mass of weight 32.0 N is suspended from its free end. The Young modulus of the material of the wire is 1.10×10^{11} Pa. Assuming that the limit of proportionality of the wire is not exceeded, calculate:

- a the applied stress
- b the strain
- c the extension of the wire.

5 An elastic band of area of cross-section $2.0\,mm^2$ has an original length of 8.0 cm. When stretched by a force of 0.40 N, its length becomes 8.3 cm. Calculate the Young modulus of the elastic.

6.2 Elastic and plastic behaviour

When an object has its shape or size changed by forces acting on it a deformation is said to have been produced. Figure 6.5 shows the force against extension graph for a wire. For small forces, when the force is removed, the wire returns to its original length. The wire is said to have undergone an **elastic deformation**.



▲ **Figure 6.5** Force against extension graph

In an elastic deformation, an object returns to its original shape and size when the force on it is removed.

The point E on Figure 6.5 is referred to as the **elastic limit** and is usually just beyond the point P the limit of proportionality.

The elastic limit is the maximum force that can be applied to a wire/spring such that the wire/spring returns to its original length when the force is removed.

If the force is increased greatly, the spring will change its shape or size permanently. The wire/spring is deformed permanently and the deformation is said to be **plastic** for points beyond E. The graph in Figure 6.5 follows the dashed line as the force is removed. At point B the wire has a permanent extension when the force is zero.

In a **plastic deformation**, an object does not return to its original shape and size when the force on it is removed.

Elastic potential energy

Work has to be done by the force acting on the object to cause its deformation and hence produce a strain. The work done to produce the strain is stored in the object as potential energy. This particular form of potential energy is called strain potential energy, or simply **strain energy**. All the stored potential energy is recovered when the force is removed from the object provided the force applied is within the elastic limit. The stored potential energy is then called **elastic potential energy**.

Elastic potential energy (strain energy) is energy stored in an object due to change of shape or size, which is completely recovered when the force causing deformation is removed.

The work done by a force that is greater than the elastic limit is stored as potential energy in the object but the energy is not completely recovered when the force is removed.

Consider the spring shown in Figure 6.1. To produce a final extension e, the force applied at the lower end of the spring increases with extension from zero to a value F. Provided the spring is deformed within its limit of proportionality, the extension is directly proportional to this maximum force and the average force is $\frac{1}{2}F$. The work done W by the force is therefore

$$W = \text{average force} \times \text{extension (see Topic 5.1)}$$
$$= \tfrac{1}{2}Fe$$

The work done is equal to the elastic potential energy stored in the spring, E_p. Hence for a spring deformed within its limit of proportionality,

$$E_p = \frac{1}{2}Fe$$

However, the force constant k is given by the equation

$$F = ke$$

Therefore, substituting for F,

$$E_p = \frac{1}{2}ke^2$$

The energy is in joules if k is in newtons per metre and e is in metres.

A graph of force (y-axis) against extension (x-axis) enables the work done to be found even when the graph is not linear (see Figure 6.6). We have shown that for a spring deformed within its limit of proportionality, work done is given by

$$W = \frac{1}{2}Fe$$

The expression $\frac{1}{2}Fe$ represents the area between the straight line on Figure 6.6 and the x-axis. This relationship only applies when the extension is proportional to the force, but the work done in deforming is still given by the area under the line if the graph is curved. This means that work done is represented by the area under the line on a graph of force (y-axis) plotted against extension (x-axis).

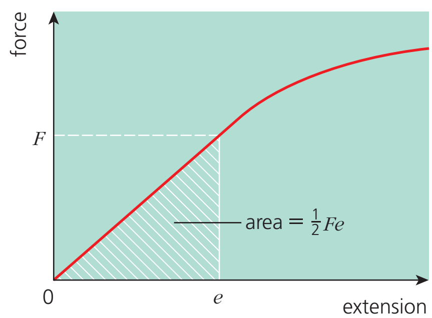


Figure 6.6 Work done is given by the area under the graph.

For any deformation, the area under the force–extension graph represents the work done.

The work done and hence the elastic potential energy is area under the graph ($\frac{1}{2}Fe$). The force (in N) × extension (in m) gives the work done (energy stored) in J, provided the limit of proportionality is not exceeded.

WORKED EXAMPLE 6C

A spring has a spring constant 65 N m^{-1} and is extended within the limit of proportionality by 1.2 cm. Calculate the elastic potential energy stored in the spring.

Answer

$$\text{elastic potential energy } W = \frac{1}{2}ke^2$$
$$= \frac{1}{2} \times 65 \times (1.2 \times 10^{-2})^2$$
$$= 4.7 \times 10^{-3}\,\text{J}$$

Questions

6 Explain what is meant by *extended elastically*.

7 A wire has a force constant of 5.5×10^4 N m^{-1}. It is extended within the limit of proportionality by 1.4 mm. Calculate the elastic potential energy stored in the wire.

8 A rubber band has a force constant of 180 N m^{-1}. The work done in extending the band is 0.16 J. Calculate the extension of the band.

SUMMARY

- Forces on an object can cause tensile deformation (stretching) or compressive deformation (squeezing).
- The limit of proportionality is the point up to which the force is proportional to the extension.
- Hooke's law states extension is proportional to force provided the limit of proportionality is not exceeded.
- The spring constant (force constant) k is the ratio of force to extension, $k = F/x$.
- Tensile strain = extension/original length.
- Tensile stress = force/cross-sectional area; stress has units $\mathrm{N\,m^{-2}}$ or Pa.
- The Young modulus of a material is defined as Young modulus = stress/strain; the units of the Young modulus are $\mathrm{N\,m^{-2}}$, or Pa.
- The Young modulus of a metal in the form of a wire can be found by applying loads to a wire and measuring the extensions caused. The original length and diameter of the wire are also measured. The Young modulus is determined using the gradient of a force-extension graph or directly from the gradient of the stress/strain curve.
- An elastic deformation occurs when an object returns to its original shape and size when the force is removed from it.
- In a plastic deformation, an object does not return to its original shape and size when the force on it is removed.
- The elastic limit is the maximum force that can be applied to a wire/spring such that the wire/spring returns to its original length when the force is removed.
- Elastic potential energy is energy stored in an object due to change of shape or size that is completely recovered when the force causing deformation is removed.
- Elastic potential energy, $E_p = \frac{1}{2}Fe = \frac{1}{2}ke^2$ for an object deformed within its limit of proportionality.
- The area under the force–extension graph represents the work done. This applies for forces within the elastic limit and greater than the elastic limit.

END OF TOPIC QUESTIONS

1 What are the SI base units of the spring constant?

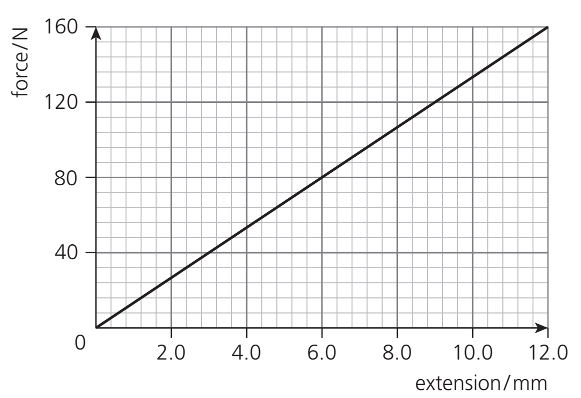
A N **B** N m **C** $\mathrm{N\,m^{-1}}$ **D** $\mathrm{N\,m^{-2}}$

2 A uniform metal wire is fixed at one end so that it hangs vertically from the ceiling. A load of 5 kg is placed on the lower end and the wire extends. The original length of the wire was 1.5 m. The diameter of the wire is 0.40 mm and the metal of the wire has a Young modulus of $2 \times 10^{11}\,\mathrm{N\,m^{-2}}$. What is the extension of the wire?

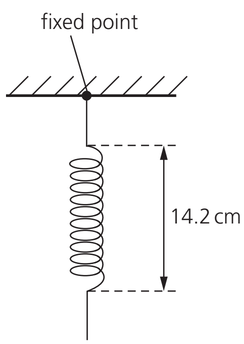
A 0.07 mm **B** 0.7 mm **C** 0.3 mm **D** 3 mm

3 Fig. 6.7 shows the extension of a wire as forces from 0 to 160 N are applied. What is the work done extending the wire with forces from 80 N to 160 N?

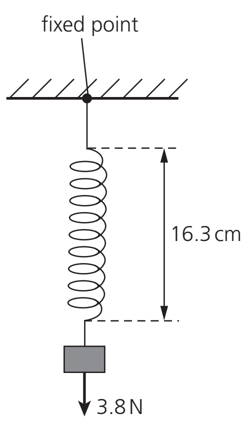
A 0.48 J **B** 0.72 J **C** 1.4 J **D** 1.9 J



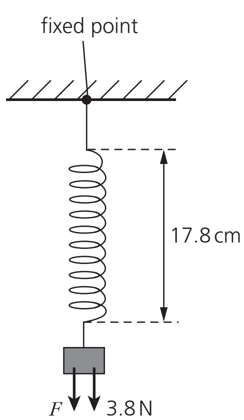
▲ **Figure 6.7**



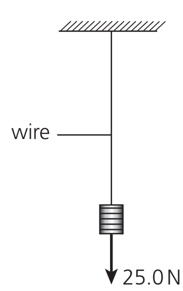
▲ **Figure 6.8**



▲ **Figure 6.9**



▲ **Figure 6.10**



▲ **Figure 6.11**

4 A spring has an original length of 12.4 cm. When a load of 4.5 N is suspended from the spring, its length becomes 13.3 cm. Calculate:

a the spring constant of the spring,

b the length of the spring for a load of 3.5 N.

5 The elastic cord of a catapult has a force constant of 700 N m^{-1}. Calculate the elastic potential energy stored in the elastic cord when it is extended by 15 cm.

6 Two wires each have length 1.8 m and diameter 1.2 mm. One wire has a Young modulus of 1.1×10^{11} Pa and the other 2.2×10^{11} Pa. One end of each wire is attached to the same fixed point and the other end of each wire is attached to the same load of 75 N so that each has the same extension. Assuming that the limit of proportionality of the wires is not exceeded, calculate the extension of the wires.

7 Explain what is meant by *plastic deformation*.

8 a Explain what is meant by *elastic potential energy (strain energy)*. [2]

b A spring that obeys Hooke's law has a spring constant k.
Show that the energy E stored in the spring when it has been extended elastically by an amount x is given by $E = \frac{1}{2}kx^2$. [3]

c A light spring of unextended length 14.2 cm is suspended vertically from a fixed point, as shown in Fig. 6.8.
A mass of weight 3.8 N is hung from the end of the spring, as shown in Fig. 6.9. The length of the spring is now 16.3 cm.
An additional force F then extends the spring so that its length becomes 17.8 cm, as shown in Fig. 6.10.
The spring obeys Hooke's law and the elastic limit of the spring is not exceeded.

i Show that the spring constant of the spring is 1.8 N cm^{-1}. [1]

ii For the extension of the spring from a length of 16.3 cm to a length of 17.8 cm,

1 calculate the change in the gravitational potential energy of the mass on the spring, [2]

2 show that the change in elastic potential energy of the spring is 0.077 J, [1]

3 determine the work done by the force F. [1]

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9 a Define, for a wire:

i *stress*, [1]

ii *strain*. [1]

b A wire of length 1.70 m hangs vertically from a fixed point, as shown in Fig. 6.11. The wire has cross-sectional area 5.74×10^{-8} m^2 and is made of a material that has a Young modulus of 1.60×10^{11} Pa. A load of 25.0 N is hung from the wire.

i Calculate the extension of the wire. [3]

ii The same load is hung from a second wire of the same material. This wire is twice the length but the **same volume** as the first wire. State and explain how the extension of the second wire compares with that of the first wire. [3]

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10 a State Hooke's law. [1]

b The variation with extension x of the force F for a spring A is shown in Fig. 6.12.
The point L on the graph is the elastic limit of the spring.

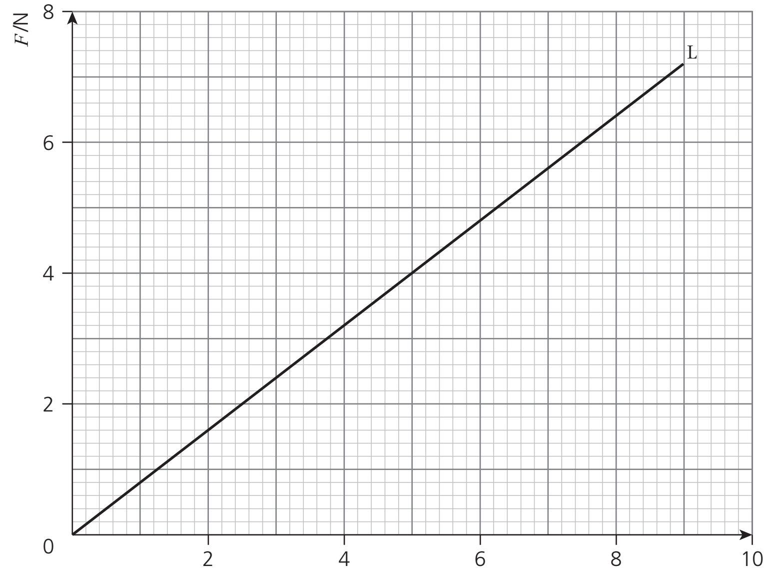

Figure 6.12

i Describe the meaning of *elastic limit*. **[1]**

ii Calculate the spring constant k_A for spring A. **[1]**

iii Calculate the work done in extending the spring with a force of 6.4 N. **[2]**

c A second spring B of spring constant $2k_A$ is now joined to spring A, as shown in Fig. 6.13.

A force of 6.4 N extends the combination of springs.

For the combination of springs, calculate:

i the total extension, **[1]**

ii the spring constant. **[1]**

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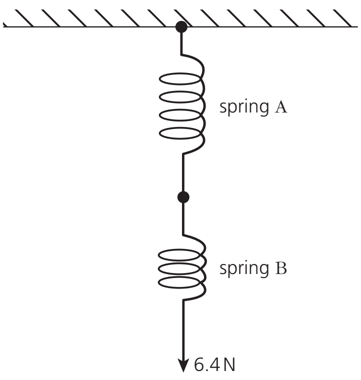

Figure 6.13

11 a For the deformation of a wire under tension, define:

i *stress*, **[1]**

ii *strain*. **[1]**

b A wire is fixed at one end so that it hangs vertically. The wire is given an extension x by suspending a load F from its free end. The variation of F with x is shown Fig. 6.14.

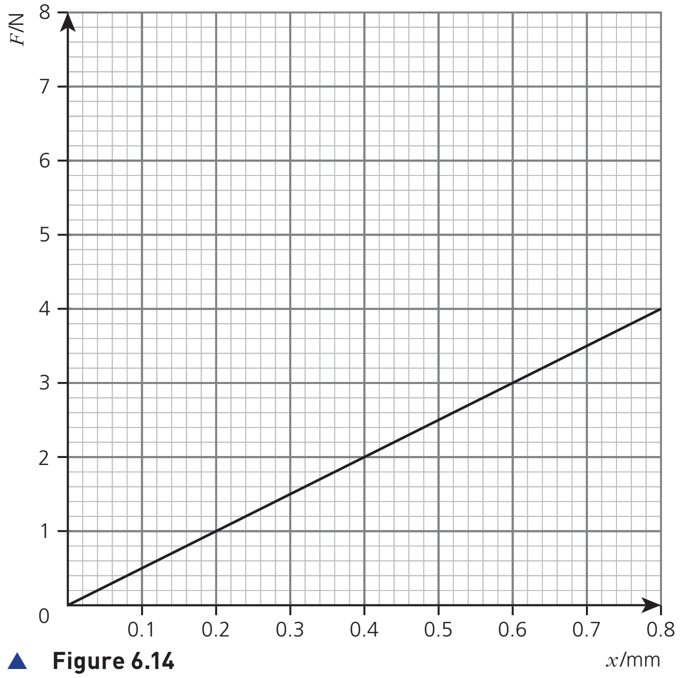

Figure 6.14

The wire has cross-sectional area 9.4×10^{-8} m^2 and original length 2.5 m.

i Describe how measurements can be taken to determine accurately the cross-sectional area of the wire. **[3]**

ii Determine the Young modulus E of the material of the wire. **[2]**

iii Use Fig. 6.14 to calculate the increase in the energy stored in the wire when the load is increased from 2.0 N to 4.0 N. **[2]**

c The wire in **b** is replaced by a new wire of the same material. The new wire has twice the length and twice the diameter of the old wire. The new wire also obeys Hooke's law. On a copy of Fig. 6.14, sketch the variation with extensions x of the load F for the new wire from $x = 0$ to $x = 0.80$ mm. **[2]**

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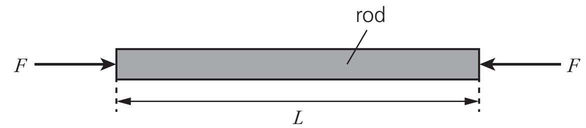
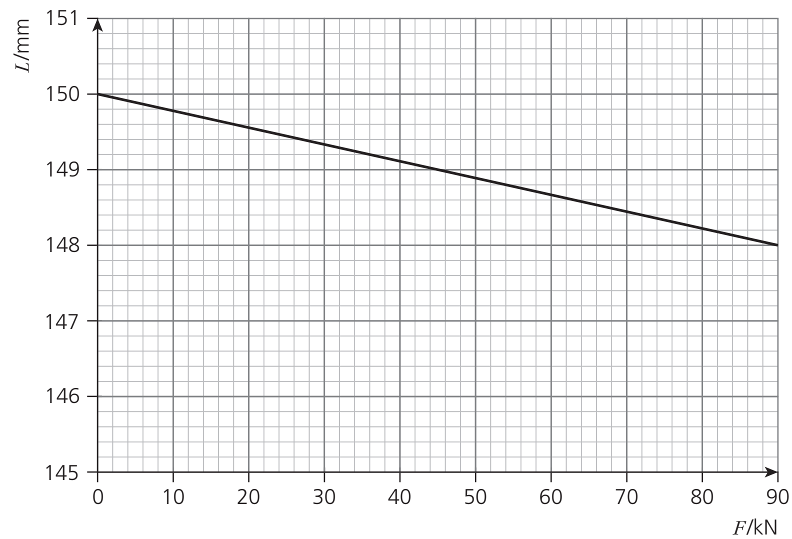
12 a Define the *Young modulus* of a material. **[1]**

b A metal rod is compressed, as shown in Fig. 6.15.



▲ **Figure 6.15**

The variation with compressive force F for the length L of the rod is shown in Fig. 6.16.



▲ **Figure 6.16**

Use Fig. 6.16 to:

i determine the spring constant k of the rod, **[2]**

ii determine the strain energy stored in the rod for $F = 90$ kN. **[3]**

c The rod in **b** has cross-sectional area A and is made of metal of Young modulus E. It is now replaced by a new rod of the same original length. The new rod has cross-sectional area $A/3$ and is made of metal of Young modulus $2E$. The compression of the new rod obeys Hooke's law. On a copy of Fig. 6.16, sketch the variation with F of the length L for the new rod from $F = 0$ to $F = 90$ kN. **[2]**

Cambridge International AS and A Level Physics (9702) Paper 23 Q4 May/June 2018

7 Waves

Learning outcomes

By the end of this topic, you will be able to:

7.1 Progressive waves

1. describe what is meant by wave motion as illustrated by vibration in ropes, springs and ripple tanks
2. understand and use the terms displacement, amplitude, phase difference, period, frequency, wavelength and speed
3. understand the use of the time-base and y-gain of a cathode-ray oscilloscope (CRO) to determine frequency and amplitude
4. derive, using the definitions of speed, frequency and wavelength the wave equation $v = f\lambda$
5. recall and use the equation $v = f\lambda$
6. understand that energy is transferred by a progressive wave
7. recall and use intensity = power/area and intensity is proportional to (amplitude)^2 for a progressive wave

7.2 Transverse and longitudinal waves

1. compare transverse and longitudinal waves
2. analyse and interpret graphical representations of transverse and longitudinal waves

7.3 Doppler effect for sound waves

1. understand that when a source of sound waves moves relative to a stationary observer, the observed frequency is different from the source frequency
2. use the expression $f_o = f_s v/(v \pm v_s)$ for the observed frequency when a source of sound waves moves relative to a stationary observer

7.4 Electromagnetic spectrum

1. state that all electromagnetic waves are transverse waves that travel with the same speed c in free space
2. recall the approximate range of wavelengths in free space of the principal regions of the electromagnetic spectrum from radio waves to γ-rays
3. recall that wavelengths in the range 400–700 nm in free space are visible to the human eye

7.5 Polarisation

1. understand that polarisation is a phenomenon associated with transverse waves
2. recall and use Malus's law ($I = I_0 \cos^2 \theta$) to calculate the intensity of a plane polarised electromagnetic wave after transmission through a polarising filter or a series of polarising filters

Starting points

- ★ The basic properties of waves, such as reflection, refraction, diffraction and interference.
- ★ The electromagnetic spectrum.
- ★ Visible light is a small part of the electromagnetic spectrum with each colour having a different frequency.

7.1 Progressive waves: transverse and longitudinal waves

This topic will introduce some general properties of waves. We will meet two broad classifications of waves, **transverse** and **longitudinal**, based on the direction in which the particles vibrate relative to the direction in which the wave transmits energy. We will define terms such as amplitude, wavelength and frequency for a wave, and derive the relationship between speed, frequency and wavelength. We will look at demonstrations of some properties of waves, such as reflection and refraction.

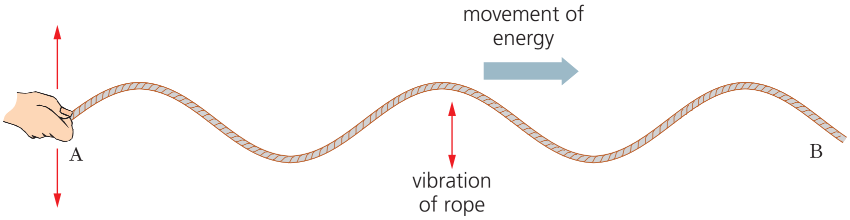
Wave motion is a means of moving energy from place to place. For example, **electromagnetic waves** from the Sun carry the energy that plants need to survive and grow. The energy carried by sound waves causes our ear drums to vibrate. The energy carried by seismic waves (earthquakes) can devastate vast areas, causing land to move and buildings to collapse. Waves which transfer energy from place to place without the transfer of matter are called **progressive waves**.

Vibrating objects act as sources of waves. For example, a vibrating tuning fork sets the air close to it into oscillation, and a sound wave spreads out from the fork. For a radio wave, the vibrating objects are electrons.

There are two main groups of waves. These are **transverse** waves and **longitudinal** waves.

A transverse wave is one in which the vibrations of the particles in the wave are at right angles to the direction in which the energy of the wave is travelling.

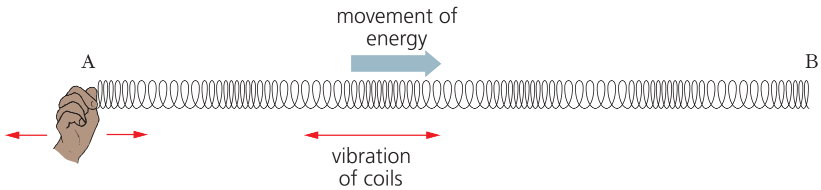
Figure 7.1 shows a transverse wave moving along a rope. The particles of the rope vibrate up and down, while the energy travels at right angles to this, from A to B. There is no transfer of matter from A to B. Examples of transverse waves are electromagnetic waves which include light waves, surface water waves and secondary seismic waves (S-waves).



▲ **Figure 7.1** Transverse wave on a rope

A longitudinal wave is one in which the direction of the vibrations of the particles in the wave is along or parallel to the direction in which the energy of the wave is travelling.

Figure 7.2 shows a longitudinal wave moving along a stretched spring (a 'slinky'). The coils of the spring vibrate along the length of the spring, whilst the energy travels along the same line, from A to B. Note that the spring itself does not move from A to B. Examples of longitudinal waves include sound waves and primary seismic waves (P-waves).



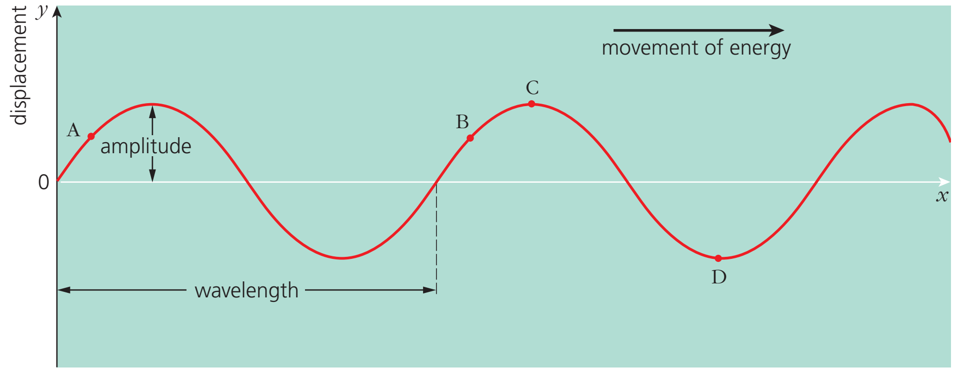
▲ **Figure 7.2** Longitudinal wave on a slinky spring

Graphical representation of waves

The **displacement** of a particle in a wave is its distance in a specified direction from its rest/equilibrium position.

Displacement is a vector quantity; it can be positive or negative. A transverse wave may be represented by plotting displacement y on the y-axis against distance x along the wave, in the direction of energy travel, on the x-axis. This is shown in

Figure 7.3. It can be seen that the graph is a snapshot of what is actually observed to be a transverse wave. The crest at C and the trough at D will be seen to move to the right with time transferring energy to the right. The particles at C and D will move towards zero displacement at right angles to the direction the energy is travelling.



▲ **Figure 7.3** Displacement–distance graph for a transverse wave

For a longitudinal wave, the displacement of the particles is along or parallel to the direction of energy travel. However, if these displacements are plotted on the y-axis of a graph of displacement against distance, the graph has exactly the same shape as for a transverse wave (see Figure 7.3). This is very useful, in that one graph can represent both types of wave. Using this graph, wave properties may be treated without reference to the type of wave.

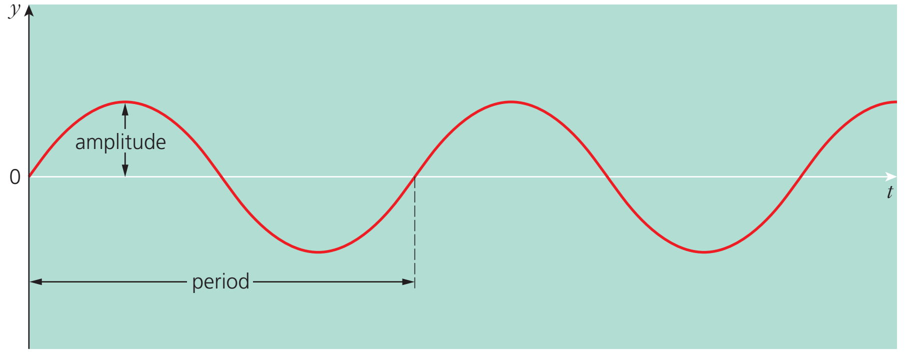
The **amplitude** of the wave motion is defined as the maximum displacement of a particle in the wave from its rest/equilibrium position.

It can be seen in Figure 7.3 that the wave repeats itself after a certain distance. That is, the wave can be constructed by repeating a particular section of the wave. The smallest distance that shows the section of the wave that is repeated is called the **wavelength**.

The motion of any particle in the wave from the maximum positive displacement (a crest) to a maximum negative displacement (a trough) back to a maximum positive displacement is called a **cycle** or **oscillation**. Note that the distance moved by a particle in the wave from crest to trough is twice the amplitude and in one oscillation the particle moves a distance that is four times the amplitude.

Another way to represent both waves is to plot a graph of displacement y of a particle in the wave against time t. This is shown in Figure 7.4. Again, the wave repeats itself, in this case after a certain interval of time. This is the time for one complete cycle or oscillation and is called the **period** T of the wave.

The **period** of the wave is the time for a particle in the wave to complete one oscillation or one cycle.



▲ **Figure 7.4** Displacement–time graph for a wave

The number of oscillations (cycles) per unit time is called the **frequency** *f* of the wave.

The displacement and amplitude are measured in mm, m or other units of length. Period is measured in seconds (s). Frequency has the unit per second (s^{-1}) or **hertz (Hz)**. The period $T = 1/f$.

A term used to compare the displacements and relative motions of particles in a wave is **phase difference**. Two particles that vibrate together are said to be **in phase**. In Figure 7.3 the particles at A and B move in phase as they have the same displacements at the same times and are moving in the same direction. Both particles will move down towards the rest position (zero displacement) as the wave moves towards the right. Particles C and D are half a cycle out of phase as C is at a crest when D is at a trough. Particle C moves down towards the rest position (zero displacement) and particle D moves up towards the rest position. The phase difference is generally stated as an angle expressed in radians or degrees.

One cycle in the wave is represented by 2π radians or 360°.

The radian is included in the following sections as angles and phase differences may be given in this unit. In AS Level, questions are set where angles and phase differences are given in degrees, and answers may be given in degrees or radians. The radian is defined in Topic 12.1.

To convert from a fraction of a cycle to an angle the fraction is multiplied by 2π for radians or 360° for degrees. Hence in Figure 7.3 C and D have a phase difference of π radians or 180°.

The **wavelength** is the minimum distance between particles which are vibrating in phase with each other.

The wavelength is, therefore, the minimum distance between two crests or two troughs.

For two-dimensional waves the term **wavefront** is used to join points that are in phase. The ripples seen on the surface of water in a ripple tank are wavefronts (see Figures 7.6 and 7.7 on page 116).

The wavelength is also the distance between adjacent wavefronts. It is the distance moved by the wavefront or the energy during one cycle/oscillation of the source of the waves.

The usual symbol for wavelength is λ, the Greek letter lambda.

The term phase difference is also used to describe the relative positions of the crests or troughs of two *different* waves of the same frequency. When the crests and troughs of the two waves are aligned, the waves are said to be in phase. When crests and troughs are not aligned the waves are said to have a phase difference. When a crest and a trough of two waves are aligned the waves are said to be in **antiphase**. Thus, when waves are out of phase with a crest aligned with a trough, one wave is half a cycle behind the other. In this case, the phase difference between waves that are exactly out of phase (in antiphase) is π radians or 180°. (Phase difference is also discussed in Topic 8 for the superposition of two progressive waves.)

Consider Figure 7.5 (overleaf), in which there are two waves of the same frequency, but with a phase difference between them. The period T corresponds to a phase angle of 2π rad or 360°. The two waves are out of step by a time t. Thus, phase difference is equal to $2\pi(t/T)$ rad = $360(t/T)$°. A similar argument may be used for waves of wavelength λ which are out of step by a distance x. In this case the phase difference is $2\pi(x/\lambda)$ rad = $360(x/\lambda)$°.

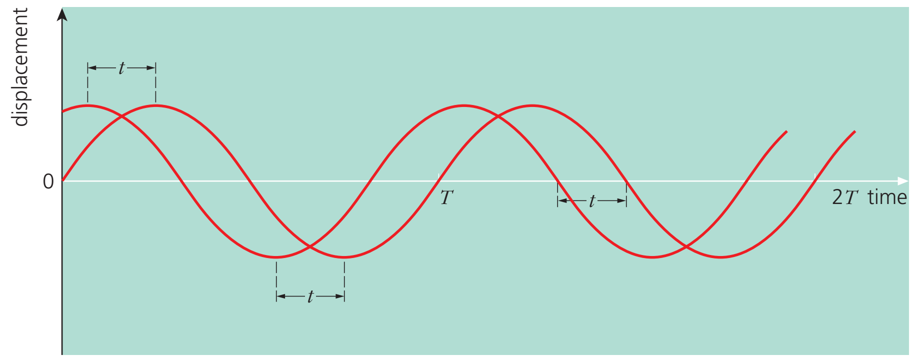


Figure 7.5 Phase difference

WORKED EXAMPLE 7A

In Figure 7.5 the time period T is 2.0 s and the time t is 0.25 s. Calculate the phase difference between the two waves in degrees.

Answer

The fraction of time t to the period T is 0.25/2 = 0.125.

Hence the phase difference is 0.125 × 360 = **45°**.

Question

1 Two waves of the same frequency have a time period of 3.0 s. The two waves are out of phase by a time difference of 0.5 s. Calculate the phase difference between the two waves in degrees.

Wave equation

The speed of a progressive wave is the distance moved per unit time by the wavefronts (or crests) or wave energy. The definition of wavelength λ is the minimum distance between wavefronts. Hence in one cycle of the source the wave energy moves a distance λ. The time taken for one cycle is the time period T. Referring to Topic 2, speed v is the distance moved per unit time. Therefore,

$$v = \lambda/T$$

If f is the frequency of the wave, then $f = 1/T$. Therefore,

$$v = f\lambda$$

or

speed = frequency × wavelength

This is an important relationship between the speed of a wave and its frequency and wavelength.

Intensity

One of the characteristics of a progressive wave is that it carries energy. The amount of energy passing through unit area per unit time is called the **intensity** of the wave.

The intensity of a wave is the power per unit area.

The intensity is proportional to the square of the amplitude of a wave for constant frequency. Thus, doubling the amplitude of a wave increases the intensity of the wave by a factor of four. The intensity also depends on the frequency: intensity is proportional to the square of the frequency.

For a wave of frequency f and amplitude A, the intensity I is proportional to A^2f^2.

If the waves from a point source spread out equally in all directions, a spherical wave is produced. As the wave travels further from the source, the energy it carries passes through an increasingly large area. Since the surface area of a sphere is $4\pi r^2$, the intensity is $W/4\pi r^2$, where W is the power of the source. The intensity of the wave thus decreases with increasing distance from the source. The intensity I is proportional to $1/r^2$, where r is the distance from the source.

This relationship assumes that there is no absorption of wave energy.

WORKED EXAMPLE 7B

1 A tuning fork of frequency 170 Hz produces sound waves of wavelength 2.0 m. Calculate the speed of sound.
2 The amplitude of a wave in a rope is 15 mm. If the amplitude were changed to 20 mm, keeping the frequency the same, by what factor would the power carried by the rope change?

Answers

1 Using $v = f\lambda$, we have $v = 170 \times 2.0 =$ **340 m s^{-1}**.
2 Intensity is proportional to the square of the amplitude. Here the amplitude has been increased by a factor of 20/15, so the power carried by the wave increases by a factor of $(20/15)^2 =$ **1.8**.

Questions

2 Water waves of wavelength 0.080 m have a frequency 5.0 Hz. Calculate the speed of these water waves.
3 The speed of sound is 340 m s^{-1}. Calculate the wavelength of the sound wave produced by a violin when a note of frequency 500 Hz is played.
4 A sound wave has twice the intensity of another sound wave of the same frequency. Calculate the ratio of the amplitudes of the waves.

Properties of wave motions

Although there are many different types of waves (light waves, sound waves, electromagnetic waves, mechanical waves, etc.) there are some basic properties which they all have in common. All waves can be reflected and refracted. All waves can be diffracted, and can produce interference patterns (diffraction and interference will be discussed in Topic 8).

These properties may be demonstrated using a ripple tank similar to that shown in Figure 7.6 (overleaf). As the motor turns, the wooden bar vibrates, creating ripples on the surface of the water. A lamp shines light onto the ripples from above. This creates shadows of the waves on the viewing screen below the tank. The shadows show the shape and movement of the waves. Each dark shadow line joins points that are in phase. These lines represent a **wavefront**. The minimum distance between the wavefronts is the wavelength of the water waves. A stroboscope can be used to 'freeze' the pattern on the screen. The stroboscope emits flashes of light at a given frequency. The frequency is adjusted until it is the same as the frequency of the waves and the shadows are seen in fixed positions.

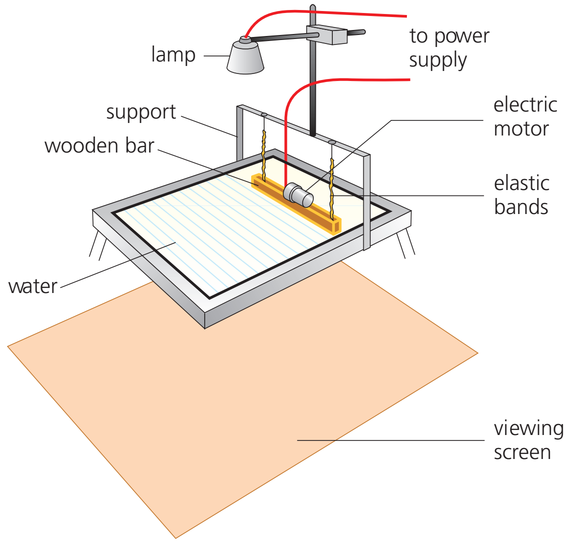


Figure 7.6 Ripple tank

Figure 7.7 illustrates the pattern of wavefronts produced by a low-frequency vibrator and one of higher frequency. Note that for the higher frequency the wavelength is shorter, since wave speed is constant and $v = f\lambda$.

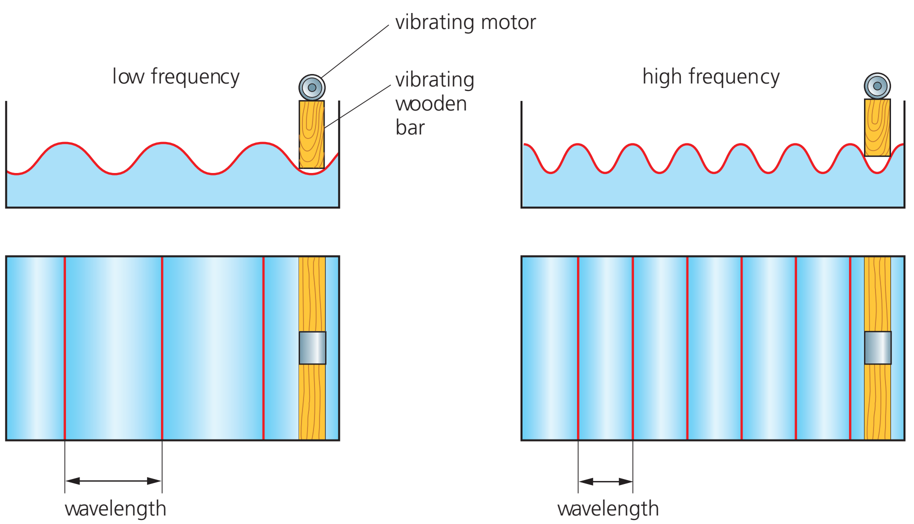


Figure 7.7 Ripple tank patterns for low and high frequency vibrations

Circular waves may be produced by replacing the vibrating bar with a small dipper, or by allowing drops of water to fall into the ripple tank. A circular wave is illustrated in Figure 7.8. This pattern is characteristic of waves spreading out from a point source.

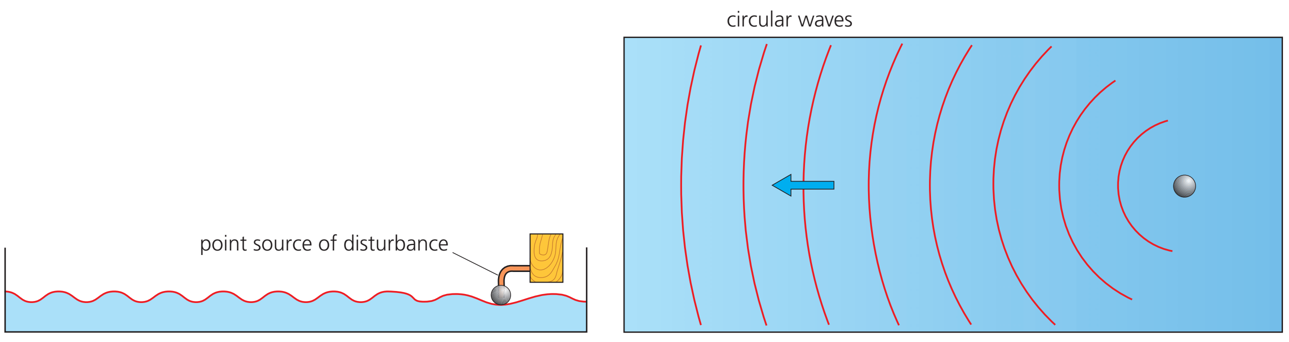
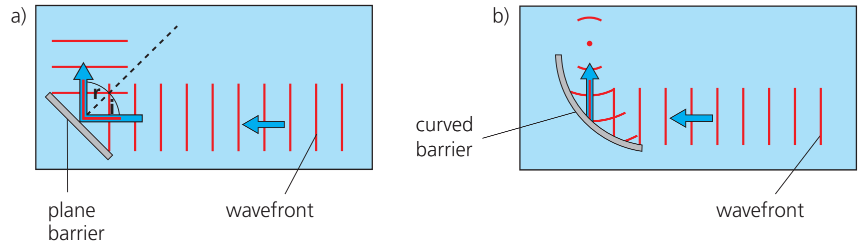


Figure 7.8 Ripple tank pattern for a point source

We shall now see how the ripple tank may be used to demonstrate the wave properties of reflection and refraction.

Reflection

As the waves strike a plane barrier placed in the water, they are reflected. The angle of reflection equals the angle of incidence, and there is no change in wavelength (see Figure 7.9a). If a curved barrier is used, the waves can be made to converge or diverge (Figure 7.9b).

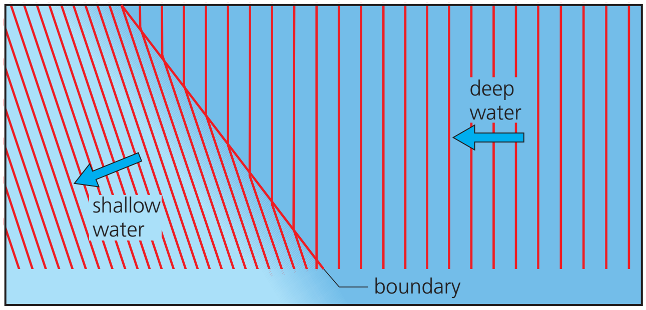


▲ **Figure 7.9** Ripple tank pattern showing reflection at a) a plane surface and b) a curved one

Refraction

If a glass block is submerged in the water, this produces a sudden change in the depth of the water. The speed of surface ripples on water depends on the depth of the water: the shallower the water, the slower the speed. Thus, the waves move more slowly as they pass over the glass block. The frequency of the waves remains constant, and so the wavelength decreases. If the waves are incident at an angle to the submerged block, they will change direction, as shown in Figure 7.10.

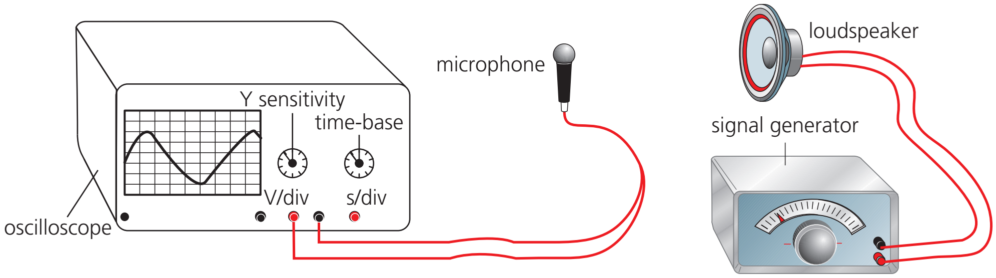
The change in direction of a wave due to a change in speed is called **refraction**.



▲ **Figure 7.10** Ripple tank pattern showing refraction

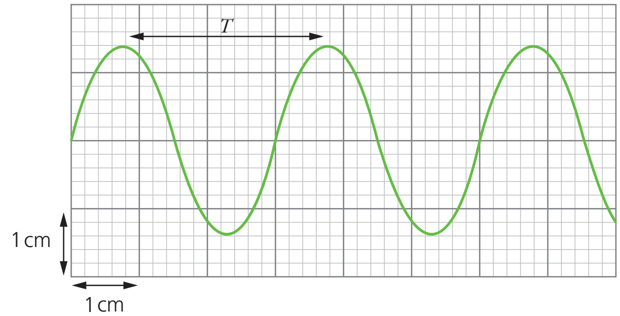
The determination of the frequency and amplitude of sound using a calibrated CRO

A cathode ray oscilloscope (CRO) with a calibrated time-base may be used to determine the frequency of sound. A method of measuring the frequency of sound waves is illustrated in Figure 7.11. A signal generator and loudspeaker are used to produce a note of a single frequency. The microphone is connected to the Y-plates of the CRO.



▲ **Figure 7.11** Measuring the frequency and amplitude of sound using a CRO

The microphone detects the sound and a trace on the CRO can be obtained by adjusting the Y-plate sensitivity and the time-base settings. A typical trace is shown in Figure 7.12.



▲ **Figure 7.12** Measurement of the time period and amplitude from the CRO trace

The distance between peaks or troughs is measured using the scale on the *x*-axis on the CRO display. The time-base setting is used to determine the time period *T* and frequency of the sound. The calculated value can be compared with that shown on the signal generator.

The amplitude of the sound is represented by the amplitude of the trace on the CRO. The distance from crest to trough along the y-axis is measured and divided by two. The Y-sensitivity, usually given in V cm^{-1}, is used to determine the amplitude of the wave shown on the CRO in volts. The intensity of different sounds can be compared by the value obtained for the amplitude on the CRO. Provided the detector produces a voltage that is proportional to the intensity of the sound, the amplitude of the trace on the CRO is directly related to the intensity. For constant frequency the intensity is proportional to the $(\text{amplitude})^2$. For example, if the intensity of the sound wave is doubled, the amplitude of the sound wave is increased by the square root of two and the trace on the CRO shows the same increase.

The CRO can be used to measure the frequency and compare amplitudes of other waves using a suitable detector in place of the microphone.

WORKED EXAMPLE 7C

1 The time-base setting for the CRO used to obtain the trace in Figure 7.12 is $2.0\,\text{ms cm}^{-1}$. Determine for the sound:
   a the time period
   b the frequency.

2 The Y-sensitivity for the CRO is set on $2.0\,\text{V cm}^{-1}$. Determine the voltage amplitude of the trace shown on the CRO.

Answers

1 a The distance for a time period is 3.0 cm.
   Hence the period = $3.0 \times 2.0 \times 10^{-3}$ = $\mathbf{6.0 \times 10^{-3}\,s}$
   b The frequency = 1/time period = $1/6.0 \times 10^{-3}$ = **170 Hz**

2 The amplitude of the trace is 1.4 cm and hence the voltage amplitude is 1.4×2.0 = **2.8 V**.

Questions

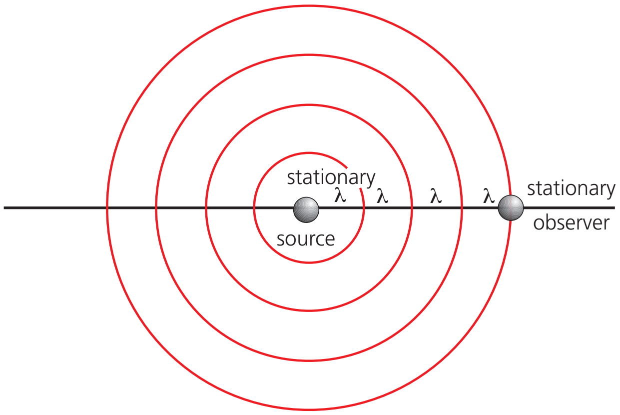
5 The time-base on a CRO is set on 0.50 ms cm^{-1}. The trace obtained for a sound wave shows three complete time periods in 7.2 cm. Calculate:
   a the time period
   b the frequency.

6 Make a copy of the trace shown in Figure 7.12. Draw a second trace on your copy for a sound wave that has half the intensity of the original sound but with the same frequency.

7.2 Doppler effect

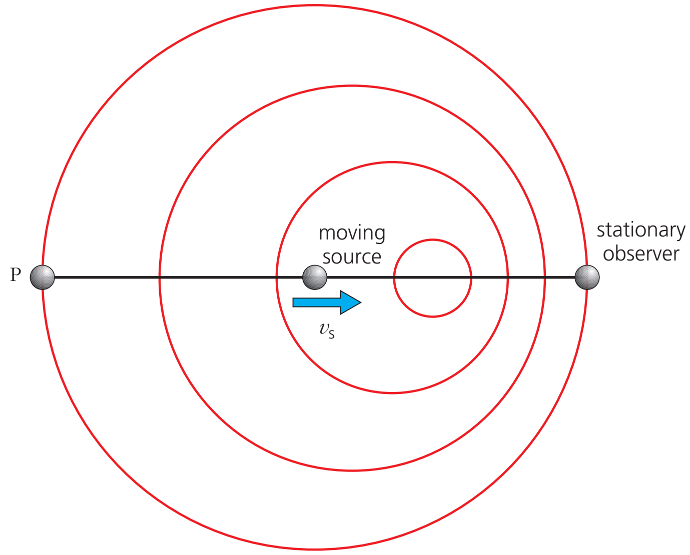
The whistle of a train or the siren of a police car appears to increase in frequency (pitch) as it moves towards a stationary observer. The frequency change due to the relative motion between a source of sound or light and an observer is known as the **Doppler effect**.

When the observer and source of sound are both stationary, the number of waves per second reaching the observer will be the same frequency as the source (see Figure 7.13).



▲ **Figure 7.13** **The source emits waves of wavelength λ. The observer is stationary and receives waves with the same wavelength λ.**

When the source moves towards the observer the effect is to shorten the wavelength of the waves reaching the observer (see Figure 7.14).



▲ **Figure 7.14** **Source of sound moving towards a stationary observer**

Let v be the speed of sound in air. A source of sound has a frequency f_s and wavelength λ. The source moves *towards* an observer at a speed v_s.

The period of oscillation of the source of sound is T (= $1/f_s$). In the time of one oscillation the source moves **towards** the observer a distance v_sT. Hence the wavelength is shortened by this distance. The wavelength of the sound received by the observer is $\lambda - v_sT$.

Hence the frequency observed $f_o = v/(\lambda - v_sT) = v/(v/f_s - v_s/f_s)$

$$f_o = \frac{f_s v}{(v - v_s)}$$

The source would move **away** from a stationary observer at position P on the left-hand side of Figure 7.14. The observed wavelengths would lengthen.

For a source of sound moving *away* from an observer the observed frequency can be shown to be

$$f_o = \frac{f_s v}{(v + v_s)}$$

The observed frequency f_o when a source of sound waves moves at speed v_s relative to a stationary observer is

$$f_o = \frac{f_s v}{(v \pm v_s)}$$

The observed frequency is *greater* than the source frequency when the source moves towards the observer and the observed frequency is *less* than the source frequency when the source moves away from the observer.

The above expressions apply only when the source of waves is sound. However, a change of frequency (Doppler shift) is observed with all waves, including light.

EXTENSION

In astronomy, the wavelength tends to be measured rather than the frequency. If the measured wavelength of the emitted light (see Topic 25) is less than that measured for a stationary source, then the distance between the source (star) and detector is decreasing (blueshift). If the measured wavelength is greater than the value of a stationary source, then the source is moving away from the detector (redshift). The blue and red shifts are referred to in this way as red has the longest wavelength in the visible spectrum and blue the shortest.

WORKED EXAMPLE 7D

A police car travels towards a stationary observer at a speed of $15\,\mathrm{m\,s^{-1}}$. The siren on the car emits a sound of frequency 250 Hz. Calculate the observed frequency. The speed of sound is $340\,\mathrm{m\,s^{-1}}$.

Answer

observed frequency $f_o = \frac{f_s v}{(v - v_s)}$

$= 250 \times \frac{340}{(340 - 15)}$

$=$ **260 Hz**

Question

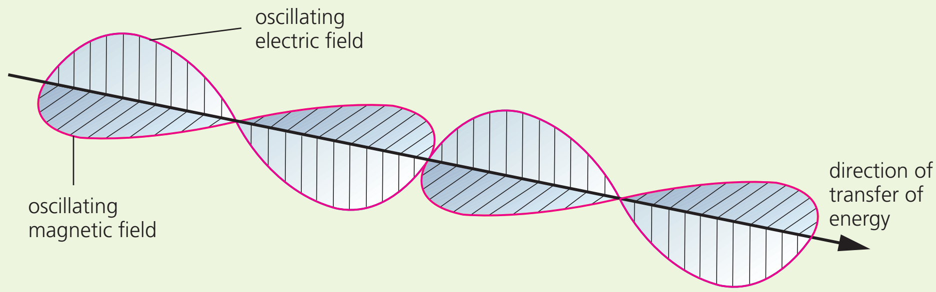
7 The sound emitted from the siren of an ambulance has a frequency of 1500 Hz. The speed of sound is $340\,\mathrm{m\,s^{-1}}$. Calculate the difference in frequency heard by a stationary observer as the ambulance travels towards and then away from the observer at a speed of $30\,\mathrm{m\,s^{-1}}$.

7.3 Electromagnetic spectrum

Electromagnetic waves are progressive transverse waves (see Topic 7.1). However, unlike other types of transverse wave they do not require a medium to travel through.

EXTENSION

Electromagnetic waves consist of electric and magnetic fields which oscillate at right angles to each other and to the direction in which the wave is travelling. This is illustrated in Figure 7.15.



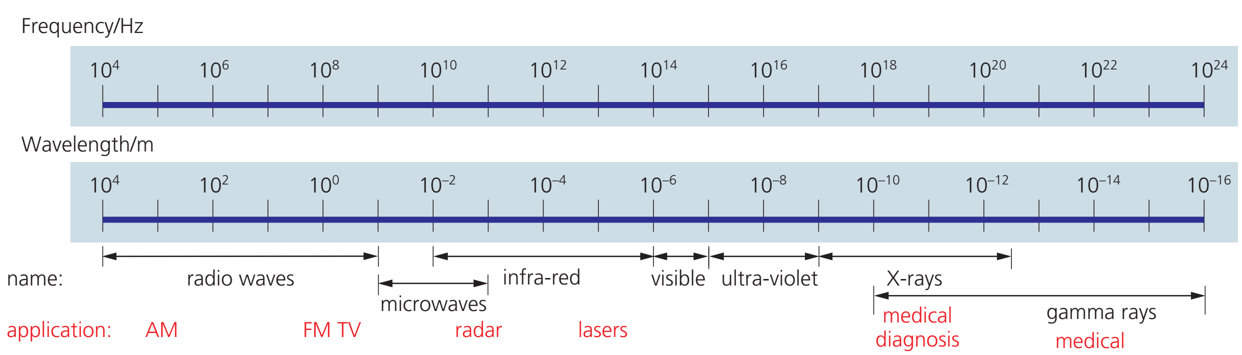
▲ **Figure 7.15** Oscillating electric and magnetic fields in an electromagnetic wave

Electromagnetic (e.m.) waves show all the properties common to wave motions: they can be reflected and refracted. They can also be diffracted and they obey the principle of superposition and produce interference patterns (as we shall see in Topic 8).

In a vacuum (free space) all electromagnetic waves travel at the same speed, $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$. However their speed is lower in other substances. When a wave slows down, its frequency is unaltered (since this depends on the wave source) but the wavelength will decrease (from $v = f\lambda$).

The complete **electromagnetic spectrum** has a continuous range of frequencies (or wavelengths) but may be divided into a series of regions based on the properties of electromagnetic waves in these regions, as illustrated in Figure 7.16. It should be noted that there is no clear boundary between regions.

Table 7.1 (overleaf) lists the wavelength range of the main regions of the electromagnetic spectrum in free space.



▲ **Figure 7.16** The electromagnetic spectrum

radiation	approximate wavelength range/m
γ-rays	10^{-10}–10^{-16} and shorter
X-rays	10^{-9}–10^{-12} and shorter
ultraviolet	10^{-7}–10^{-9}
visible	4×10^{-7}– 7×10^{-7}
infrared	10^{-2}–10^{-6}
microwaves	10^{-3}–10^{-1}
radio waves	10^{-1}–10^{4} and longer

▲ **Table 7.1** Wavelength range of the principal regions of the electromagnetic spectrum in free space

Visible light is just a small region of the electromagnetic spectrum with wavelengths of 400–700 nm in free space, where 1 nm = 1×10^{-9} m (Topic 1). Blue light has a shorter wavelength than red light.

WORKED EXAMPLE 7E

1 Calculate the frequency in MHz of a radio wave of wavelength 250 m. The speed of all electromagnetic waves is $3.00 \times 10^{8}\,\text{m s}^{-1}$.

2 Calculate the wavelength in nm of an X-ray wave of frequency 2.0×10^{18} Hz.

Answers

1 For a wave $v = f\lambda$

$$f = 3.00 \times \frac{10^8}{250} = 1.2 \times 10^6\,\text{Hz}$$
$$= \mathbf{1.2\,MHz}$$

2 For a wave $v = f\lambda$

$$\lambda = 3.00 \times 10^8/2 \times 10^{18} = 1.5 \times 10^{-10}$$
$$= \mathbf{0.15\,nm}$$

Questions

8 The speed of light is $3.00 \times 10^{8}\,\text{m s}^{-1}$. Calculate the frequency of red light of wavelength 650 nm. Give your answer in THz.

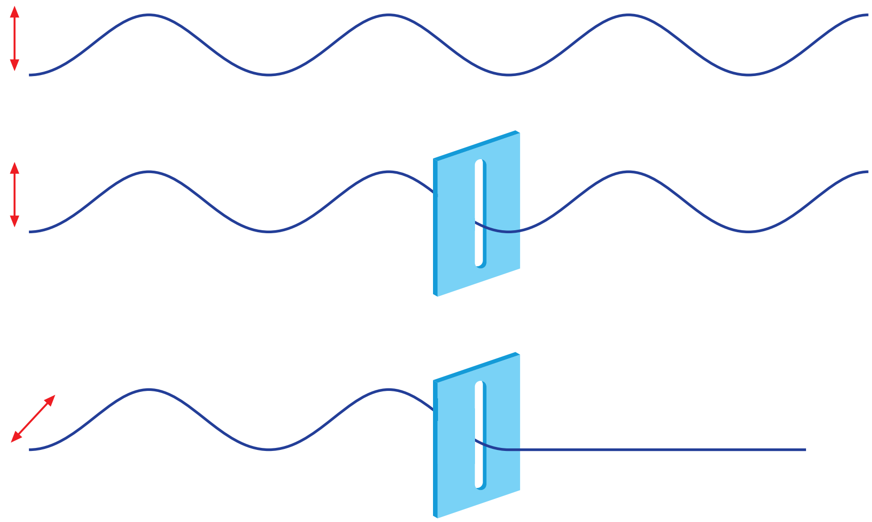
9 A beam of red light has an amplitude that is 2.5 times the amplitude of a second beam of the same colour. Calculate the ratio of the intensities of the waves.

10 Calculate the wavelength of microwaves of frequency 8.0 GHz.

7.4 Polarisation

Consider generating waves in a rope by moving your hand holding the stretched rope up and down, or from side to side. The transverse vibrations of the rope will be in just one plane – the vertical plane if your hand is moving up and down, or the horizontal plane if it moves from side to side. In each case the plane is that containing both the direction of vibration of the rope and the direction in which the wave energy is travelling. These vibrations are said to be **plane polarised** in either a vertical plane or a horizontal plane.

However, there are an infinite number of directions for the vibrations of the rope to be at right angles to the direction the wave energy is travelling (the condition for the wave produced to be a transverse wave). If the direction of the vibration of the rope is continually changed but kept at right angles to the direction of energy travel then the waves produced are said to be **unpolarised**. This is similar to that shown for unpolarised electromagnetic waves in Figure 7.18.

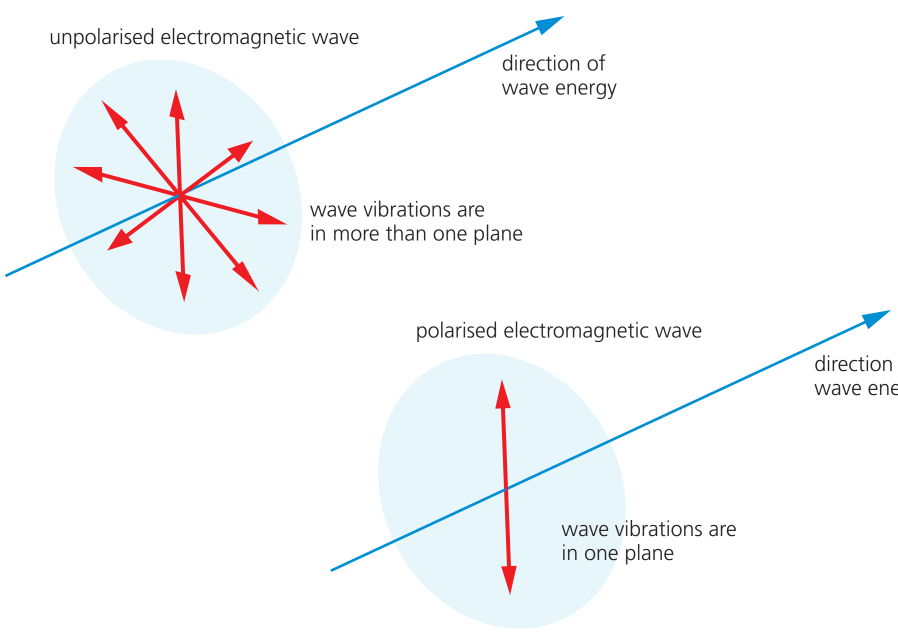


▲ **Figure 7.17** Transverse waves on a rope

A polarised wave is a transverse wave in which vibrations occur in only one of the directions at right angles to the direction in which the wave energy is travelling.

The condition for a wave to be plane polarised is for the vibrations to be in just one plane, which contains the direction in which the wave energy is travelling.

If the rope passes through a vertical slot, then only if you move your hand up and down will the wave pass through the slot. This is illustrated in Figure 7.17. If the rope passes through a horizontal slot, only waves generated by a side-to-side motion of the hand will be transmitted. If the rope passes through a slot of one direction followed by another at right angles, the wave is totally blocked whether it was initially vibrating in the vertical or the horizontal plane. This is similar to that shown for electromagnetic waves in Figure 7.20a on page 125.



▲ **Figure 7.18** Unpolarised and polarised electromagnetic waves

Clearly, polarisation can apply only to transverse waves. In longitudinal waves the vibrations are parallel to the direction of wave travel, and whatever the direction of the slit, it would make no difference to the transmission of the waves.

The Sun and domestic light bulbs emit unpolarised light; that is, over a short period of time, the vibrations take place in many directions in a plane at right angles to the direction of the wave energy, instead of in a single direction of vibration required for plane-polarised electromagnetic waves (see Figure 7.18).

Many radio waves, microwaves and television waves are plane polarised.

Polarising light waves

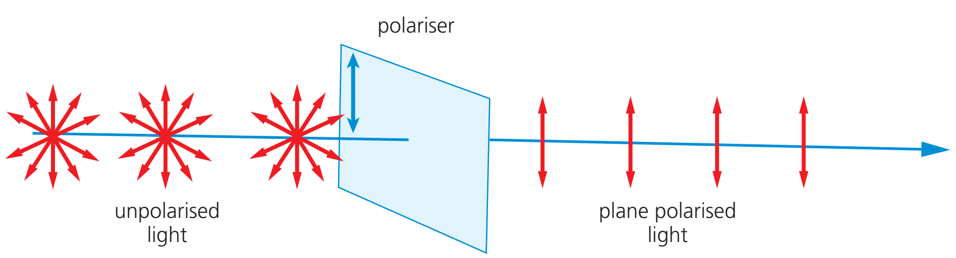
Some transparent materials, such as a Polaroid sheet (used in some types of sunglasses), allow vibrations of light to pass through in one direction only. A Polaroid sheet contains long chains of organic molecules aligned parallel to each other.

EXTENSION

The fact that light can be polarised was understood only in the early 1800s. This was a most important discovery. It showed that light is a transverse wave motion, and opened the way, 50 years later, to Maxwell's theory of light as electromagnetic radiation. James Clerk Maxwell described light in terms of oscillating electric and magnetic fields, at right angles to each other and at right angles to the direction of travel of the wave energy (see Figure 7.15). When we talk about the direction of polarisation of a light wave, we refer to the direction of the electric field component of the electromagnetic wave.

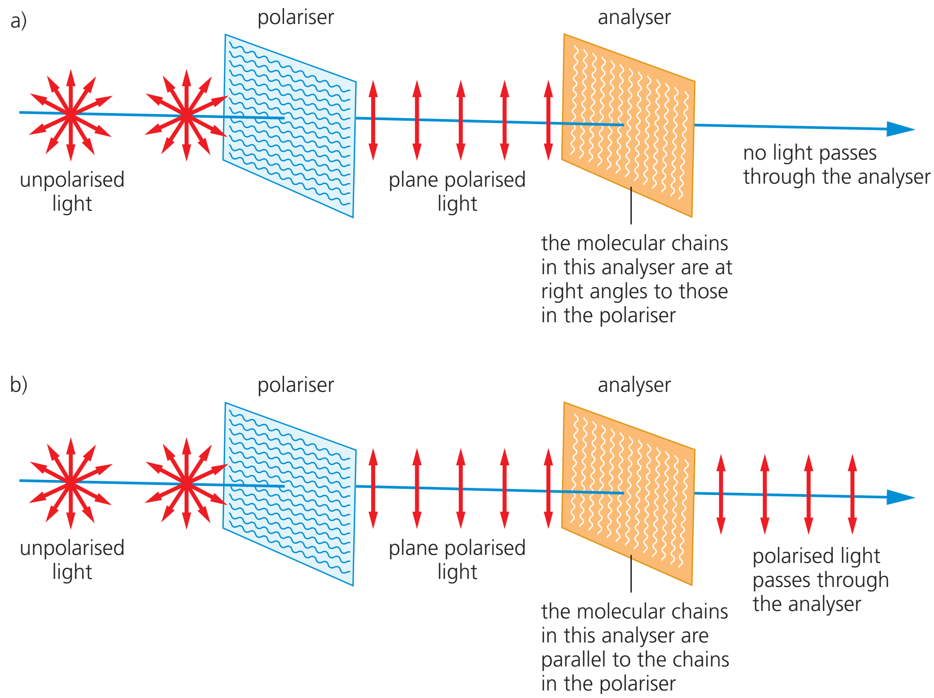
When unpolarised light arrives at a Polaroid sheet, the component of the electric field of the incident light which is parallel to the molecules is strongly absorbed, whereas light with its electric field perpendicular to the molecules is transmitted through the sheet.

The Polaroid sheet acts as a **polariser**, producing plane-polarised light from light that was originally unpolarised. Figure 7.19 illustrates unpolarised light entering the polariser, and polarised light leaving it. The blue arrow in the polariser represents the direction of vibration of the light waves allowed by the polariser.



▲ **Figure 7.19**

If you try to view plane-polarised light through a second sheet of Polaroid which is placed so that its polarising direction is at right angles to the polarising direction of the first sheet, it will be found that no light is transmitted. In this arrangement, the Polaroids are said to be 'crossed'. The second Polaroid sheet is acting as an **analyser**. If the two Polaroids have their polarising directions parallel, then plane-polarised light from the first Polaroid can pass through the second. These two situations are illustrated in Figure 7.20 on the next page. Although the action of the Polaroid sheet is not that of a simple slit, the arrangement of the crossed Polaroids has the same effect as the crossed slits in the rope-and-slits experiment (Figure 7.17).

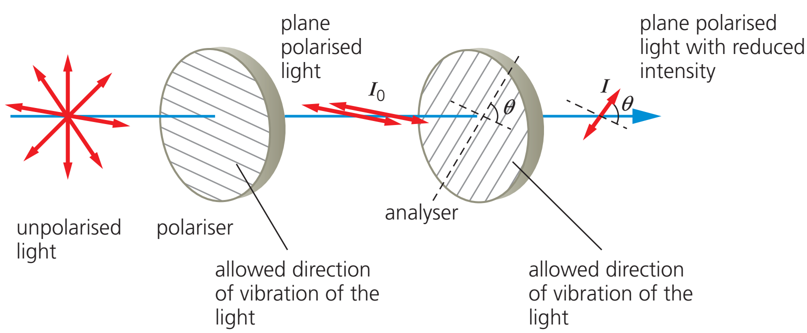


▲ **Figure 7.20 Polariser and analyser in a) crossed and b) parallel situations**

Malus's law

When two Polaroid sheets are crossed, none of the polarised light transmitted through the first sheet can be transmitted through the second Polaroid sheet. However, if the two Polaroid sheets are arranged so that the polarising direction of the second sheet is at an angle of less than 90° to the polarising direction of the first sheet, some light can be transmitted. The brightness or intensity of light emerging from the second sheet depends on the angle between the polarising directions of the two sheets.

Consider the maximum light intensity transmitted through the polariser to have amplitude A_0 and intensity I_0. The plane of polarisation of the light from the polariser is at an angle θ to the polarising direction of the analyser. See Figure 7.21.



▲ **Figure 7.21 Action of an analyser**

The analyser polarises the light in a direction parallel to its polarising direction – the plane of polarisation is rotated through an angle θ. The amplitude of the light emerging is $A_0 \cos\theta$. Since intensity is proportional to the amplitude squared ($I \propto A^2$) the transmitted intensity $I \propto A_0{}^2 \cos^2\theta$.

and therefore,

$$I = I_0 \cos^2\theta$$

This is known as **Malus's law**.

Hence, when $\theta = 0$, the maximum intensity I_0 is transmitted, and when $\theta = 90°$, no light is transmitted.

WORKED EXAMPLE 7F

The polariser in Figure 7.19 is slowly rotated through 360°. Describe the intensity of the transmitted plane polarised light as the polariser is rotated.

Answer

The intensity does not vary. (The incident light is unpolarised and, therefore, the vibrations take place in many different directions. On average there are the same number of vibrations in each direction.)

Questions

11 The analyser shown in Figure 7.20b is adjusted to give maximum intensity of transmitted light.

Describe the intensity transmitted by the analyser as it is slowly rotated through 360°. Draw a graph to show how the intensity varies with the angle of rotation of the analyser.

12 The polariser and analyser have their planes of polarisation parallel (as shown in Figure 7.20b).

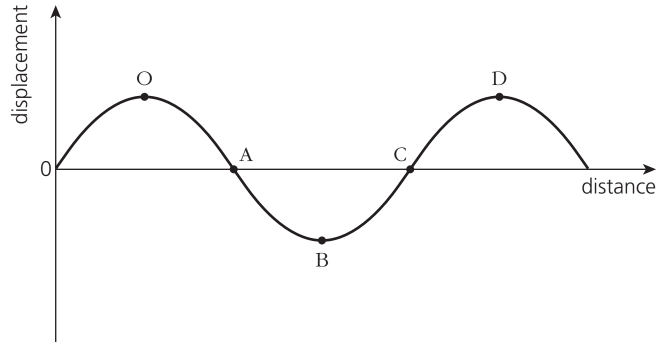
Calculate the angle the analyser must be turned through to reduce the transmitted light intensity by a quarter.

SUMMARY

- A progressive wave travels outwards from the source, carrying energy but without transferring matter.
- In a transverse wave, the oscillations of the particles are at right angles to the direction in which the wave carries energy.
- In a longitudinal wave, the oscillations of the particles are in the same direction as the direction in which the wave carries energy.
- Transverse and longitudinal waves can be represented by a graph of displacement against distance along the wave at a particular moment in time, or as a graph of displacement against time to show how the displacement changes at a particular position.
- The relative positions of two points on the same wave, or of two waves of the same frequency can be stated as a phase difference in degrees.
- An oscilloscope can display sound waves as a voltage–time graph from which the frequency and amplitude of the wave can be determined.
- Properties of wave motion (reflection, refraction, diffraction and interference) can be observed for water waves in a ripple tank.
- The intensity of a wave is the power per unit area. Intensity is proportional to the square of the amplitude.
- The speed v, frequency f and wavelength λ of a wave are related by $v = f\lambda$.
- When a source of sound waves moves relative to a stationary observer there is a change in the observed frequency compared with the frequency emitted by the source. This is the Doppler effect.
- The observed frequency f_0 is related to the source frequency f and the speed of the source, v, by $f_0 = f_s v/(v \pm v_s)$.
- All electromagnetic waves have the same speed in free space $c = 3.00 \times 10^8\,\text{m s}^{-1}$.
- All electromagnetic waves are transverse waves.
- The wavelengths of electromagnetic waves in free space visible to the human eye are in the range 400 nm to 700 nm.
- In a plane-polarised wave, the vibrations of the wave are in one direction only, which is at right-angles to the direction of travel of the wave.
- Transverse waves can be polarised; longitudinal waves cannot.
- Plane-polarised light can be produced from unpolarised light by using a polariser, such as a sheet of Polaroid.
- Rotating an analysing Polaroid in a beam of plane-polarised light prevents transmission of the polarised light when the polarising directions of polariser and analyser are at right angles.
- Malus's law ($I = I_0 \cos^2\theta$) is used to calculate the intensity of plane-polarised electromagnetic waves after transmission through a polarising filter or series of filters.

END OF TOPIC QUESTIONS

1 Fig. 7.22 represents the displacement against distance for a transverse wave. Which point, A, B, C or D vibrates out of phase with point O by 180°?

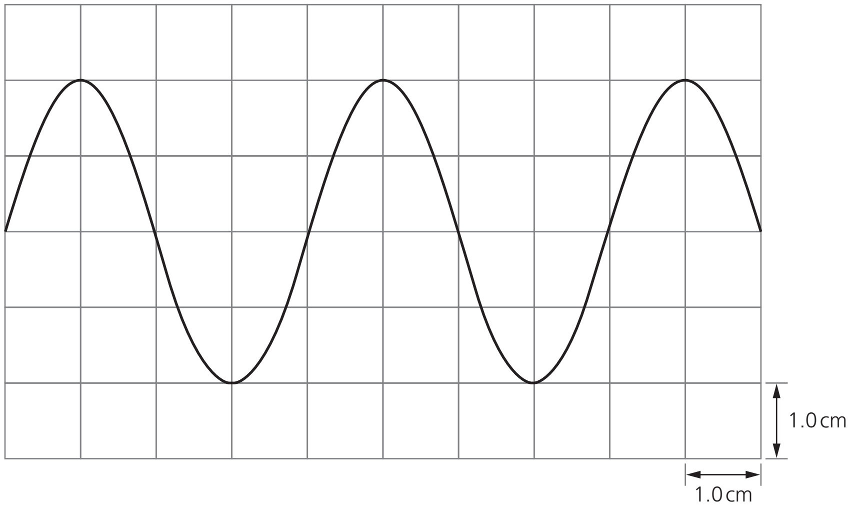


▲ **Figure 7.22**

2 Fig. 7.23 represents the screen of a cathode-ray oscilloscope (c.r.o.). The c.r.o. shows the waveform of a sound wave.

The time-base setting is 0.40 ms cm^{-1}. What is the frequency of the sound wave?

A 0.63 Hz **B** 1.3 Hz **C** 630 Hz **D** 1300 Hz



▲ **Figure 7.23**

3 A monochromatic source of electromagnetic waves is viewed through two pieces of polarising filter whose preferred direction of vibration are parallel to each other.

What angle should one sheet of polarising filter be turned to reduce the amplitude of the observed wave to half its original value?

A 30° **B** 45° **C** 60° **D** 90°

4 A certain sound wave in air has a speed 340 m s^{-1} and wavelength 1.7 m. For this wave, calculate:

a the frequency,

b the period.

5 The speed of electromagnetic waves in vacuum (or air) is 3.00×10^8 m s^{-1}.

a The visible spectrum extends from a wavelength of 400 nm (blue light) to 700 nm (red light).

Calculate the range of frequencies of visible light.

b A typical frequency for v.h.f. television transmission is 250 MHz. Calculate the corresponding wavelength.

6 Two waves travel with the same speed and have the same amplitude, but the first has twice the wavelength of the second. Calculate the ratio of the intensities transmitted by the waves.

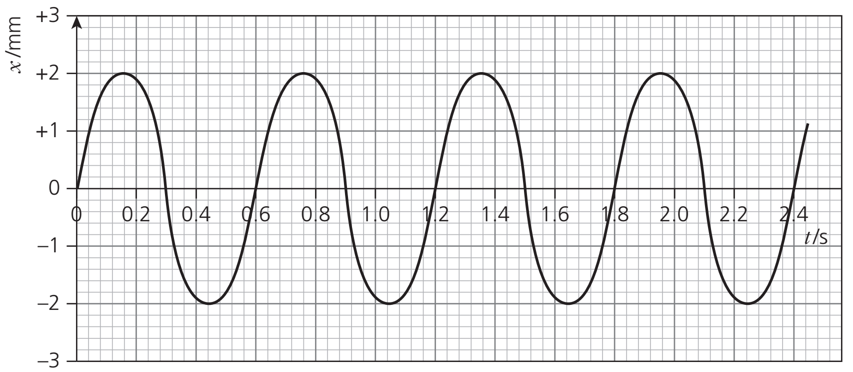
7 A student stands at a distance of 5.0 m from a point source of sound, which is radiating uniformly in all directions. The intensity of the sound wave at her ear is $6.3 \times 10^{-6}\,W\,m^{-2}$.

a The receiving area of the student's ear canal is $1.5\,cm^2$. Calculate how much energy passes into her ear in 1 minute.

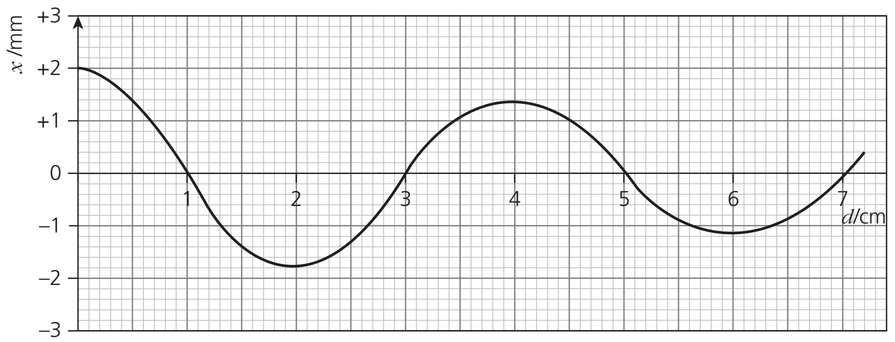
b The student moves to a point 1.8 m from the source. Calculate the new intensity of the sound.

8 Assume that waves spread out uniformly in all directions from the epicentre of an earthquake. The intensity of a particular earthquake wave is measured as $5.0 \times 10^6\,W\,m^{-2}$ at a distance of 40 km from the epicentre. What is the intensity at a distance of only 2 km from the epicentre?

9 A student is studying a water wave in which all the wavefronts are parallel to one another. The variation with time t of the displacement x of a particular particle in the wave is shown in Fig. 7.24. The distance d of the oscillating particles from the source of the waves is measured. At a particular time, the variation of the displacement x with this distance d is shown in Fig. 7.25.



▲ **Figure 7.24**



▲ **Figure 7.25**

a Define, for a wave, what is meant by:

  i *displacement*, [1]

  ii *wavelength*. [1]

b Use Figs. 7.24 and 7.25 to determine, for the water wave:

  i the period T of vibration, [1]

  ii the wavelength λ, [1]

  iii the speed v. [2]

c **i** Use Figs. 7.24 and 7.25 to state and explain whether the wave is losing power as it moves away from the source. [2]

  ii Determine the ratio $\dfrac{\text{intensity of wave at source}}{\text{intensity of wave 6.0 cm from source}}$ [3]

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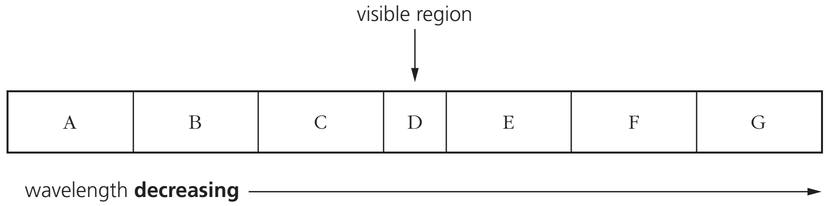
10 a State one property of electromagnetic waves that is **not** common to other transverse waves. **[1]**

b The seven regions of the electromagnetic spectrum are represented by blocks labelled A to G in Fig. 7.26. A typical wavelength for the visible region D is 500 nm.



▲ **Figure 7.26**

i Name the principle radiations and give a typical wavelength for each of the regions B, E and F. **[3]**

ii Calculate the frequency corresponding to a wavelength of 500 nm. **[2]**

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11 a A loudspeaker oscillates with frequency f to produce sound waves of wavelength λ. The loudspeaker makes N oscillations in time t.

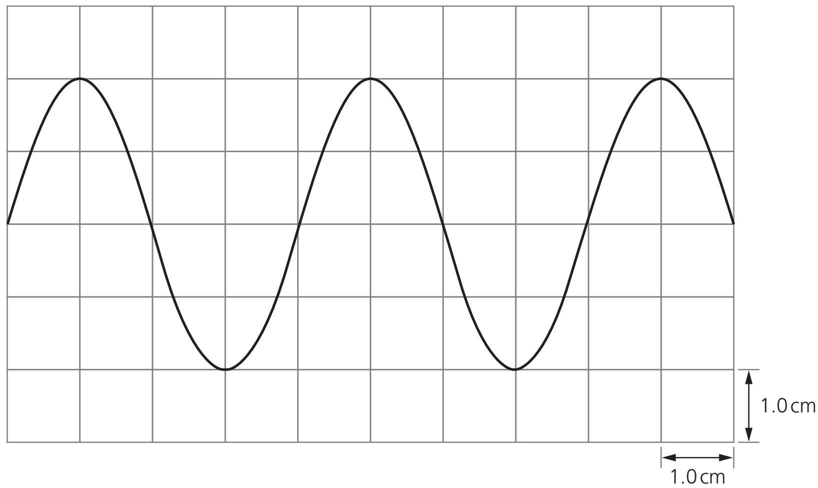
i State expressions, in terms of some or all of the symbols f, λ and N, for:

1 the distance moved by a wavefront in time t,

2 time t. **[2]**

ii Use your answers in **i** to deduce the equation relating the speed v of the sound wave to f and λ. **[1]**

b The waveform of a sound wave is displayed on the screen of a cathode-ray oscilloscope (c.r.o.), as shown in Fig. 7.27.

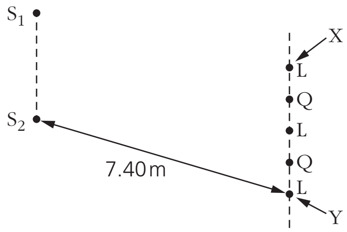


▲ **Figure 7.27**

The time-base setting is 0.20 ms cm^{-1}.

Determine the frequency of the sound wave. **[2]**

c Two sources S_1 and S_2 of sound waves are positioned as shown in Fig. 7.28. The sources emit coherent sound waves of wavelength 0.85 m. A sound detector is moved parallel to the line S_1S_2 from a point X to a point Y. Alternate positions of maximum loudness L and minimum loudness Q are detected, as illustrated in Fig. 7.28.



▲ **Figure 7.28** (not to scale)

Distance S_1X is equal to distance S_2X. Distance S_2Y is 7.40 m.

i Explain what is meant by *coherent waves*. **[1]**

ii State the phase difference between the two waves arriving at the position of minimum loudness Q that is closest to point X. **[1]**

iii Determine the distance S_1Y. **[2]**

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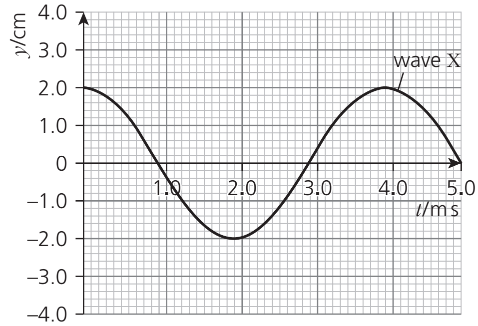
12 The variation with time t of the displacement y of a wave X, as it passes a point P, is shown in Fig. 7.29.



▲ **Figure 7.29**

The intensity of wave X is I.

a Use Fig. 7.29 to determine the frequency of wave X. **[2]**

b A second wave Z with the same frequency as wave X also passes point P. Wave Z has intensity $2I$. The phase difference between the two waves is 90°. On a copy of Fig. 7.29, sketch the variation with time t of the displacement y of wave Z. Show your working. **[3]**

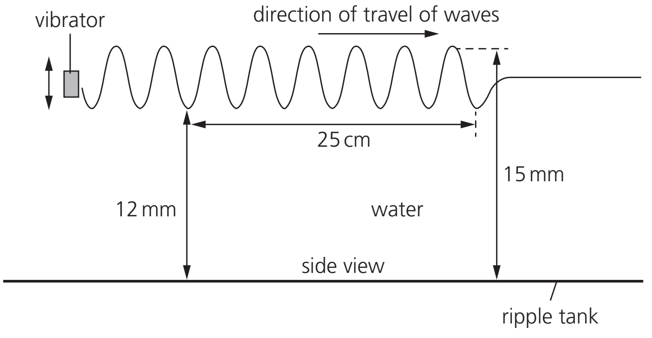
Cambridge International AS and A Level Physics (9702) Paper 21 Q5a and b May/June 2016

13 a Explain what is meant by the following quantities for a wave on the surface of water:

i displacement and amplitude, **[2]**

ii frequency and time period. **[2]**

b Fig. 7.30 represents waves on the surface of water in a ripple tank at one particular instant of time.



▲ **Figure 7.30**

A vibrator moves the surface of the water to produce the waves of frequency f. The speed of the waves is 7.5 cm s^{-1}. Where the waves travel on the water surface, the maximum depth of the water is 15 mm and the minimum depth is 12 mm.

i Calculate, for the waves:

1 the amplitude, **[1]**

2 the wavelength. **[2]**

ii Calculate the time period of the oscillations of the vibrator. **[2]**

c State and explain whether the waves on the surface of the water shown in Fig. 7.30 are:

i progressive or stationary, **[1]**

ii transverse or longitudinal. **[1]**

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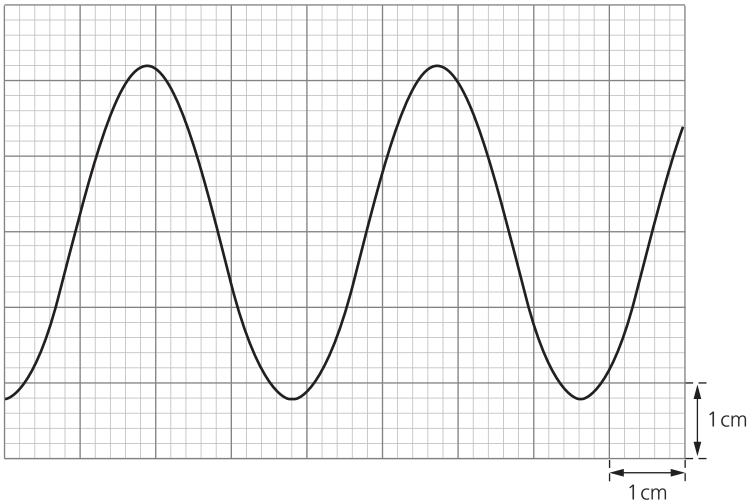
14 A microphone detects a musical note of frequency f. The microphone is connected to a cathode-ray oscilloscope (c.r.o.). The signal from the microphone is observed on the c.r.o. as illustrated in Fig. 7.31.



▲ **Figure 7.31**

The time-base setting of the c.r.o. is 0.50 ms cm^{-1}. The Y-plate setting is 2.5 mV cm^{-1}.

a Use Fig. 7.31 to determine:

i the amplitude of the signal, **[2]**

ii the frequency f, **[3]**

iii the actual uncertainty in f caused by reading the scale on the c.r.o. **[2]**

b State f with its actual uncertainty. **[1]**

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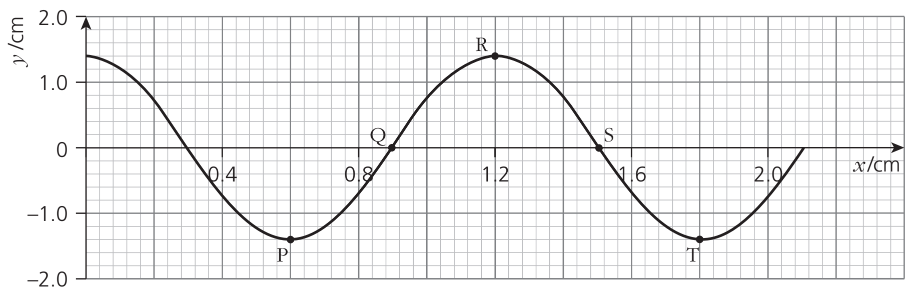
15 a i Explain what is meant by a *progressive transverse wave*. **[2]**

ii Define *frequency*. **[1]**

b The variation with distance x of displacement y for a transverse wave is shown in Fig. 7.32.



▲ **Figure 7.32**

On Fig. 7.32 five points are labelled. Use Fig. 7.32 to state any two points having a phase difference of:

i zero, **[1]**

ii 270°. **[1]**

c The frequency of the wave in **b** is 15 Hz. Calculate the speed of the wave in **b**. **[3]**

d Two waves of the same frequency have amplitudes 1.4 cm and 2.1 cm.

Calculate the ratio $\dfrac{\text{intensity of wave of amplitude 1.4 cm}}{\text{intensity of wave of amplitude 2.1 cm}}$ **[2]**

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16 A source of light of one frequency is viewed through two sheets of Polaroid. The preferred directions of vibration of the light for the two sheets of Polaroid are parallel. The intensity of the observed light after passing through the two sheets of Polaroid is I.

a One sheet of Polaroid is rotated until the amplitude of the vibration of the observed light is reduced to a third of its original value. Calculate the angle rotated by the sheet of Polaroid.

b State the effect of this rotation on the intensity of the light transmitted.

c Calculate the angle of rotation of one sheet to reduce the intensity of the transmitted light to one third of its original value.

17 Light reflected from the surface of smooth water may be described as a polarised transverse wave.

a By reference to the direction of propagation of energy, explain what is meant by:

i a transverse wave, **[1]**

ii polarisation. **[1]**

Cambridge International AS and A Level Physics (9702) Paper 02 Q5 part a May/June 2007

8 Superposition

Learning outcomes

By the end of this topic, you will be able to:

8.1 Stationary waves

1. explain and use the principle of superposition
2. show an understanding of experiments that demonstrate stationary waves using microwaves, stretched strings and air columns
3. explain the formation of a stationary wave using a graphical method and identify nodes and antinodes
4. understand how wavelength may be determined from the positions of nodes and antinodes of a stationary wave

8.2 Diffraction

1. explain the meaning of the term diffraction
2. show an understanding of experiments that demonstrate diffraction including the qualitative effect of the gap width relative to the wavelength of the wave, for example, diffraction of water waves in a ripple tank

8.3 Interference

1. understand the terms interference and coherence
2. show an understanding of experiments that demonstrate two-source interference using water waves, sound, light and microwaves
3. understand the conditions required if two-source interference fringes are to be observed
4. recall and use $\lambda = ax/D$ for double-slit interference using light

8.4 The diffraction grating

1. recall and use $d\sin\theta = n\lambda$
2. describe the use of a diffraction grating to determine the wavelength of light

Starting points

- ★ Basic properties of waves such as reflection and refraction.
- ★ Knowledge of the terms used to describe waves.
- ★ Graphical representation of waves.

8.1 Superposition and interference of waves

Any moment now the unsuspecting fisherman in Figure 8.1 is going to experience the effects of interference. The amplitude of oscillation of his boat will be significantly affected by the two approaching waves and their interaction when they reach his position.

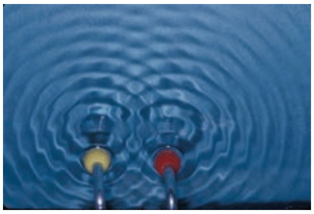
▲ **Figure 8.1**

If two or more waves overlap, the waves are said to interfere. The resultant at any point is the sum of the individual displacements of each of the waves. Remember that displacement is a vector quantity. This may lead to a resultant wave of either a larger or a smaller amplitude than either of the two component waves.

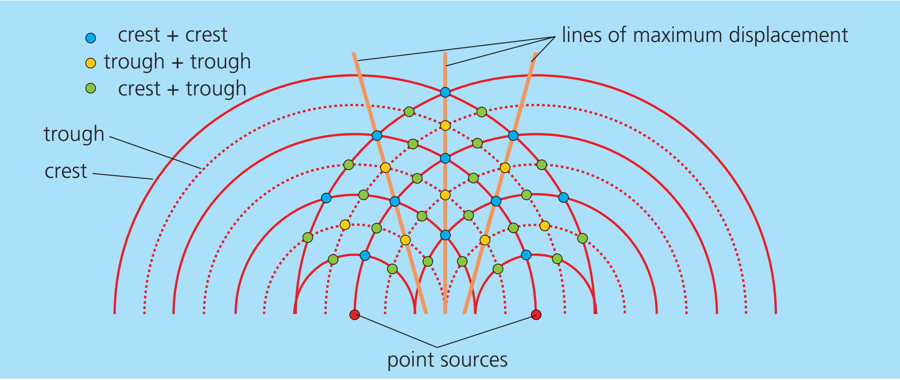
Interference is where two or more waves meet or overlap to form a resultant wave. The resultant displacement at any point is the sum of displacements of the individual waves.

Interference effects can be observed with all types of waves, for example, surface water waves, sound and electromagnetic waves.

Interference can be demonstrated in a ripple tank (see Figure 7.6 in Topic 7) by using two point sources. Figure 8.2 shows the effect produced, and Figure 8.3 shows how it arises.



▲ **Figure 8.2** The interference pattern obtained using two point sources to produce circular water waves in a ripple tank

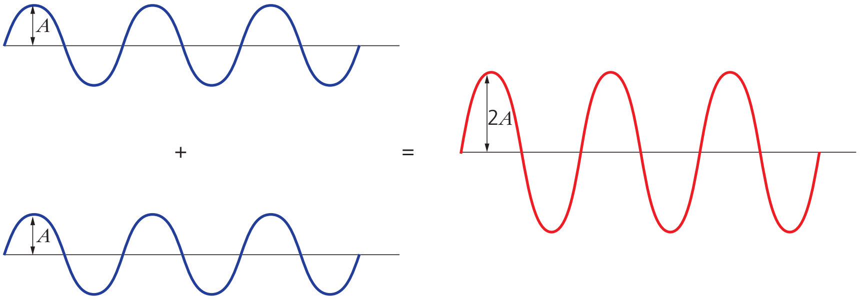


▲ **Figure 8.3** Two-source interference of circular waves

Constructive and destructive interference

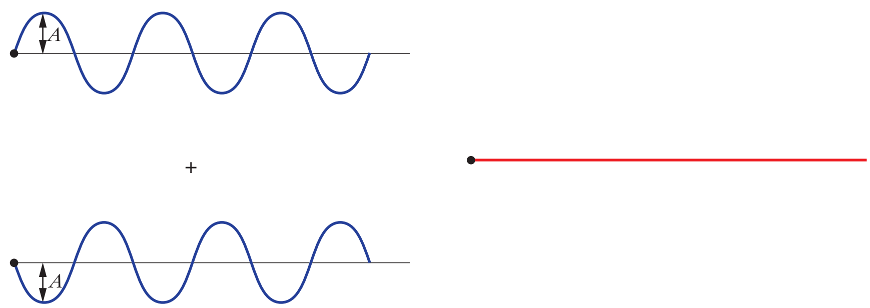
The overlapping of two waves of the same frequency may produce an interference pattern.

Figure 8.4 shows two waves of the same frequency arriving at a point **in phase**. The waves interfere **constructively**. The frequency of the resultant is the same as that of the incoming waves. A resultant wave will be produced which has crests much higher than either of the two individual waves, and troughs which are much deeper. In this case the incoming waves have equal amplitude A, so the resultant wave produced by constructive interference has an amplitude of $2A$.



▲ **Figure 8.4** Constructive interference

In the case shown in Figure 8.5, the incoming waves arrive with a phase difference of π radians or 180° (**in antiphase**). The peaks of one wave arrive at the same time as the troughs from the other and they will interfere **destructively.** Where the incoming waves have equal amplitude, as in Figure 8.5, the resultant wave has zero amplitude.



▲ **Figure 8.5** Destructive interference

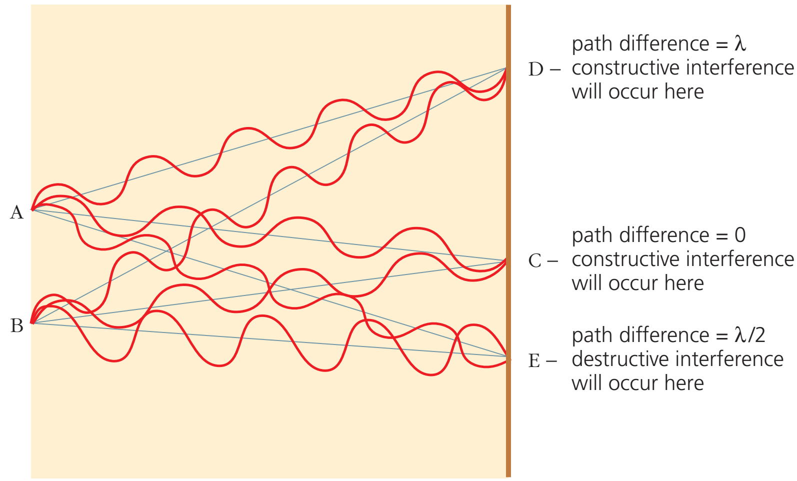
This situation is an example of the **principle of superposition of waves**. The principle describes how waves, which meet at the same point in space, interact.

The principle of superposition states that, when two or more waves meet at a point, the resultant displacement at that point is equal to the sum of the displacements of the individual waves at that point.

Because displacement is a vector, we must remember to add the individual displacements taking account of their directions. The principle applies to all types of wave.

If we consider the effect of superposition at a number of points in space, we can build up a pattern showing some areas where there is constructive interference, and hence a large wave disturbance, and other areas where the interference is destructive, and there is little or no wave disturbance.

Figure 8.6 illustrates the interference of waves from two point sources A and B. The point C is equidistant from A and B: a wave travelling to C from A moves through the same distance as a wave travelling to C from B (their **path difference** is zero). If the waves started in phase at A and B, they will arrive in phase at C (**phase difference** is zero). They combine constructively, producing a maximum disturbance at C.



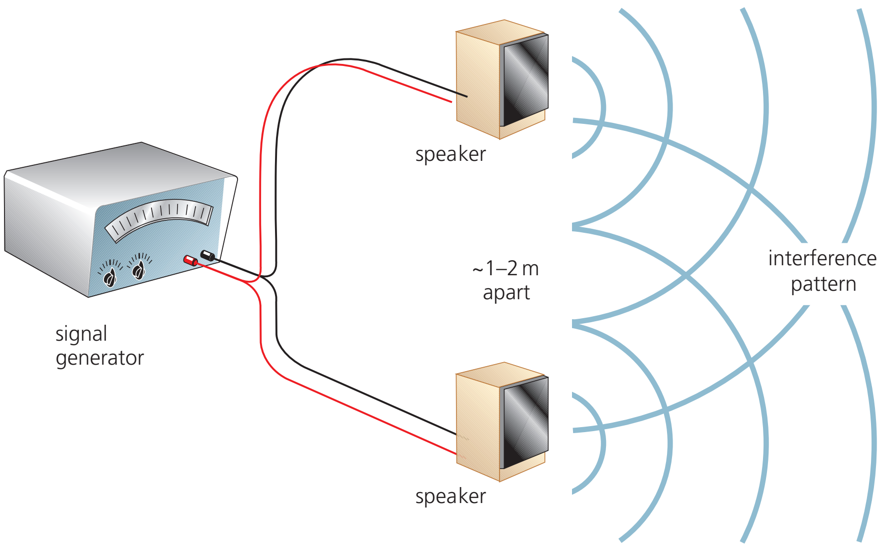
▲ **Figure 8.6** Producing an interference pattern

At other places, such as D, the waves will have travelled different distances from the two sources. There is a path difference between the waves arriving at D. If this path difference is a whole number of wavelengths (1λ, 2λ, 3λ, etc.) the waves arrive in phase and interfere constructively, producing maximum disturbance again. The equivalent phase differences between the waves are 2π radians, 4π radians, 6π radians, etc. or 360°, 720°, 1080°, etc. However, at places such as E, the path difference is an odd number of half-wavelengths ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.). The waves arrive at E in antiphase, and interference is destructive, producing a minimum resultant disturbance. The equivalent phase differences between the waves are π radians, 3π radians , 5π radians etc. or 180°, 540°, 900°, etc. The maxima and minima disturbances are called **fringes**. The collection of fringes produced by the superposition of overlapping waves is called an **interference pattern**. One is shown in Figure 8.2 for water waves.

Producing an interference pattern with sound waves

Figure 8.7 shows an experimental arrangement to demonstrate interference with sound waves from two loudspeakers connected to the same signal generator, so that each speaker produces a note of the same frequency. The demonstration is best carried out in the open air (on playing-fields, for example) to avoid reflections from walls, but it should be a windless day. Moving about in the space in front of the speakers, you pass through places where the waves interfere constructively and you can hear a loud sound. In places where the waves interfere destructively, the note is much quieter than elsewhere in the pattern.

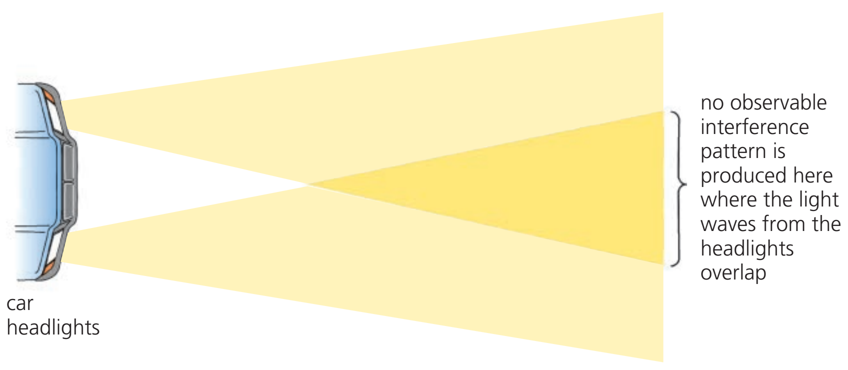
The positions of maximum and minimum sound can also be detected using a microphone and CRO similar to the arrangement shown in Figure 7.11 in Topic 7.



▲ **Figure 8.7** Demonstration of interference with sound waves

Producing an interference pattern with light waves

If you try to set up a demonstration with two separate light sources, such as car headlights, you will find that it is not possible to produce an observable interference pattern (Figure 8.8). A similar demonstration works with sound waves from two loudspeakers, each connected to separate signal generators. What has gone wrong?



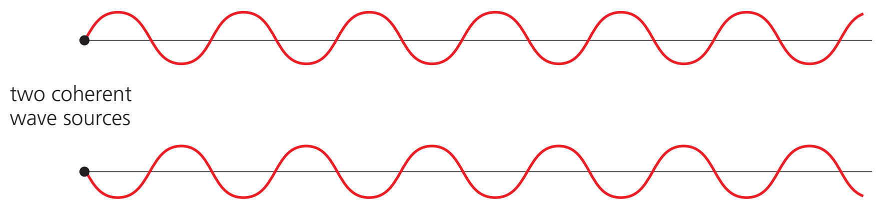
▲ **Figure 8.8** Failure of an interference demonstration with light

To produce an observable interference pattern, the two wave sources must have the same **single frequency**, not a mixture of frequencies as is the case for light from car headlights. They must also have a **constant phase difference**. In the sound experiment, the waves from the two loudspeakers have the same frequency and a constant phase difference because the loudspeakers are connected to the same oscillator and amplifier. The waves emitted from the speakers are in phase when the experiment begins and they stay in phase for the whole experiment.

Wave sources which maintain a constant phase difference are described as **coherent** sources.

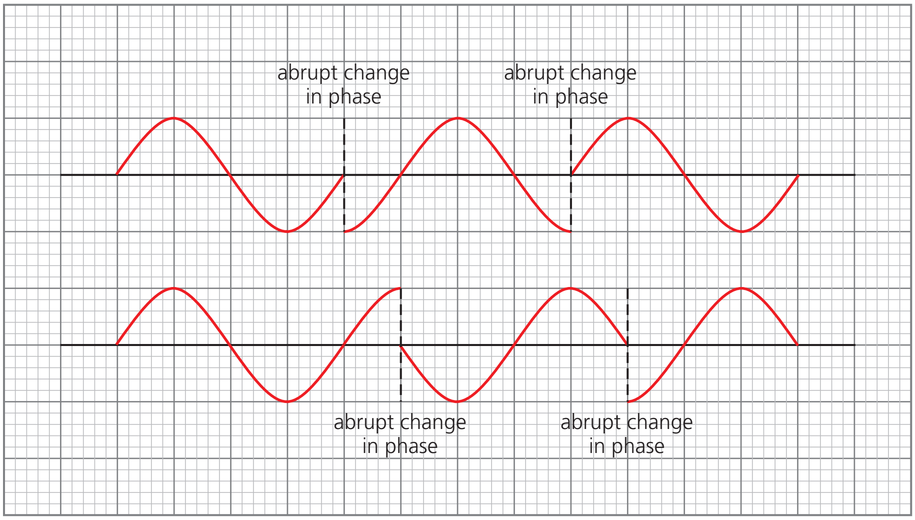
Two or more waves are coherent if they have a constant phase difference.

Two waves from coherent sources are illustrated in Figure 8.9.



▲ **Figure 8.9** Coherent waves

Light is emitted from sources as a series of pulses or packets of energy. These pulses last for a very short time, about a nanosecond (10^{-9} s). Between each pulse there is an abrupt change in the phase of the waves. Waves from two separate sources may be in phase at one instant, but out of phase in the next nanosecond. The human eye cannot cope with such rapid changes, so the pattern is not observable. Separate light sources, even of the same frequency, produce incoherent waves (Figure 8.10).

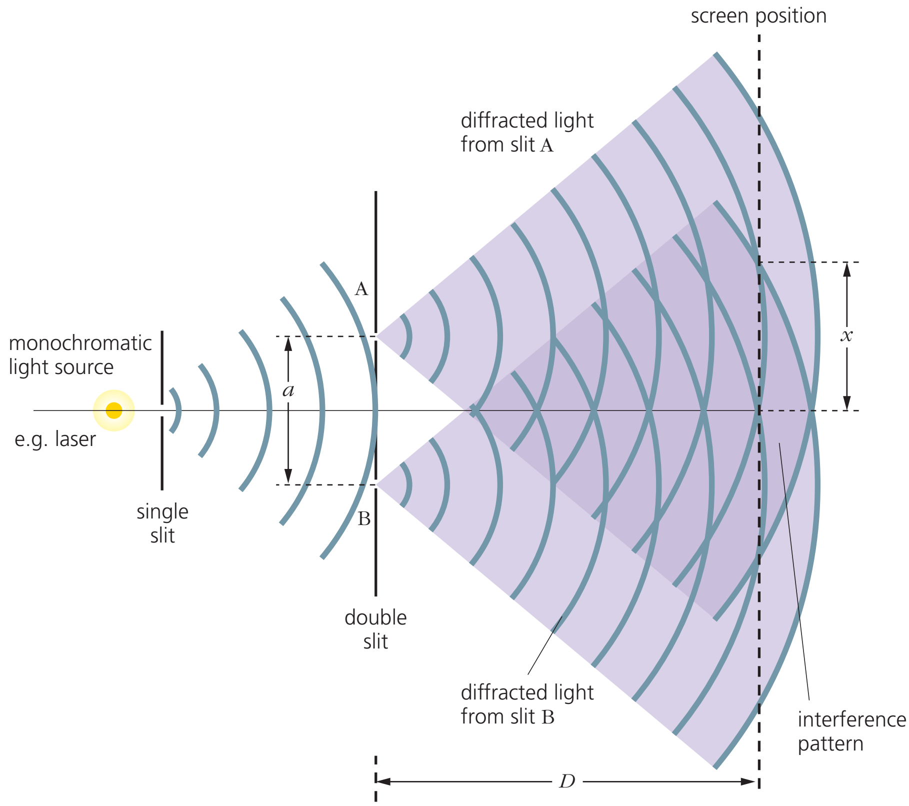


▲ **Figure 8.10** Incoherent light waves

To obtain observable interference patterns, it is not essential for the amplitudes of the waves from the two sources to be the same. However, if the amplitudes are not equal, a completely dark fringe will never be obtained, and the contrast of the pattern is reduced.

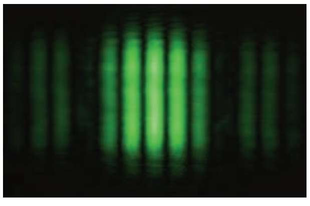
Young's double-slit experiment

In 1801 Thomas Young (1773–1829) demonstrated how light waves could produce an interference pattern. The experimental arrangement is shown in Figure 8.11 (not to scale).



▲ **Figure 8.11** Young's double-slit experiment

A monochromatic light source (a source of one colour, and hence one wavelength λ) is placed behind a single slit to create a small, well-defined source of light. Light from this source is diffracted at the slit (diffraction of waves at a gap is described in more detail in Topic 8.3), producing two light sources at the double slit A and B. Because these two light sources originate from the same primary source, they are coherent and create a sustained and observable interference pattern, as seen in the photograph of the dark and bright interference fringes in Figure 8.12. Bright fringes are seen where constructive interference occurs – that is, where the path difference between the two diffracted waves from the sources A and B is $n\lambda$, where n is a whole number. Dark fringes are seen where destructive interference occurs. The condition for a dark fringe is that the path difference should be $\left(n+\frac{1}{2}\right)\lambda$.



▲ **Figure 8.12** Fringe pattern in Young's experiment

The distance x on the screen between successive bright fringes (or between successive dark fringes) is called the fringe width or fringe separation. The fringe width is the same for bright and dark fringes and is related to the wavelength λ of the light source by the equation

$$\lambda = ax/D$$

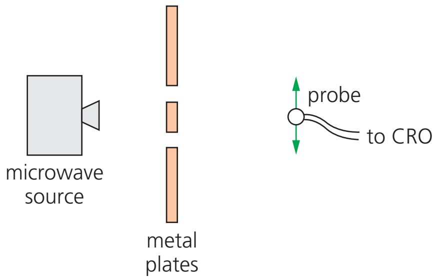
where x is the fringe width, D is the distance from the double slit to the screen and a is the distance between the centres of the slits. Note that, because the wavelength of light is so small (of the order of 10^{-7} m), to produce observable fringes D needs to be large (about 1 to 2 m) and a as small as possible (about 0.5 to 1 mm). (This is another reason why you could never see an interference pattern from two sources such as car headlamps.) See Worked Example 8A overleaf.

Note that the fringe width (x) is proportional to D (a and λ constant), proportional to λ (D and a constant) and inversely proportional to a (λ and D constant).

Although Young's original double-slit experiment was carried out with light, the conditions for constructive and destructive interference apply for any two-source situation. The same formula applies for all types of wave, including sound waves, surface water waves and microwaves, provided that the fringes are detected at a distance of many wavelengths from the two sources.

Double-slit interference with microwaves

Figure 8.13 shows an experimental arrangement to demonstrate interference with microwaves. The microwaves of a single frequency are emitted from a microwave source and detected with a microwave probe after passing through two slits. Metal plates are used to produce the double slit arrangement. Microwaves with a wavelength of a few centimetres are generally used. The microwaves are diffracted (see Topic 8.3) at each slit and then the waves from each slit overlap to produce an interference pattern.



▲ **Figure 8.13** Demonstration of interference with microwaves

The probe detects places of constructive interference (maxima) and destructive interference (minima) and these are shown as a trace on the CRO. If the probe is moved parallel to the metal plates the fringe width can be measured and the wavelength of the microwaves determined.

The conditions for interference fringes to be observed

The interference pattern (system of maxima and minima fringes) is detected when the sources are coherent. The sources are coherent if they have a constant phase difference. This implies they must have the same frequency and wavelength.
The contrast between the fringes is improved if the amplitudes are the same or similar. If the waves are transverse they must be unpolarised or polarised in the same plane (see Topic 7.4).

Table 8.1 shows typical dimensions for two-source interference with different types of waves.

type of wave	λ/cm	D/cm	a/cm	x/cm
water ripples	1	20	4	5
sound	60	100	150	40
light	5×10^{-5}	200	0.05	0.2
microwaves	3	50	5	30

▲ **Table 8.1** Typical dimensions for two-source interference of waves

WORKED EXAMPLE 8A

Calculate the observed fringe width for a Young's double-slit experiment using light of wavelength 600 nm and slits 0.50 mm apart. The distance from the slits to the screen is 0.80 m.

Answer

Using $x = \lambda D/a$,

$$x = 600 \times 10^{-9} \times 0.80/0.50 \times 10^{-3} = 9.6 \times 10^{-4}\,\text{m}$$

$= \mathbf{0.96\,mm}$

Questions

1 Calculate the wavelength of light which produces fringes of width 0.50 mm on a screen 60 cm from two slits 0.75 mm apart.

2 Radar waves of wavelength 50 mm are emitted from two aerials and create a fringe pattern 1.0 km from the aerials. Calculate the distance between the aerials if the fringe spacing is 80 cm.

White light fringes

If the two slits in Young's experiment are illuminated with white light, each of the different wavelengths making up the white light produces their own fringe pattern. These fringe patterns overlap. At the centre of the pattern, where the path difference for all waves is zero, there will be a white maximum with a black fringe on each side. Thereafter, the maxima and minima of the different colours overlap in such a way as to produce a pattern of white fringes with coloured edges. Blue appears on the edge nearest the central white fringe and red on the edge furthest from the central white fringe. Only a few will be visible; a short distance from the centre so many wavelengths overlap that they combine to produce what is effectively white light again.

8.2 Superposition and stationary waves

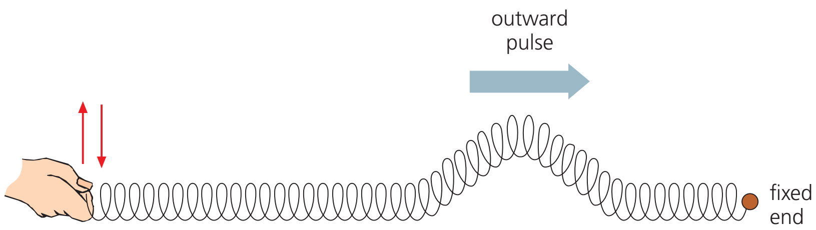
The notes we hear from a cello are created by the vibrations of its strings (Figure 8.14). The wave patterns on the vibrating strings are called **stationary waves** (or **standing waves**). The waves in the air which carry the sound to our ears transfer energy and are, therefore, **progressive waves** (see Topic 7.1).

▲ **Figure 8.14** Cello being bowed

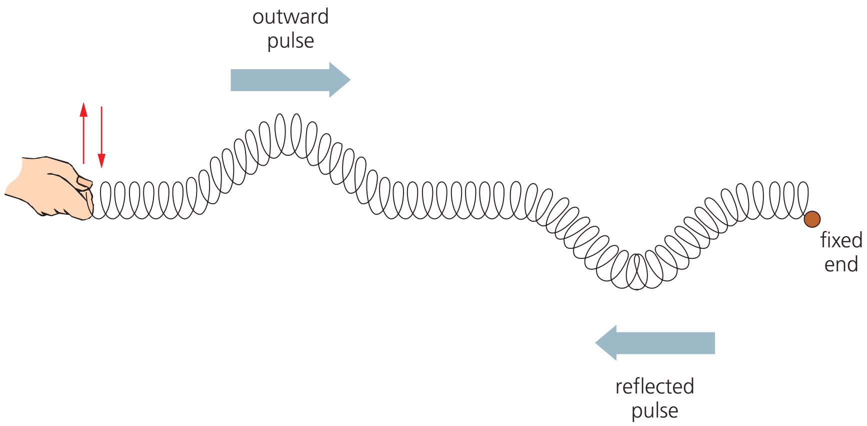
Figure 8.15 shows a single transverse pulse travelling along a 'slinky' spring. The pulse is reflected when it reaches the fixed end. If a second pulse is sent along the slinky (Figure 8.16), the reflected pulse will pass through the outward-going pulse, creating a new pulse shape. Interference will take place between the outward and reflected pulses.

If the interval between outward pulses is reduced, a progressive wave is generated. When the wave reaches the fixed end, it is reflected. We now have two progressive waves of equal frequency and amplitude travelling in opposite directions on the same spring. The waves overlap and interfere, producing a wave pattern (Figure 8.17) in which the crests and troughs do not move. This pattern is called a stationary or standing wave because it does not move.

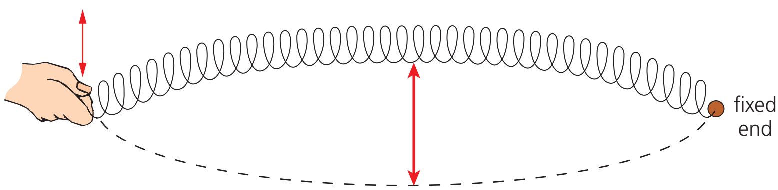
A stationary wave is the result of the overlapping and hence interference of two waves of equal frequency and amplitude, travelling along the same line with the same speed but in opposite directions.



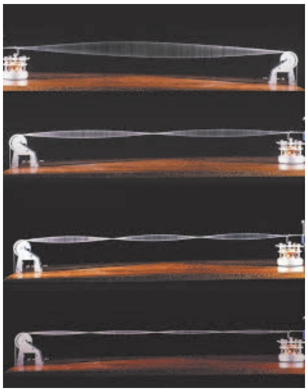
▲ **Figure 8.15** Single transverse pulse travelling along a slinky



▲ **Figure 8.16** Reflected pulse about to meet an outward-going pulse



▲ **Figure 8.17** A stationary wave is created when two waves travelling in opposite directions interfere.



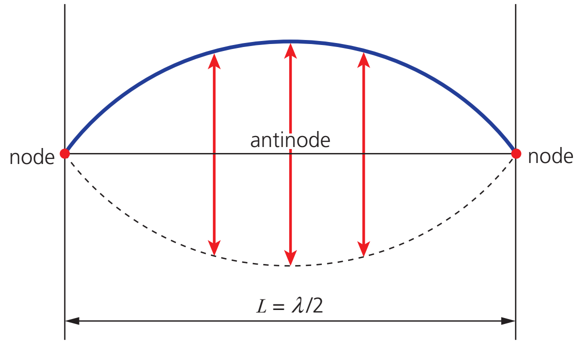
▲ **Figure 8.18** First four modes of vibration of a string

Stationary waves on strings

If a string is plucked and allowed to vibrate freely, there are certain frequencies at which it will vibrate. The amplitude of vibration at these frequencies is large. This is known as a **resonance** effect.

It is possible to investigate stationary waves in a more controlled manner using a length of string under tension and a vibrator driven by a signal generator. As the frequency of the vibrator is changed, different standing wave patterns are formed. Some of these are shown in Figure 8.18.

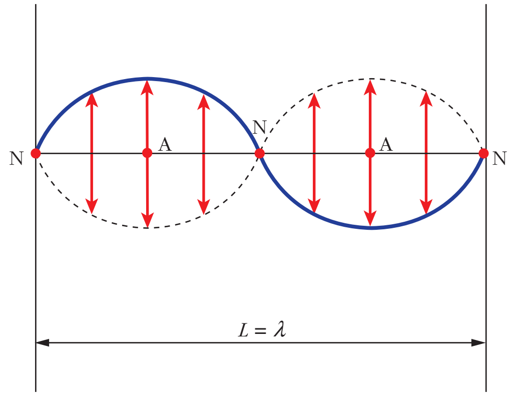
Figure 8.19 on the next page shows the simplest way in which a stretched string can vibrate. The wave pattern has a single loop. This is called the **fundamental mode** of vibration, also called the **first harmonic**. At the ends of the string there is no vibration. These points have zero amplitude and are called **nodes**. At the centre of the string, the amplitude is a maximum. A point of maximum amplitude is called an **antinode**. Nodes and antinodes do not move along the string.



▲ **Figure 8.19** Fundamental mode of vibration of a stretched string

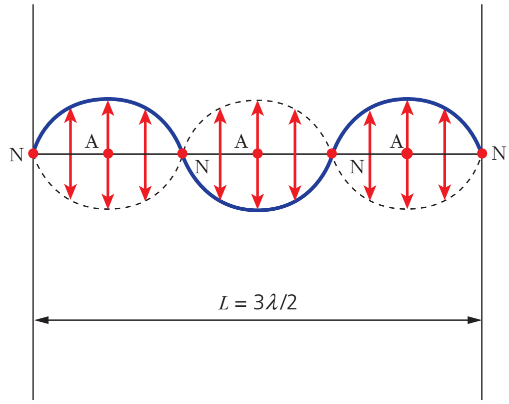
The wavelength λ of the standing wave in the fundamental mode is $2L$. From the wave equation $c = f\lambda$, the frequency f_1 of the note produced by the string vibrating in its fundamental mode is given by $f_1 = c/2L$, where c is the speed of the progressive waves which have interfered to produce the stationary wave.

Figure 8.20 shows the next resonant frequency of the string. The stationary wave pattern has two loops. This frequency is sometimes called the **first overtone**, or the **second harmonic** (don't be confused!). The wavelength is L. Applying the wave equation, the frequency f_2 is found to be c/L.



▲ **Figure 8.20** Second mode of vibration of a stretched string

Figure 8.21 shows the next resonant frequency (the second overtone, or third harmonic). This is a pattern with three loops. The wavelength is $2L/3$, and the frequency f_3 is $3c/2L$.



▲ **Figure 8.21** Third mode of vibration of a stretched string

EXTENSION

The general expression for the frequency f_n of the nth mode (or the nth harmonic, or $(n-1)$th overtone) is

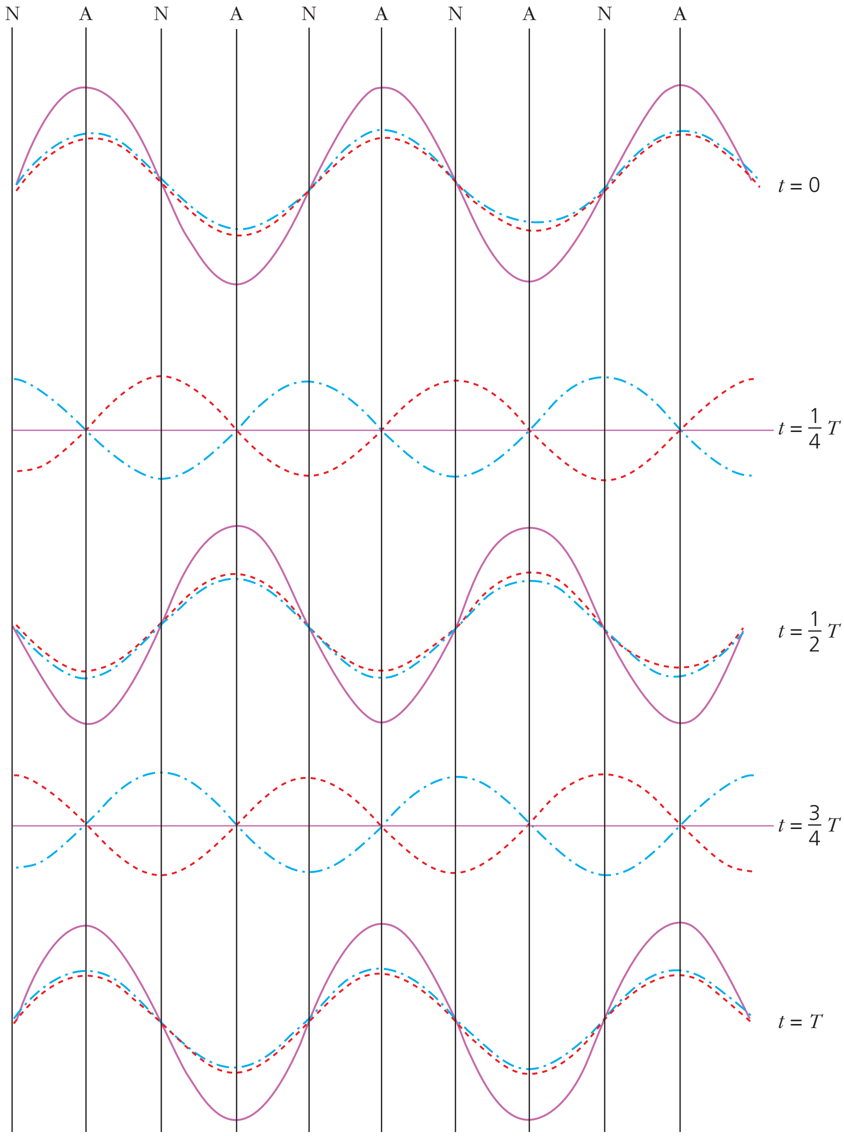
$$f_n = nc/2L \qquad n = 1, 2, 3, \ldots$$

The key features of a stationary wave pattern on a string, which distinguish it from a progressive wave, are as follows.

- The amplitude of vibration varies with position along the string: it is zero at a node, and maximum at an antinode. In a progressive wave, all points have the same amplitude.
- The wavelength is twice the distance between adjacent nodes or antinodes. In a progressive wave it is the distance between adjacent points having the same phase (crest to adjacent crest or trough to adjacent trough).
- The nodes and antinodes do not move along the string, whereas in a progressive wave, the waveform (crests and troughs) moves with the velocity of the wave.
- There is no translational movement of energy but there is energy associated with the wave. In a progressive wave there is energy translation in the direction of movement of the waveform.
- Between adjacent nodes, all points of the stationary wave vibrate in phase. That is, all particles of the string are at their maximum displacement at the same instant. The particles between two adjacent nodes are π radians or 180° out of phase with the next two adjacent nodes. In a progressive wave, phase varies continuously along the wave. All neighbouring points along one wavelength are out of phase with each other, they reach their maximum displacement at different times.
- There are only certain frequencies of stationary waves possible on the string. The allowed frequencies depend on the length L of the string and c, the speed of the progressive waves that form the stationary wave.

Explaining the formation of stationary waves

Let us explain the formation of a stationary wave using the principle of superposition. The set of graphs in Figure 8.22 overleaf represents displacement–distance graphs for two progressive waves of equal amplitude and frequency travelling in opposite directions, such as along a plucked string. The red-dashed line is for the wave travelling from left to right, and the blue-dashed line is for the wave going from right to left. This represents a string clamped at the right-hand end, producing a reflection (in blue) of the wave travelling from left to right (red). The effect of the clamp is to change the phase of the reflected wave by π radians (180°).



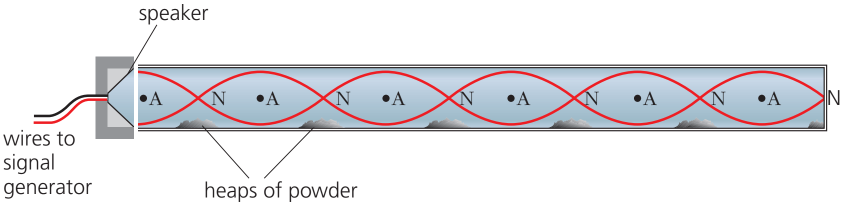
▲ **Figure 8.22** Formation of a stationary wave by superposition of two progressive waves travelling in opposite directions

The top graph shows the waves at an instant at which they are in phase. Superposition gives the purple curve, which has twice the amplitude of either of the progressive graphs. The second graph is the situation a quarter of a period (cycle) later, when the two progressive waves have each moved a quarter of a wavelength in opposite directions. This has brought them to a situation where the movement of one relative to the other is half a wavelength, so that the waves are exactly out of phase. The resultant, obtained by superposition, is zero everywhere. In the third graph, half a period from the start, the waves are again in phase, with maximum displacement for the resultant. The process continues through the fourth graph, showing the next out-of-phase situation, with zero displacement of the resultant everywhere. Finally, the fifth graph, one period on from the first, brings the waves into phase again.

We can see how there are some positions, the nodes N, where the displacement of the resultant is zero *throughout* the cycle. The displacement of the resultant at the antinodes A fluctuates from a maximum value when the two progressive waves are in phase to zero when they are out of phase. This explains the stationary waves shown in Figures 8.19, 8.20 and 8.21.

Stationary waves in air

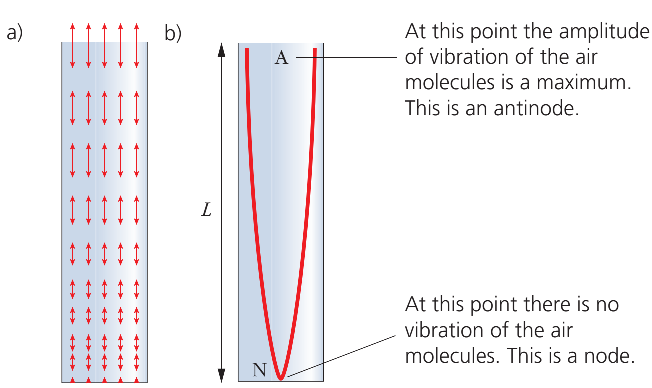
Figure 8.23 shows an experiment to demonstrate the formation of stationary waves in air. A fine, dry powder (such as cork dust or lycopodium powder) is sprinkled evenly along the transparent tube. A loudspeaker powered by a signal generator is placed at the open end. The frequency of the sound from the loudspeaker is gradually increased. At certain frequencies, the powder forms itself into evenly spaced heaps along the tube. A stationary wave has been set up in the air, caused by the interference of the sound wave from the loudspeaker and the wave reflected from the closed end of the tube. At nodes (positions of zero amplitude) there is no disturbance, and the powder can settle into a heap. At antinodes the disturbance is at a maximum, and the powder is dispersed.



▲ **Figure 8.23** Demonstration of stationary waves in air

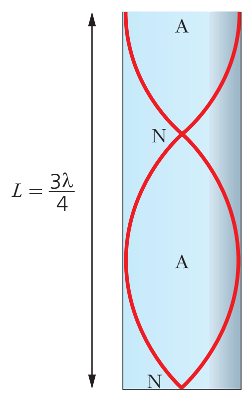
For stationary waves in a closed pipe, the air cannot move at the closed end, and so this must always be a node N. However, the open end is a position of maximum disturbance, and this is an antinode A. (In fact, the antinode is slightly outside the open end. The distance of the antinode from the end of the tube is called the end-correction. The value of the end-correction depends on the diameter of the tube.)

Figure 8.24 shows the simplest way in which the air in a pipe, closed at one end, can vibrate. Figure 8.24a illustrates the motion of some of the air particles in the tube. Their amplitude of vibration is zero at the closed end, and increases with distance up the tube to a maximum at the open end. This representation is tedious to draw, and Figure 8.24b is the conventional way of showing the amplitude of vibration: the amplitudes along the axis of the tube are plotted as a continuous curve. One danger of using diagrams like Figure 8.24b is that they give the impression that the sound wave is transverse rather than longitudinal. So be warned! The mode illustrated in Figure 8.24 is the fundamental mode (the first harmonic). The wavelength of this stationary wave (ignoring the end-correction) is $4L$, where L is the length of the pipe. Using the wave equation, the frequency f_1 of the fundamental mode is given by $f_1 = c/4L$, where c is the speed of the sound in air.

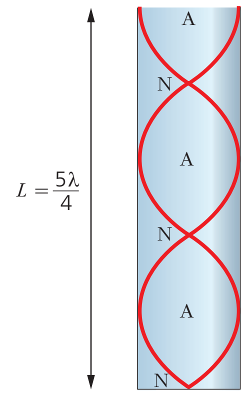


▲ **Figure 8.24** Fundamental mode of vibration of air in a closed pipe

Other modes of vibration are possible. Figures 8.25 and 8.26 (overleaf) show the second mode (the first overtone, or second harmonic) and the third mode (the second overtone, or third harmonic). The corresponding wavelengths are $4L/3$ and $4L/5$, and the frequencies are $f_2 = 3c/4L$ and $f_3 = 5c/4L$.



▲ **Figure 8.25** Second mode of vibration of air in a closed pipe



▲ **Figure 8.26** Third mode of vibration of air in a closed pipe

EXTENSION

The general expression for the frequency f_n of the nth mode of vibration of the air in the closed tube (the nth harmonic, or the $(n-1)$th overtone) is

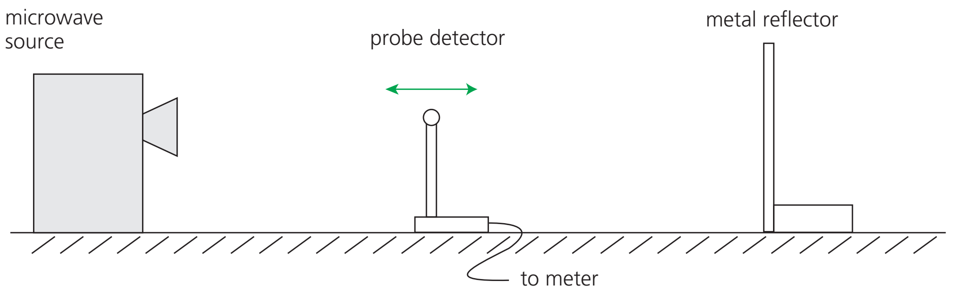
$$f_n = \frac{(2n-1)c}{4L}$$

This is another example of resonance. The particular frequencies at which stationary waves are obtained in the pipe are also known as the resonant frequencies of the pipe. The particular frequencies of stationary waves possible in the pipe depend on the length L of the pipe and c, the speed of sound in air.

Measuring the wavelength of stationary waves

Stationary waves using microwaves

Stationary waves can be demonstrated using microwaves. A source of microwaves faces a metal reflecting plate, as shown in Figure 8.27. The microwaves from the source reflect off the metal reflector and overlap with the incoming waves from the source to produce stationary waves. A small probe detector is placed between source and reflector. The reflector is moved towards or away from the source until the signal picked up by the detector fluctuates regularly as it is moved slowly back and forth. The minima are nodes of the stationary wave pattern, and the maxima are antinodes. The distance moved by the detector between successive nodes or successive antinodes is half the wavelength of the microwaves.

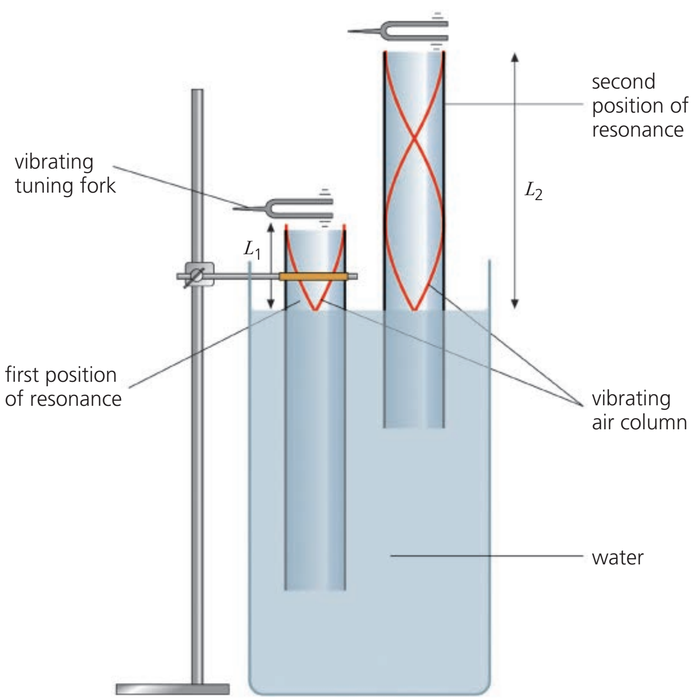


▲ **Figure 8.27** Using microwaves to demonstrate stationary waves

Stationary waves using sound

The principle of resonance in a tube closed at one end can be used to measure the wavelength of sound in air and hence the speed of sound in air. A glass tube is placed in a cylinder of water. By raising the tube, the length of the column of air can be increased. A vibrating tuning fork of known frequency f is held above the open end of the glass tube, causing the air in it to vibrate and make a sound of the same frequency. A loud speaker connected to a signal generator can be used in place of the tuning fork to produce a known frequency of sound. The tube is gradually raised, increasing the length of the air column. At a certain position the note becomes much louder.

The air in the tube vibrates with a loud sound (resonance) when a stationary wave is produced in air in the tube. The lowest frequency that produces resonance occurs when the stationary wave is formed as shown on the left hand side of Figure 8.28. This is known as the first position of resonance, and occurs when a stationary wave corresponding to the fundamental mode is established inside the tube with one of its antinodes at the top of the tube. The length L_1 of the air column is noted. The tube is raised further until the next position at which the sound is much louder is found. This position of second resonance corresponds to the second mode of vibration. The length L_2 at this position is also noted. The two tube positions and the stationary waves corresponding to these positions are illustrated in Figure 8.28.



▲ **Figure 8.28 Wavelength of sound by the resonance tube method**

At the first position of resonance, $\lambda/4 = L_1 + e$, where e is the end-correction of the tube (to allow for the fact that the antinode is slightly above the open end of the tube). At the second position of resonance, $3\lambda/4 = L_2 + e$. By subtracting these equations, we can eliminate e

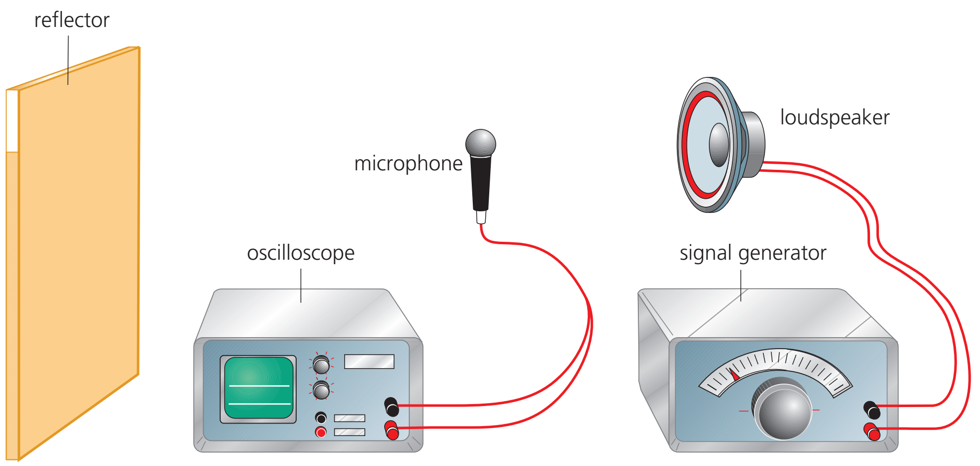
$$\lambda/2 = L_2 - L_1$$

from which the wavelength of sound can be found. Note that the distance between the two successive antinodes is $\lambda/2$.

From the wave equation, the speed of sound c is given by $c = f\lambda$. Thus

$$c = 2f(L_2 - L_1)$$

Figure 8.29 (overleaf) illustrates a method of measuring the wavelength and speed of sound using stationary waves in free air, rather than in the tube shown in Figure 8.28. The signal generator and loudspeaker produce a note of known frequency f.



▲ **Figure 8.29** Wavelength and speed of sound using stationary waves in free air

The reflector is moved slowly back and forth until the trace on the oscilloscope has a minimum amplitude. When this happens, a stationary wave has been set up with one of its nodes in the same position as the microphone. The microphone is now moved along the line between the loudspeaker and the reflector. The amplitude of the trace on the oscilloscope will increase to a maximum, and then decrease to a minimum. The microphone has been moved from one node, through an antinode, to the next node. The distance d between these positions is measured. We know that the distance between nodes is $\lambda/2$, so the wavelength can be easily found. The speed of sound can then be calculated using $c = f\lambda$, giving $c = 2fd$.

WORKED EXAMPLE 8B

1 A string 75 cm long is fixed at one end. The other end is moved up and down with a frequency of 15 Hz. This frequency gives a stationary wave pattern with three complete loops on the string. Calculate the speed of the progressive waves which have interfered to produce the stationary wave.

2 Find the fundamental frequency and next two possible resonant frequencies for an organ pipe which is 0.17 m long and closed at one end. The speed of sound in air is 340 m s^{-1}.

Answers

1 The three-loop pattern corresponds to the situation where the length L of the string is $3\lambda/2$ (see Figure 8.21). The wavelength λ is thus $2 \times 0.75/3 = 0.50$ m. The frequency of the wave is 15 Hz, so by the wave equation $c = f\lambda = 15 \times 0.50 =$ **7.5 m s^{-1}**.

2 The frequencies of the fundamental and next two possible resonant frequencies of a tube of length L, closed at one end, are $c/4L$, $3c/4L$ and $5c/4L$ (see Figures 8.24–8.26). The frequencies are thus $340/4 \times 0.17 =$ **500 Hz**, $3 \times 340/4 \times 0.17 =$ **1500 Hz** and $5 \times 340/4 \times 0.17 =$ **2500 Hz**.

Questions

3 A violin string vibrates with the lowest possible (fundamental) frequency of 440 Hz. What are the frequencies of the next two possible resonant frequencies?

4 The speed of waves on a certain stretched string is 48 m s^{-1}. When the string is vibrated at frequency of 64 Hz, stationary waves are set up. Find the separation of successive nodes in the stationary wave pattern.

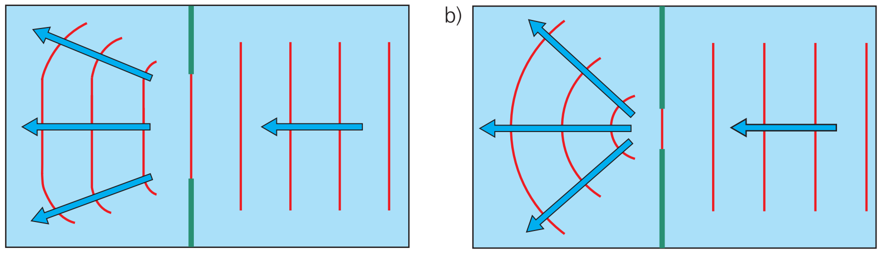
5 You can produce a musical note from an empty lemonade bottle by blowing across the top. This is an example of resonance. What fundamental frequency of vibration would you expect for a bottle 25 cm deep? The speed of sound in air is 340 m s^{-1}.

6 A certain organ pipe, closed at one end, can produce loud sounds only at the following consecutive frequencies: 640 Hz, 896 Hz and 1152 Hz. Deduce its fundamental frequency.

8.3 Diffraction and the diffraction grating

When waves pass through a narrow gap, they spread out. This spreading out is called **diffraction**. The extent of diffraction depends on the width of the gap compared with the wavelength. It is most noticeable if the width of the gap is approximately equal to the wavelength. Diffraction can be demonstrated in a ripple tank (see Topic 7) by using the apparatus shown in Figures 7.6 and 7.7.

Diffraction is illustrated in Figure 8.30. Note that diffraction may also occur at an edge.



▲ **Figure 8.30** Ripple tank pattern showing diffraction at a) a wide gap, b) a narrow gap

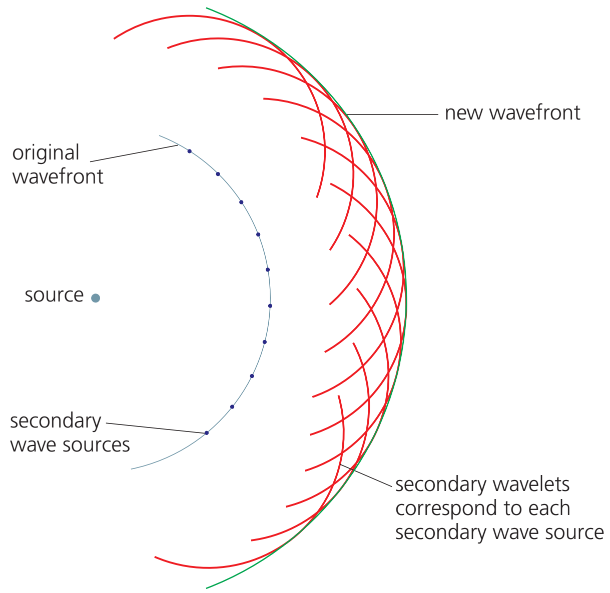
> Diffraction is defined as the spreading of a wave into regions where it would not be seen if it moved only in straight lines after passing through a narrow slit or past an edge.

Although we often hear the statement 'light travels in straight lines', there are occasions when this appears not to be the case. Newton tried to explain the fact that when light travels through an aperture (gap), or passes the edge of an obstacle, it deviates from the straight-on direction and appears to spread out. We have seen from the ripple tank demonstration (Figure 8.30) that water waves spread out when they pass through an aperture. This shows that water waves can be diffracted. The fact that light undergoes diffraction is powerful evidence that light has wave properties. Newton's attempt to explain diffraction was not, in fact, based on a wave theory of light. The Dutch scientist Christian Huygens, a contemporary of Newton, favoured the wave theory, and used it to account for reflection, refraction and diffraction. (It was not until 1815 that the French scientist Augustin Fresnel developed the wave theory of light to explain diffraction in detail.)

The experiment illustrated in Figure 8.30 shows that the degree to which waves are diffracted depends upon the size of the obstacle or aperture and the wavelength of the wave. The greatest effects occur when the wavelength is about the same size as the aperture. The wavelength of light is very small (green light has wavelength 5×10^{-7} m), and, therefore, diffraction effects can be difficult to detect.

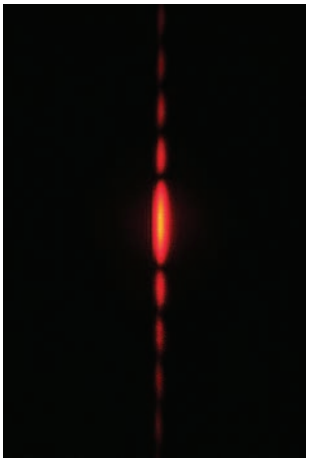
Huygens' explanation of diffraction

If we let a single drop of water fall into a ripple tank, it will create a circular wavefront which will spread outwards from the disturbance (Figure 7.8). Huygens put forward a wave theory of light which was based on the way in which circular wavefronts advance. He suggested that, at any instant, all points on a wavefront could be regarded as secondary disturbances, giving rise to their own outward-spreading circular wavelets. The envelope, or tangent curve, of the wavefronts produced by the secondary sources gives the new position of the original wavefront. This construction is illustrated in Figure 8.31 (overleaf) for a circular wavefront.



▲ **Figure 8.31 Huygens' construction for a circular wavefront**

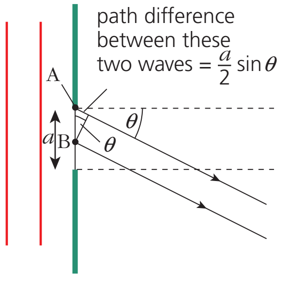
Now think about a plane (straight) wavefront. If the wavefront is restricted in any way, for example, by passing through an aperture in the form of a slit, some of the wavelets making up the wavefront are removed, causing the edges of the wavefront to be curved. If the wavelength is small compared with the size of the aperture, the wavefronts which pass through the aperture show curvature only at their ends, and the diffraction effect is relatively small. If the aperture is comparable with the wavelength, the diffracted wavefronts become circular, centred on the slit. Note that there is no change of wavelength with diffraction. This effect is illustrated in Figure 8.30.



▲ **Figure 8.32 Diffraction of light at a single slit**

Figure 8.32 shows the diffraction pattern created by a single slit illuminated by monochromatic light. The central region of the pattern is a broad, bright area with narrow, dark fringes on either side. Beyond these is a further succession of bright and dark areas. The bright areas become less and less intense as we move away from the centre.

This single-slit diffraction pattern has many features that we associate with an interference pattern. But how can a single slit produce an interference-type pattern? The explanation that follows is based on Huygens' wavelet idea.

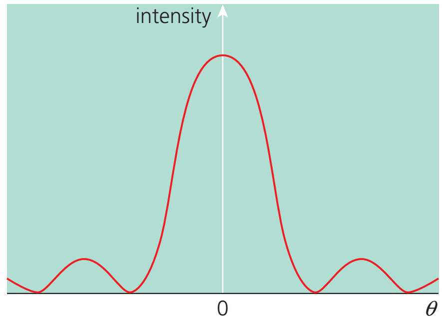


▲ **Figure 8.33 Light leaving a single slit**

Figure 8.33 shows plane wavefronts arriving at a single slit of width a. Each point on the wavefront passing through the slit can be considered to be a source of secondary wavelets.

One such source is at A, at the top edge of the slit, and a second is at B, at the centre of the slit, a distance $a/2$ along the wavefront from A. These two sources behave like the sources in a two-source interference experiment. The wavelets spreading out from these points overlap and create an interference pattern. In the straight-on direction, there is no path difference between the waves from A and B. Constructive interference occurs in this direction, giving a bright fringe in the centre of the pattern. To either side of the central fringe there are directions where the path difference between the waves from A and B is an odd number of half-wavelengths. This is the condition for destructive interference, resulting in dark fringes. The condition for constructive interference is that the path difference should be a whole number of wavelengths. Thus, the dark fringes alternate with bright fringes.

This argument can be applied to the whole of the slit. Every wavelet spreading out from a point in the top half of the slit can be paired with one coming from a point $a/2$ below it in the lower half of the slit. When wavelets from points right across the aperture are added up, we find that there are certain directions in which constructive interference occurs, and other directions in which the interference is destructive. Figure 8.34 is a graph of the intensity of the diffraction pattern as a function of the angle θ at which the light is viewed. It shows that most of the intensity is in the central area, and that this is flanked by dark and bright fringes.



▲ **Figure 8.34** Light intensity graph for single slit diffraction

EXTENSION

We can use Figure 8.33 to derive an expression for the angle at which the first dark fringe is obtained. Remember that the condition for destructive interference is that the path difference between the two rays should be half a wavelength. The path difference between the two rays shown in Figure 8.33 is $\frac{1}{2}a\sin\theta$. If this is to be $\lambda/2$, we have

$$\sin\theta = \lambda/a$$

This is the condition to observe the first dark fringe at angle θ. More generally,

$$\sin\theta = n\lambda/a$$

where n is a whole number called the *order* of the dark fringe being considered, counting outwards from the centre.

Although we have been concentrating on a diffraction pattern obtained with light, the derivation above applies to any type of wave passing through a rectangular aperture.

The wavelength of light is generally small compared with the width of slits or other apertures, so the diffraction angle θ is also small. Provided that θ is only a few degrees (less than about 5°), the approximation $\sin\theta = \theta$ may be used (remember that θ must be in radians!). Very often, the single-slit diffraction equation for light is expressed in the form

$$\theta = n\lambda/a$$

making use of the $\sin\theta$ approximation. But take care! This approximate form may not apply for the diffraction of other types of wave, such as sound or water waves, where the wavelength may be closer in magnitude to the width of the aperture, and diffraction angles are larger.

WORKED EXAMPLE 8C (EXTENSION)

Calculate the angle between the centre of the diffraction pattern and the first minimum when light of wavelength 600 nm passes through a slit 0.10 mm wide.

Answer

Using $a\sin\theta = n\lambda$, we have $\sin\theta = n\lambda/a$. Substituting, $\sin\theta = 1 \times 6.0 \times 10^{-7}/1.0 \times 10^{-4}$
(Don't forget to convert the nm and mm to m.) $= 0.0060$, and $\theta =$ **0.34°**

Using the $\sin\theta = \theta$ approximation, we would have obtained

$\theta = n\lambda/a = 1 \times 6.0 \times 10^{-7}/1.0 \times 10^{-4} =$ **0.0060 rad** (which is equal to 0.34°).

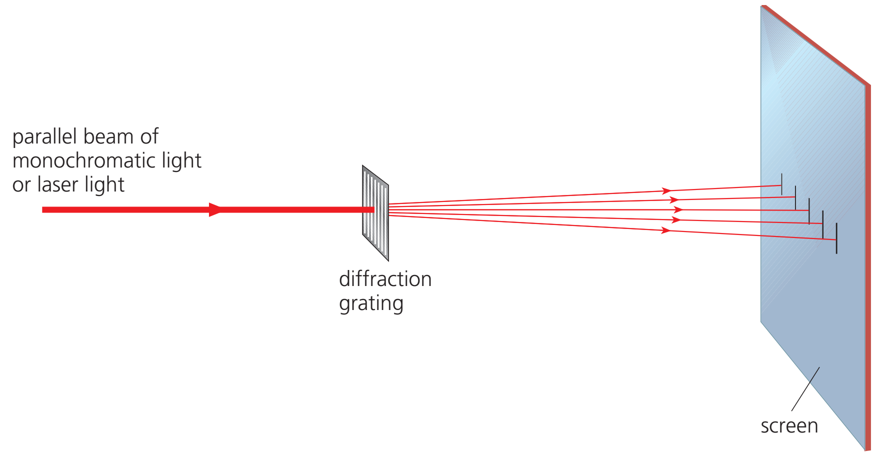
Questions

7 Calculate the angle between the centre of the diffraction pattern and the first minimum when a sound wave of wavelength 1.0 m passes through a door 1.2 m wide.

8 Calculate the wavelength of water waves which, on passing through a gap 50 cm wide, create a diffraction pattern such that the angle between the centre of the pattern and the second-order minimum is 60°.

The diffraction grating

A **diffraction grating** is a plate on which there is a very large number of parallel, identical, very closely spaced slits. If monochromatic light is incident on this plate, a pattern of narrow bright fringes is produced (Figure 8.35).



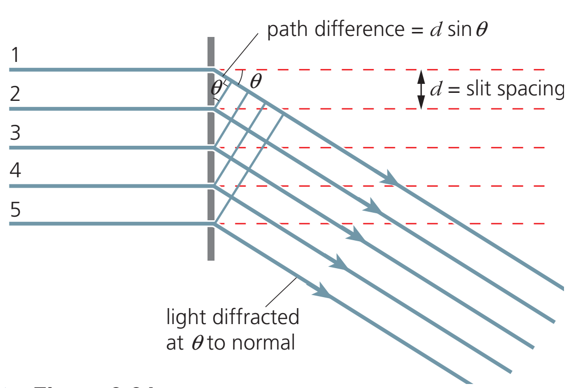
▲ **Figure 8.35** Arrangement for obtaining a fringe pattern with a diffraction grating

Although the device is called a *diffraction* grating, we shall use straightforward superposition and interference ideas in obtaining an expression for the angles at which the maxima of intensity are obtained.

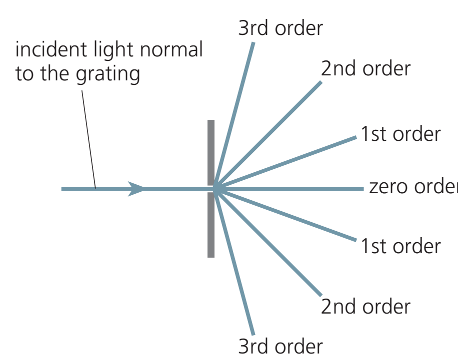
Figure 8.36 shows a parallel beam of light incident normally on a diffraction grating in which the spacing between adjacent slits is d. Consider first rays 1 and 2 which are incident on adjacent slits. The path difference between these rays when they emerge at an angle θ is $d \sin\theta$. To obtain constructive interference in this direction from these two rays, the condition is that the path difference should be a whole number of wavelengths. The path difference between rays 2 and 3, 3 and 4, and so on, will also be $d \sin\theta$. The condition for constructive interference is the same. Thus, the condition for a maximum of intensity at angle θ is

$$d \sin\theta = n\lambda$$

where λ is the wavelength of the monochromatic light used, and n is an integer.



▲ **Figure 8.36**



▲ **Figure 8.37** Maxima in the diffraction pattern of a diffraction grating

When $n = 0$, $\sin\theta = 0$ and θ is also zero; this gives the straight-on direction, or what is called the zero-order maximum. When $n = 1$, we have the first-order diffraction maximum, and so on (Figure 8.37).

WORKED EXAMPLE 8D

Monochromatic light is incident normally on a grating with 7.00×10^5 lines per metre. A second-order maximum is observed at an angle of diffraction of 40.0°. Calculate the wavelength of the incident light.

Answer

The slits on a diffraction grating are created by drawing parallel lines on the surface of the plate. The relationship between the slit spacing d and the number N of lines per metre is $d = 1/N$. For this grating, $d = 1/7.00 \times 10^5 = 1.43 \times 10^{-6}$ m. Using $n\lambda = d\sin\theta$,

$$\lambda = (d/n)\sin\theta = (1.43 \times 10^{-6}/2)\sin 40.0° = \mathbf{460\,nm}$$

Questions

9 Monochromatic light is incident normally on a grating with 5.00×10^5 lines per metre. A third-order maximum is observed at an angle of diffraction of 78.0°. Calculate the wavelength of the incident light.

10 Light of wavelength 5.90×10^{-7} m is incident normally on a diffraction grating with 8.00×10^5 lines per metre. Calculate the diffraction angles of the first- and second-order diffraction images.

11 Light of wavelength 590 nm is incident normally on a grating with spacing 1.67×10^{-6} m. How many orders of diffraction maxima can be obtained?

The diffraction grating with white light

If white light is incident on a diffraction grating, each wavelength λ making up the white light is diffracted by a different amount, as described by the equation $d\sin\theta = n\lambda$. Red light, because it has the longest wavelength in the visible spectrum, is diffracted through the largest angle. Blue light has the shortest wavelength, and is diffracted the least. Thus, the white light is split into its component colours, producing a continuous spectrum (Figure 8.38). The spectrum is repeated in the different orders of the diffraction pattern. Depending on the grating spacing, there may be some overlapping of different orders. For example, the red component of the first-order image may overlap with the blue end of the second-order spectrum. The angular separation of the blue and red ends of each spectrum is greatest for the highest order as indicated in Figure 8.38.

An important use of the diffraction grating is in a **spectrometer**, a piece of apparatus used to investigate spectra. By measuring the angle at which a particular diffracted image appears, the wavelength of the light producing that image may be determined.

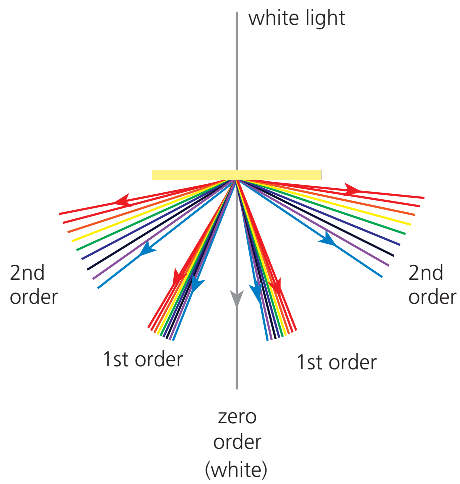


Figure 8.38 Production of the spectrum of white light with a diffraction grating

SUMMARY

- The principle of superposition of waves states that, when waves meet at the same point in space, the resultant displacement is given by the sum of the displacements of the individual waves.
- Constructive interference is obtained when the waves that meet are in phase, so that the resultant wave is of greater amplitude than any of its constituents.
- Destructive interference is obtained when the waves that meet are out of phase by π radians or 180° out of phase (in antiphase).
- Interference is where two or more waves overlap to form a resultant wave given by the principle of superposition. The resultant wave may have an amplitude that is the same, smaller or greater than the overlapping waves.
- To produce a sustained and observable interference pattern the sources must be monochromatic (single frequency), same frequency and coherent (have a constant phase difference).
- Coherent sources have a constant phase difference between the vibrations of the sources.
- Coherent waves have a constant phase difference.
- Two-source interference can be demonstrated using water ripples, sound, light and microwaves

  For two-source interference fringes to be observed, the waves from the two sources must be coherent (constant phase difference) and must meet (superpose) with a path difference of $n\lambda$ ($n = 0, 1, 2$ ...) or phase difference of 0, 360°, 720° ... or 0, 2π radians, 4π radians ... for maxima and $\left(n+\frac{1}{2}\right)\lambda$ ($n = 0, 1, 2$ ...) or phase difference of 180°, 540°, 900° ... or π radians, 3π radians, 5π radians ... for minima.
- Young's double-slit experiment:
  - condition for constructive interference: path difference = $n\lambda$
  - condition for destructive interference: path difference $(n+\frac{1}{2})\lambda$
  - wavelength can be found from $\lambda = ax/D$, where a is the separation of the source slits, x is the fringe width and D is the distance of the screen from the slits.
- A stationary wave is the result of the overlapping and hence interference between two progressive waves of equal frequency and similar amplitude travelling along the same line with the same speed, but in opposite directions.
- Points of zero amplitude on a stationary wave are called nodes; points of maximum amplitude are called antinodes.
- For stationary waves on a stretched string, frequency f_n of the nth mode is given by $f_n = nc/2L$, where c is the speed of progressive waves on the string and L is the length of the string.
- For stationary waves in a gas in a tube closed at one end, frequency f_n of the nth mode is given by $f_n = (2n-1)c/4L$, where c is the speed of sound in air and L is the length of the tube.
- The distance between two successive antinodes is half a wavelength.
- The distance between two successive nodes is half a wavelength.
- Diffraction is the spreading out of waves after passing through an aperture or passing the edge of an obstacle. There is greater diffraction when the size of the aperture and the wavelength of the wave are approximately the same.
- Properties of wave motion (diffraction and interference) can be observed in a ripple tank.
- Interference and diffraction of light is evidence that light has wave properties.
- The condition for a diffraction maximum in a diffraction grating pattern is $d\sin\theta = n\lambda$, where d is the grating spacing, θ is the angle at which the diffraction maximum is observed, n is an integer (the order of the image), and λ is the wavelength of the light.

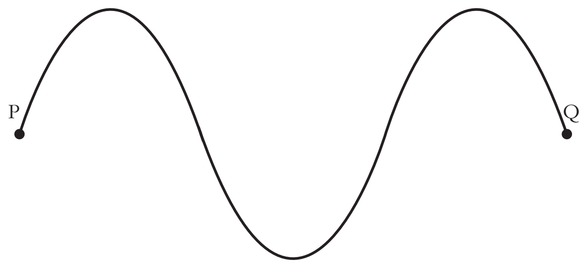
END OF TOPIC QUESTIONS

1 A transverse wave has an amplitude of 2.0 cm and intensity I at a particular point P. A second wave of the same type and frequency as the first wave overlaps the first wave. At point P the second wave has an intensity of $I/2$. At point P the two waves are 180° out of phase. What is the intensity of the resultant wave at point P?

A 0.09 I **B** 0.5 I **C** 1.5 I **D** 2.9 I

2 Fig. 8.39 represents a stationary wave on a string PQ. All the points on the string are shown at their maximum displacement. What are the number of nodes (first number) and the number of antinodes (second number)?

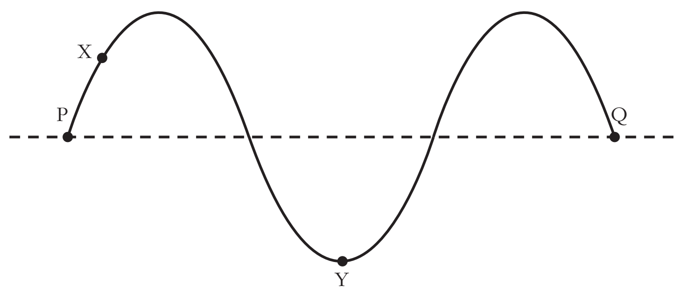
A 2, 3 **B** 3, 2 **C** 3, 4 **D** 4, 3



▲ **Figure 8.39**

3 Fig. 8.40 represents a stationary wave on a string PQ. All the points on the string are shown at their maximum displacement. What could be the phase difference between points X and Y?

A 0° **B** 180° **C** 225° **D** 450°



▲ **Figure 8.40**

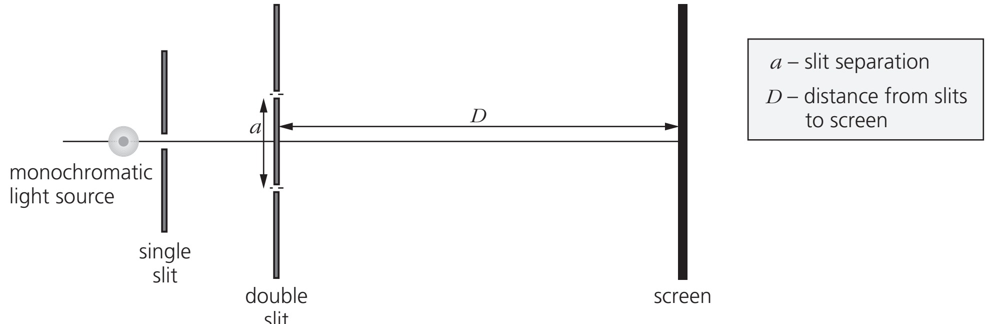
4 In the double-slit interference of light experiment which single change causes the fringe width to decrease?

A reduce the slit separation
B decrease the width of each slit
C decrease the distance from the double slit to the screen where the fringes are observed
D increase the wavelength of the light

5 Compare a two-source experiment to demonstrate the interference of sound waves with a Young's double-slit experiment using light. What are the similarities and differences between the two experiments?

6 **a** Explain the term *coherence* as applied to waves from two sources.
b Describe how you would produce two coherent sources of light.
c A double-slit interference pattern is produced using slits separated by 0.45 mm, illuminated with light of wavelength 633 nm from a laser. The pattern is projected on to a wall 2.50 m from the slits. Calculate the fringe separation.

7 Fig. 8.41 shows the arrangement for obtaining interference fringes in a Young's double-slit experiment. Describe and explain what will be seen on the screen if the arrangement is altered in each of the following ways:
a the slit separation a is halved,
b the distance D from slits to screen is doubled,
c the monochromatic light source is replaced with a white-light source.



▲ **Figure 8.41**

8 State and explain four ways in which stationary waves differ from progressive waves.

9 A source of sound of frequency 2000 Hz is placed in front of a flat wall. When a microphone is moved away from the source towards the wall, a series of maxima and minima are detected.

a Explain what has happened to create these maxima and minima.

b The speed of sound in air is 340 m s^{-1}. Calculate the distance between successive minima.

10 A string is stretched between two fixed supports separated by 1.20 m. Stationary waves are generated on the string. It is observed that two stationary wave frequencies are 180 Hz and 135 Hz; there is no resonant frequency between these two. Calculate:

a the speed of progressive waves on the stretched string,

b the lowest resonant frequency of the string.

11 Blue and red light, with wavelengths 450 nm and 650 nm respectively, is incident normally on a diffraction grating which has 4.0×10^5 lines per metre.

a Calculate the grating spacing.

b Calculate the angle between the second-order maxima for these wavelengths.

c For each wavelength, find the maximum order that can be observed.

12 Discuss any difference between the interference patterns formed by:

a two parallel slits 1 μm apart,

b a diffraction grating with grating spacing 1 μm, when illuminated with monochromatic light.

13 Light of wavelength 633 nm passes through a slit 50 μm wide. Calculate the angular separation between the central maximum and the first minimum of the diffraction pattern.

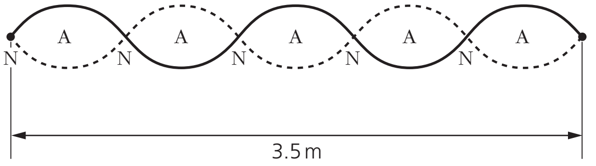
14 A string is stretched between two fixed supports 3.5 m apart. Stationary waves are generated by disturbing the string. One possible mode of vibration of the stationary waves is shown in Fig. 8.42. The nodes and antinodes are labelled N and A respectively.

a Distinguish between a node and an antinode in a stationary wave.

b State the phase difference between the vibrations of particles of the string at any two neighbouring antinodes.

c Calculate the ratio of the frequency of the mode of vibration shown in Fig. 8.42 to the frequency of the fundamental mode of vibration of the string.

d The frequency of the mode of vibration shown in Fig. 8.42 is 160 Hz. Calculate the speed of the progressive waves which produced this stationary wave.

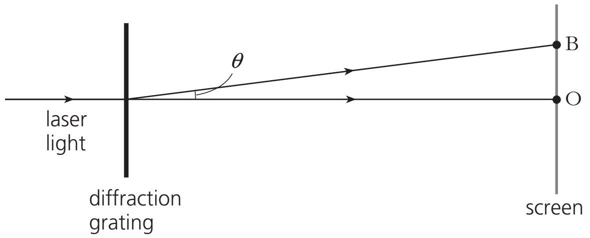


▲ **Figure 8.42**

15 A vibrating tuning fork of frequency 320 Hz is held over the open end of a resonance tube. The other end of the tube is immersed in water. The length of the air column is gradually increased until resonance first occurs. Taking the speed of sound in air as 340 m s^{-1}, calculate the length of the air column. (Neglect any end-correction.)

16 We can hear sounds round corners. We cannot see round corners. Both sound and light are waves. Explain why sound and light seem to behave differently.

17 Fig. 8.43 shows a narrow beam of monochromatic laser light incident normally on a diffraction grating. The central bright spot is formed at O.



▲ **Figure 8.43**

a Write down the relationship between the wavelength λ of the light and the angle θ for the first diffraction image formed at B. Identify any other symbol used.

b The screen is 1.1 m from the diffraction grating and the grating has 300 lines per mm. The laser light has wavelength 6.3×10^{-7} m. Find the distance OB from the central spot to the first bright image at B.

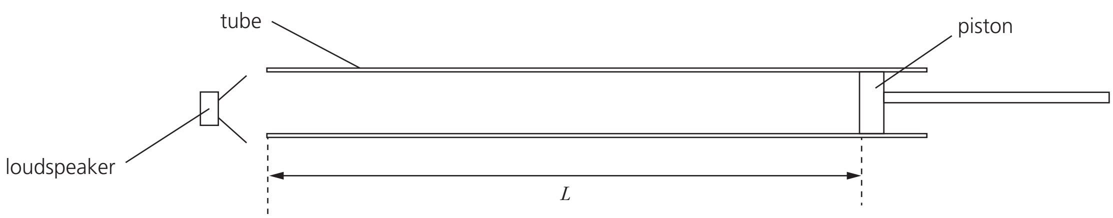
c The diffraction grating is now replaced by one which has 600 lines per mm. For this second grating, calculate the distance from the central spot to the first bright image.

18 a State two features of a stationary wave that distinguish it from a progressive wave. **[2]**

b A long tube is open at one end. It is closed at the other end by means of a piston that can be moved along the tube, as shown in Fig. 8.44.

A loudspeaker producing sound of frequency 550 Hz is held near the open end of the tube.

The piston is moved along the tube and a loud sound is heard when the distance L between the piston and the open end of the tube is 45 cm.



▲ **Figure 8.44**

The speed of sound in the tube is 330 m s^{-1}.

i Show that the wavelength of the sound in the tube is 60 cm. **[1]**

ii On a copy of Fig. 8.44, mark all the positions along the tube of:

1 the displacement nodes (label these with the letter N), **[1]**

2 the displacement antinodes (label these with the letter A). **[2]**

c The frequency of the sound produced by the loudspeaker in **b** is gradually reduced.

Determine the lowest frequency at which a loud sound will be produced in the tube of length L = 45 cm. **[3]**

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19 a A diffraction grating is used to determine the wavelength of light.

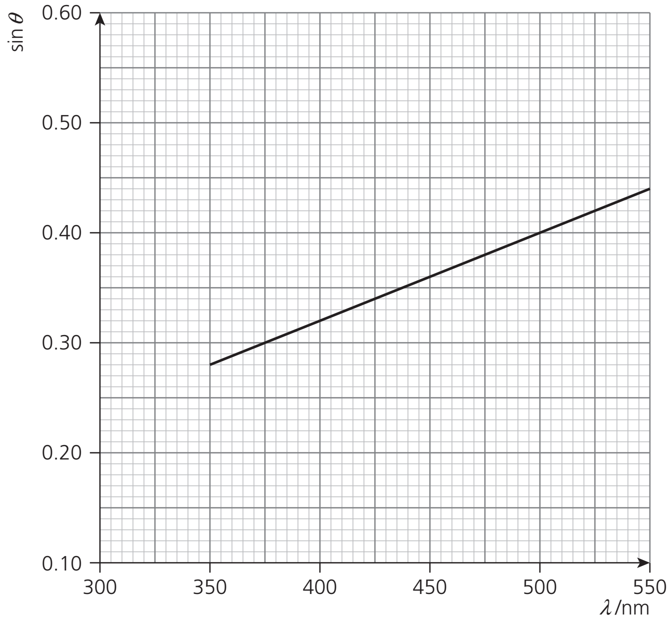
i Describe the diffraction of light at a diffraction grating. **[1]**

ii By reference to interference, explain:

1 the zero order maximum,

2 the first order maximum. **[3]**

b A diffraction grating is used with different wavelengths of light. The angle θ of the second order maximum is measured for each wavelength. The variation with wavelength λ of sin θ is shown in Fig. 8.45.



▲ **Figure 8.45**

i Determine the gradient of the line shown in Fig. 8.45. **[1]**

ii Use the gradient determined in **i** to calculate the slit separation d of the diffraction grating. **[1]**

iii On a copy of Fig. 8.45, sketch a line to show the results that would be obtained for the first order maxima. **[1]**

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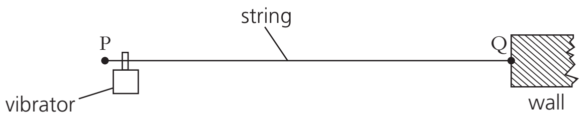
20 Fig. 8.46 shows a string stretched between two fixed points P and Q.



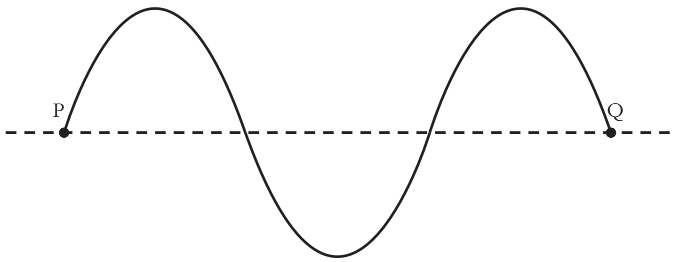
▲ **Figure 8.46**

A vibrator is attached near end P of the string. End Q is fixed to a wall. The vibrator has a frequency of 50 Hz and causes a transverse wave to travel along the string at a speed of $40\,\text{m}\,\text{s}^{-1}$.

a **i** Calculate the wavelength of the transverse wave on the string. **[2]**

ii Explain how this arrangement may produce a stationary wave on the string. **[2]**

b The stationary wave produced on PQ at one instant of time t is shown in Fig. 8.47. Each point on the string is at its maximum displacement.



▲ **Figure 8.47**

i On a copy of Fig. 8.47, label all the nodes with the letter N and all the antinodes with the letter A. **[2]**

ii Use your answer in **a i** to calculate the length of string PQ. **[1]**

iii On a copy of Fig. 8.47, draw the stationary wave at time (t + 5.0 ms). Explain your answer. **[3]**

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21 a By reference to two waves, state:

i the principle of superposition, [2]

ii what is meant by *coherence*. [1]

b Two coherent waves P and Q meet at a point in phase and superpose. Wave P has an amplitude of 1.5 cm and intensity I. The resultant intensity at the point where the waves meet is $3I$. Calculate the amplitude of wave Q. [2]

c The apparatus shown in Fig. 8.48 is used to produce an interference pattern on a screen.

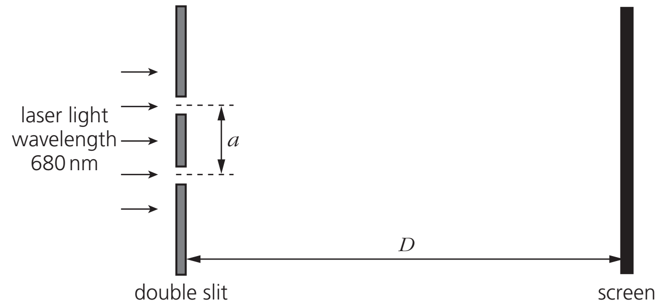

Figure 8.48

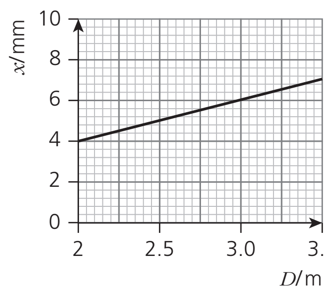

Figure 8.49

Light of wavelength 680 nm is incident on a double slit. The slit separation is a. The separation between adjacent fringes is x. Fringes are viewed on a screen at distance D from the double slit. Distance D is varied from 2.0 m to 3.5 m. The variation with D of x is shown in Fig. 8.49.

i Use Fig. 8.49 to determine the slit separation a. [3]

ii The laser is now replaced by another laser that emits light of a shorter wavelength. On a copy of Fig. 8.49, sketch a possible line to show the variation with D of x for the fringes that are now produced. [2]

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22 a State the principle of superposition. [2]

b An arrangement for demonstrating the interference of light is shown in Fig. 8.50.

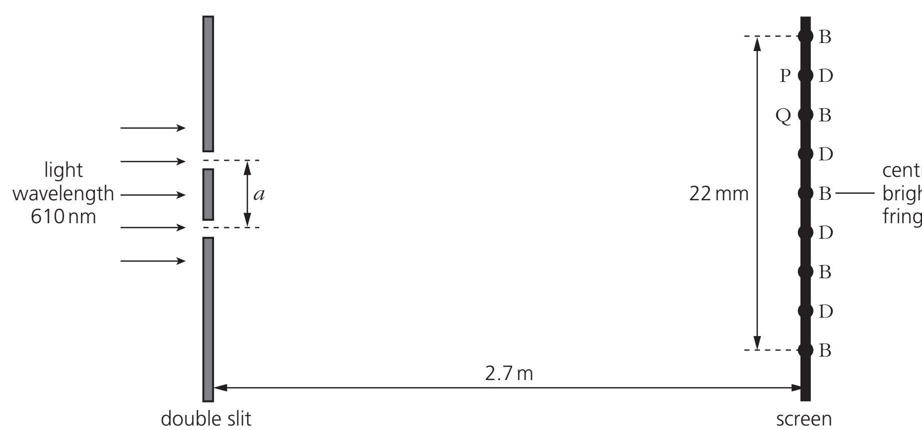

Figure 8.50

The wavelength of the light is 610 nm. The distance between the double slit and the screen is 2.7 m. An interference pattern of bright fringes and dark fringes is observed on the screen. The centres of the bright fringes are labelled B and centres of the dark fringes are labelled D. Point P is the centre of a particular dark fringe and point Q is the centre of a particular bright fringe, as shown in Fig. 8.50. The distance across five bright fringes is 22 mm.

i The light waves leaving the two slits are coherent. State what is meant by *coherent*. [1]

ii 1 State the phase difference between the waves meeting at Q.

2 Calculate the path difference, in nm, of the waves meeting at P. [2]

iii Determine the distance a between the two slits. **[3]**

iv A higher frequency of visible light is now used. State and explain the change to the separation of the fringes. **[1]**

v The intensity of the light incident on the double slit is now increased without altering its frequency. Compare the appearance of the fringes after this change with their appearance before this change. **[2]**

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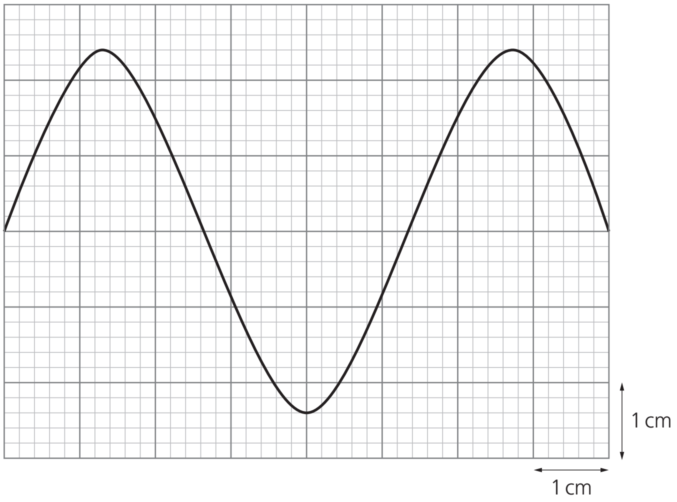
23 a State the conditions required for the formation of a stationary wave. **[2]**

b The sound from a loudspeaker is detected by a microphone that is connected to a cathode-ray oscilloscope (c.r.o.). Fig. 8.51 shows the trace on the screen of the c.r.o.



▲ Figure 8.51

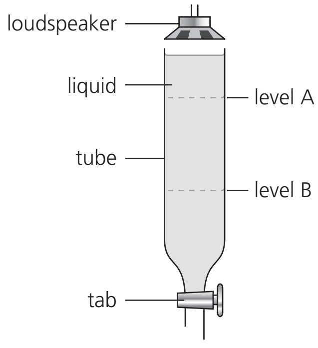
In air, the sound wave has a speed of $330\,m\,s^{-1}$ and a wavelength of 0.18 m.

i Calculate the frequency of the sound wave. **[2]**

ii Determine the time-base setting, in $s\,cm^{-1}$, of the c.r.o. **[2]**

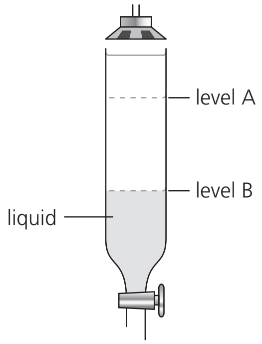
iii The intensity of the sound from the loudspeaker is now halved. The wavelength of the sound is unchanged. Assume that the amplitude of the trace is proportional to the amplitude of the sound wave. On a copy of Fig. 8.51, sketch the new trace shown on the screen of the c.r.o. **[2]**

c The loudspeaker in **b** is held above a vertical tube of liquid, as shown in Fig. 8.52.



▲ Figure 8.52

A tap at the bottom of the tube is opened so that liquid drains out at a constant rate. The wavelength of the sound from the loudspeaker is 0.18 m. The sound that is heard first becomes much louder when the liquid surface reaches level A. The next time that the sound becomes much louder is when the liquid surface reaches level B, as shown in Fig. 8.53.



▲ Figure 8.53

i Calculate the vertical distance between level A and level B. **[1]**

ii On a copy of Fig. 8.53, label with the letter N the positions of the nodes of the stationary wave that is formed in the air column when the liquid surface is at level B. **[1]**

iii The mass of liquid leaving the tube per unit time is $6.7\,g\,s^{-1}$. The tube has an internal cross-sectional area of $13\,cm^2$. The density of the liquid is $0.79\,g\,cm^{-3}$. Calculate the time taken for the liquid to move from level A to level B. **[2]**

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9 Electricity

Learning outcomes

By the end of this topic, you will be able to:

9.1 Electric current

1. understand that an electric current is a flow of charge carriers
2. understand that the charge on charge carriers is quantised
3. recall and use $Q = It$
4. use, for a current-carrying conductor, the expression $I = Anvq$, where n is the number density of charge carriers

9.2 Potential difference and power

1. define the potential difference across a component as the energy transferred per unit charge
2. recall and use $V = W/Q$
3. recall and use $P = VI$, $P = I^2R$ and $P = V^2/R$

9.3 Resistance and resistivity

1. define resistance
2. recall and use $V = IR$
3. sketch the I–V characteristics of a metallic conductor at constant temperature, a semiconductor diode and a filament lamp
4. explain that the resistance of a filament lamp increases as the current increases because its temperature increases
5. state Ohm's law
6. recall and use $R = \rho L/A$
7. understand that the resistance of a light-dependent resistor (LDR) decreases as the light intensity increases
8. understand that the resistance of a negative temperature coefficient thermistor decreases as the temperature increases (it will be assumed that thermistors have a negative temperature coefficient)

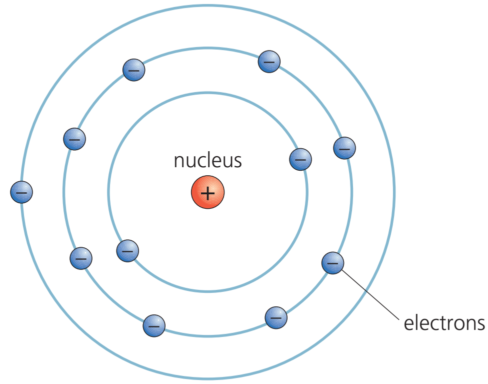
Starting points

★ This topic considers fundamental ideas about electric charge and electric current.
★ Examples of electric currents are in household wiring and electrical appliances.
★ A potential difference is required for energy changes to occur in a circuit.
★ When work is done energy is transferred.
★ Power is defined as the rate at which work is done, or energy is transferred.
★ Resistance controls the flow of charge in a circuit.

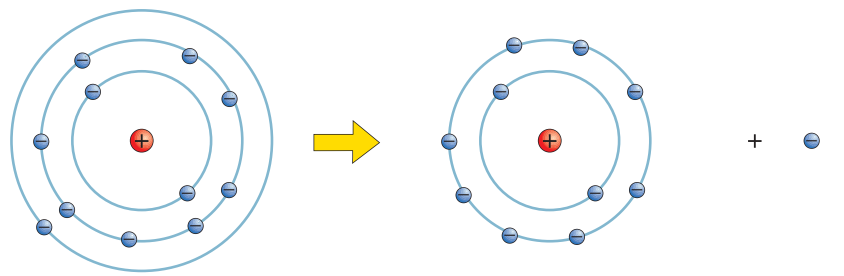
9.1 Electric current

All matter is made up of tiny particles called atoms, each consisting of a positively charged nucleus with negatively charged electrons moving around it.

The unit of charge is the **coulomb** (symbol C). The charge on an electron e is -1.60×10^{-19} C. Normally atoms have equal numbers of positive and negative charges, so that their overall charge is zero. But for some atoms it is relatively easy to remove an electron, leaving an atom with an unbalanced number of positive charges. This is called a **positive ion**.



▲ **Figure 9.1** Atoms consist of a positively charged nucleus with negative electrons outside.



▲ **Figure 9.2** An atom with one or more electrons missing is a positive ion

Robert Millikan (1868–1953) performed an experiment in 1912 to determine the charge on a single electron using charged oil droplets. The experimental result showed that, no matter what the charge on the droplets, it seemed to occur only in integer multiples of a particular value, which he deduced was the charge on an electron, e. The conclusion was that charge is not continuous but **quantised**, that is it exists only in discrete amounts, integral multiples of the charge on an electron. The photon is another example of a quantised physical quantity, introduced in the A Level section in Topic 22.

Since charge is quantised, ions formed by the removal or addition of electrons, from or to atoms, also have quantised charges of $\pm e$, $\pm 2e$, $\pm 3e$, etc. The charge on the proton is $+e$.

Atoms in metals have one or more outer electrons which are not held tightly to the nucleus. These **free** (or mobile) **electrons** wander at random throughout the metal. However, when a battery is connected across the ends of the metal, the free electrons drift towards the positive terminal of the battery, producing an **electric current**.

Charge carriers in an electric current can be any charged particles. In a metal the charge carriers are electrons but in a solution or in a plasma (ionised gas) the charge carriers are positive and negative ions.

The size of the electric current is given by the rate of flow of charge. Electric current is an SI base quantity (see Topic 1.2). The SI base unit of current is the **ampere** (or amp for short), with symbol A. The SI units of all the other electrical quantities are derived from the SI base units.

A current of 3 amperes means that 3 coulombs pass a point in the circuit every second. In 5 seconds, a total charge of 15 coulombs will have passed the point. So, the charge Q that flows (in coulombs) is given by

$$\text{charge} = \text{current} \times \text{time}$$

or

$$Q = It$$

when the current I is in amperes and the time t is in seconds.

The unit ampere second (A s) is called the coulomb (C). 1 C = 1 A s.

WORKED EXAMPLE 9A

The current in the filament of a torch bulb is 0.03 A. How much charge flows through the bulb in 1 minute?

Answer

Using $Q = It$, $Q = 0.03 \times 60$ (remember the time must be in seconds), so $Q =$ **1.8 C**.

Questions

1 Calculate the current when a charge of 240 C passes a point in a circuit in a time of 2 minutes.

2 In a silver-plating experiment, 9.65×10^4 C of charge is needed to deposit a certain mass of silver.

Calculate the time taken to deposit this mass of silver when the current is 0.20 A.

3 The current in a wire is 200 mA. Calculate:

a the charge which passes a point in the wire in 5 minutes

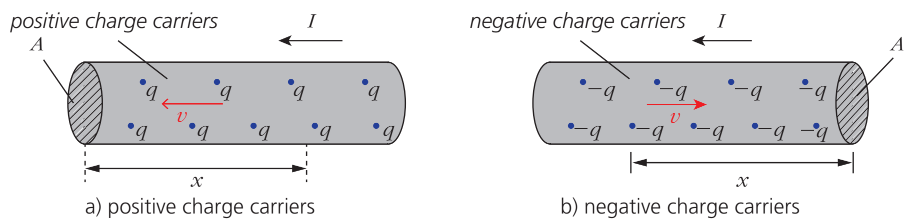
b the number of electrons needed to carry this charge. (Electron charge $e = -1.60 \times 10^{-19}$ C.)

Conventional current

Early studies of the effects of electricity led scientists to believe that an electric current is the flow of 'something'. In order to develop a further understanding of electricity, they needed to know the direction of flow. It was decided that this flow in the circuit should be from the positive terminal of the battery or power supply to the negative. This current is called the **conventional current**, and is in the direction of flow of positive charge. We now know, in a metal, that the electric current is the flow of electrons in the opposite direction, from the negative terminal to the positive terminal. However, laws of electricity had become so firmly fixed in the minds of people that the idea of conventional current has persisted. But be warned! Occasionally we need to take into account the fact that electron flow is in the *opposite* direction to conventional current, for example, in Topic 20.2 when determining the direction of the force on a charge or current-carrying conductor in a magnetic field.

Conduction in a current-carrying conductor

Figure 9.3 shows part of a conductor of cross-sectional area A through which there is a current I. In Figure 9.3a the charge carriers are positive and in Figure 9.3b the charge carriers are negative. The current in each of the conductors is from right to left but the charge carriers move in opposite directions as shown by the average **drift speed**, v.



▲ **Figure 9.3** Conduction in a current-carrying conductor

The current is due to the movement of the charge carriers along the conductor. The flow of charge carriers is characterised by an average drift speed. The average drift speed of the charge carriers in the conductor is v and the number density (the number per unit volume) of the charge carriers is n. The charge on each of the charge carriers is q. (See Figure 9.3a.)

The number of charge carriers in a length x of the conductor is Axn. The amount of charge that leaves this volume through the left-hand side of the conductor in a time t is $Axnq$.

Where the time interval t is x/v

the current I = charge/time = $(Axnq)/(x/v)$

Therefore,

$$I = Anvq$$

The same expression is obtained if the negative charge carriers are considered in the conductor shown in Figure 9.3b.

WORKED EXAMPLE 9B

A copper wire has 8.5×10^{28} charge carriers (free electrons) m^{-3}. The wire has a current of 2.0 A and a cross-sectional area of 1.2 mm^2. Calculate the average drift speed of the electrons.

Answer

$$v = \frac{I}{nAe} = \frac{2}{(8.5 \times 10^{28} \times 1.2 \times 10^{-6} \times 1.6 \times 10^{-19})}$$

$$= \mathbf{1.2 \times 10^{-4}\,m\,s^{-1}}$$

Question

4 The average drift speed of the electrons in a metal wire is 6.5×10^{-4} m s^{-1} when the current is 0.80 A. The diameter of the wire is 0.50 mm. Calculate the number of ‘free’ electrons per unit volume in the wire (number density).

9.2 Potential difference and power

A cell makes one end of the circuit positive and the other negative. The cell is said to set up a **potential difference** across the circuit. Potential difference (p.d. for short) is measured in **volts** (symbol V), and is often called the voltage. You should never talk about the potential difference or voltage *through* a device, because it is in fact a difference *across* the ends of the device. The potential difference provides the energy to move charge through the device.

The potential difference between any two points in a circuit is a measure of the energy transferred, or the work done, by each coulomb of charge as it moves from one point to the other. We already know that the unit of potential difference is the volt (V). Energy W is measured in joules, and charge Q in coulombs.

$$\text{potential difference} = \frac{\text{energy transferred (or work done)}}{\text{charge}}$$

or

$$V = \frac{W}{Q}$$

The unit of potential difference, a joule coulomb^{-1} is called a **volt** (V). 1 V = 1 J C^{-1}.

We can rearrange this equation to get an expression for the energy transferred or converted when a charge Q is moved through a potential difference V:

energy transferred (work done) = potential difference × charge

$W = VQ$

In Figure 9.4, one lamp is connected to a 240 V mains supply and the other to a 12 V car battery. Both lamps have the same current, yet the 240 V lamp glows more brightly. This is because the energy supplied to each coulomb of charge in the 240 V lamp is 20 times greater than for the 12 V lamp.

▲ **Figure 9.4** A 240 V, 100 W mains lamp is much brighter than a 12 V, 5 W car light, but both have the same current. (Do not try this experiment yourself as it involves a large voltage.)

WORKED EXAMPLE 9C

Electrons in a particular television tube are accelerated by a potential difference of 20 kV between the filament and the screen. The charge of the electron is -1.60×10^{-19} C.

Calculate the gain in kinetic energy of each electron.

Answer

Since $V = W/Q$, then $W = VQ$. The energy is transferred to the electron increasing its kinetic energy. Thus,

kinetic energy gained = $VQ = 20 \times 10^3 \times 1.60 \times 10^{-19}$

$= \mathbf{3.2 \times 10^{-15}\,J}$

(Don't forget to convert the 20 kV into volts.)

Questions

5 An electron in a particle accelerator is accelerated through a potential difference of 10^6 V. Calculate the energy, in joules, gained by the electron.

6 A torch bulb is rated 2.2 V, 0.25 A. Calculate:

a the charge passing through the bulb in one second

b the energy transferred by the passage of each coulomb of charge.

Electrical power

In Topic 5.1 we defined **power** P as the rate of doing work, or of transferring energy, $P = W/t$.

The definition of potential difference V gives energy transferred per unit charge, $V = W/Q$.

Therefore, $P = VQ/t$

and since $Q/t = I$

$$P = VI$$

power = potential difference × current

The power is measured in **watts** (W) when the potential difference is in **volts** (V) and the current is in amperes (A). A voltmeter can measure the p.d. across a device and an ammeter the current through it; the equation above can then be used to calculate the power in the device.

9.3 Resistance and resistivity

Connecting wires in circuits are often made from copper, because copper offers little opposition to the movement of electrons. The copper wire is said to have a low electrical **resistance**. In other words, copper is a good conductor.

Some materials, such as plastics, are poor conductors. These materials are said to be insulators, because under normal circumstances they conduct little or no current.

The resistance R of a conductor is defined as the ratio of the potential difference V across the conductor to the current I in it.

or

$$R = \frac{V}{I}$$

where the resistance is in ohms when the potential difference is in volts and the current in amperes. The unit of resistance, a volt ampere^{-1} is called an **ohm** (Ω). The symbol for ohms is the Greek capital letter omega, Ω. $1\,\Omega = 1\,\text{V}\,\text{A}^{-1}$.

We have defined resistance for a conductor, but many devices have resistance. The general term for such a device is a **resistor**. (Note that the resistance of a resistor is measured in ohms, just as the volume of a tank is measured in m^3. We do not refer to the 'm^3' of a tank, or to the 'ohms' of a resistor.)

The relationship between resistance, potential difference and current means that, for a given potential difference, the resistance controls the size of the current in a circuit. A high resistance means a small current, while a low resistance means a large current.

WORKED EXAMPLE 9D

The current in an electric immersion heater in a school experiment is 6.3 A when the p.d. across it is 12 V. Calculate the resistance of the heater.

Answer

Since $R = V/I$, the resistance $R = 12/6.3 = \mathbf{1.9\,\Omega}$.

Question

7 The current in a light-emitting diode is 20 mA when it has a potential difference of 2.0 V across it. Calculate its resistance.

Electrical heating

When an electric current passes through a resistor, it gets hot. This heating effect is sometimes called **Joule heating**. The electrical power P produced (dissipated) is given by $P = VI$. We can obtain two alternative expressions for power in terms of the resistance R of the resistor. Since $R = V/I$, then

$$P = I^2R$$

and

$$P = \frac{V^2}{R}$$

For a resistor of constant resistance, the power dissipated depends on the square of the current. Therefore, if the current is doubled (by doubling the voltage across the resistor), the power will be four times as great. Hence, a doubling of voltage, doubles the current and this increases the power by a factor of four.

WORKED EXAMPLE 9E

1 An electric immersion heater used in a school experiment has a current of 6.3 A when the p.d. across it is 12 V. Calculate the power of the heater.

2 The p.d. across the immersion heater in question 1 is reduced to 6.0 V. Calculate the new power of the heater (assume the resistance of the heater remains constant).

Answers

1 Since $P = VI$, power = 12×6.3 = **76 W**.

2 Since the resistance is constant the power is proportional to the square of the potential difference ($P = V^2/R$).
The p.d. V is halved hence the power is reduced by $\left(\frac{1}{2}\right)^2$ or 76/4 = **19 W**.

Questions

8 Show that a 100 W lamp connected to a mains supply of 240 V will have the same current as a 5 W car lamp connected to a 12 V battery. (See Figure 9.4.)

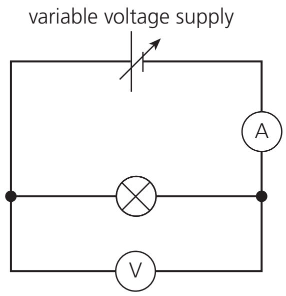
9 An electric kettle has a power of 2.2 kW at 240 V. Calculate:

a the current in the kettle

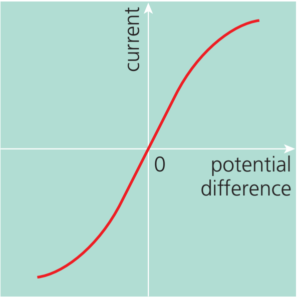
b the resistance of the kettle element.

Current–voltage (*I*–*V*) characteristics

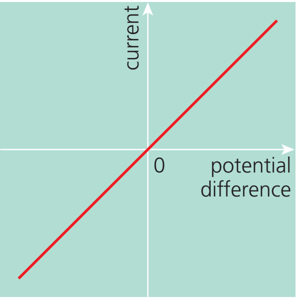
The relationship between the potential difference across an electrical component and the current through it may be investigated using the circuit of Figure 9.5 shown overleaf. For example, if a filament lamp is to be investigated, adjust the power supply for a range of potential differences and measure the corresponding currents in and potential differences across the lamp. The variation of current with potential difference is shown in Figure 9.6. This graph is known as an *I*–*V* characteristic.



▲ **Figure 9.5** Circuit for plotting graphs of current against potential difference for a circuit component



▲ **Figure 9.6** Current against potential difference for a filament lamp



▲ **Figure 9.7** Current against potential difference for a constantan wire

The resistance *R* of the lamp can be calculated from $R = V/I$. At first, the resistance is constant (where the graph is a straight line through the origin), but then the resistance increases with current (where the graph curves).

If the lamp is replaced by a length of constantan wire, the graph of the results is as shown in Figure 9.7. It is a straight line through the origin. This shows that, for constantan wire, the current is proportional to the potential difference. The resistance of the wire is found to be constant as the current increases. The difference between Figures 9.6 and 9.7 is that the temperature of the constantan wire was constant for all currents used in the experiment, whereas the temperature of the filament of the lamp increased to about 1500°C as the current increased.

▲ **Figure 9.8** Georg Ohm (1789–1854)

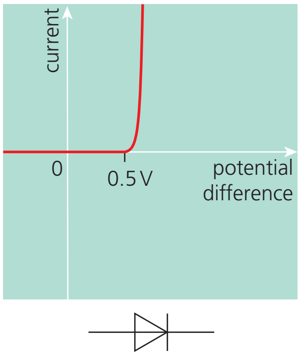
Ohm's law

Graphs like Figure 9.7 would be obtained for wires of any metal, provided that the temperature of the wires did not change during the experiment. The graph illustrates a law discovered by the German scientist Georg Simon Ohm (Figure 9.8). (Ohm's name is now used for the unit of resistance.)

Ohm's law states that, for a metallic conductor at constant temperature, the current in the conductor is proportional to the potential difference across it.

Conductors where the current against potential difference graph is a straight line through the origin, like that in Figure 9.7, are said to obey Ohm's law. It is found that Ohm's law applies to metal wires, provided that the current is not too large. What does 'too large' mean here? It means that the current must not be so great that there is a pronounced heating effect, causing an increase in temperature of the wire.

A lamp filament consists of a thin metal wire. Why does it not obey Ohm's law? (Figure 9.6 shows that the current against potential difference graph is not a straight line.) This is because, as stated previously, the temperature of the filament does not remain constant. The increase in current causes the temperature to increase so much that the filament glows. The reason for this is explained opposite in the section Resistance and temperature,



▲ **Figure 9.9** Current against potential difference for a diode

Current–voltage characteristics of a diode

When a diode is tested in the same way as the filament lamp (see the circuit in Figure 9.5), the current–potential difference graph shown in Figure 9.9 is obtained. Diodes are made from semiconducting material. The diode conducts when the current is in the direction of the arrowhead on the symbol. This condition is called **forward bias**. The potential on the left-hand side of the diode is more positive than the potential on the right-hand side. When the potential difference is reversed, there is negative bias. This is called **reverse bias**. Figure 9.9 shows this important difference in the current–potential difference graph when the p.d. is reversed. The diode does not conduct when in the reverse bias condition. As a result, diodes are used to change alternating current into direct current in devices called **rectifiers** (see Topic 21.2).

Diodes do not obey Ohm's law. The resistance of the diode is very high for low voltages in the forward bias condition. The diode conducts with a forward bias voltage of about 0.5 V. The resistance of the diode decreases as the voltage is increased with forward bias. The straight line part of the graph in this region does not follow Ohm's law as the line does not go through the origin and, therefore, the resistance is not constant. The current is not proportional to the p.d.

Resistance and temperature

All solids are made up of atoms that constantly vibrate about their equilibrium positions. The higher the temperature, the greater the amplitude of vibration.

Electric current is the flow of electrons through a metal. As the electrons move, they collide with the vibrating metal ions, so their movement is impeded. The more the ions vibrate, the greater is the chance of collision. This means that the current is less and the resistance of metals increases with temperature.

EXTENSION

A temperature rise can cause an increase in the number of free electrons. If there are more electrons free to move, this may outweigh the effect due to the vibrating ions, and thus the flow of electrons, or the current, will increase. The resistance is therefore reduced. This is the case in semiconductors. Insulators, too, show a reduction in resistance with temperature rise.

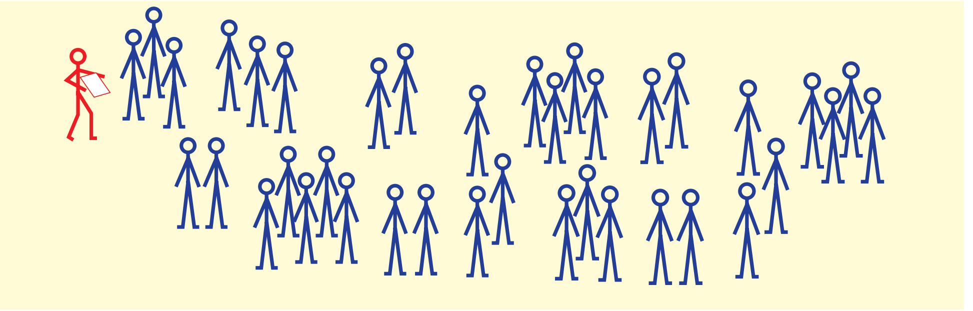
For metals there is no increase in the number of free electrons. The increased amplitude of vibration of the atoms makes the resistance of metals increase with temperature.

Resistivity

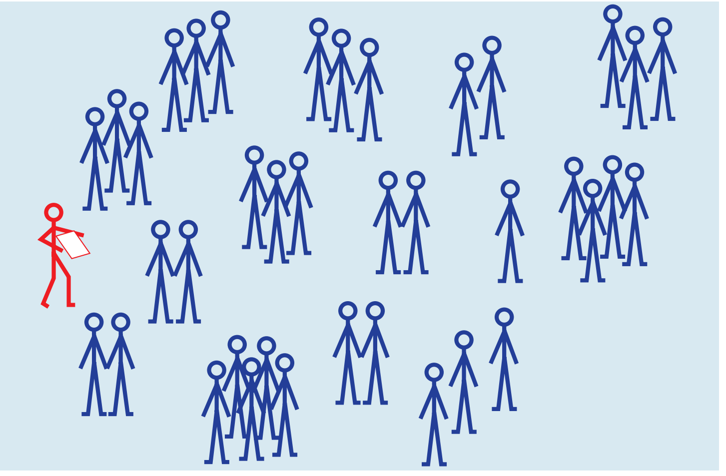
All materials have some resistance to a flow of charge. A potential difference across the material causes free charges inside to accelerate. As the charges move through the metal, they collide with the fixed vibrating ions of the metal which get in their way. They transfer some or all of their kinetic energy, and then accelerate again due to the potential difference across the material. It is this transfer of energy on collision that causes electrical heating in a resistor.

As you may have thought, the longer a wire, the greater its resistance. This is because the charges have further to go through the metal; there is more chance of collisions with the fixed vibrating ions. In fact the resistance of a conductor is proportional to its length, or $R \propto L$. Also, the thicker a conductor, the smaller its resistance. This is because there is a bigger area for the charges to travel through, with less chance of collision. In fact the resistance is inversely proportional to the cross-sectional area of the conductor, or $R \propto 1/A$.

These relations are illustrated by the analogy for a metal wire in Figures 9.10 and 9.11 (overleaf). The waiter delivering his order represents an electron attracted to the positive terminal and the other people represent the lattice of positive ions in a metal. In each case the overall 'number density' of people (the number per unit volume) is the same (since the rooms represent the same type of material).



▲ **Figure 9.10** The longer the room, the greater the resistance the waiter meets.



▲ **Figure 9.11** The wider the room, the easier it is for the waiter to pass through.

Finally, the resistance depends on the type of material. As previously stated, copper is a good conductor, whereas plastics are good insulators. Putting all of this together gives

$$R = \frac{\rho L}{A}$$

where ρ is a constant for a particular material at a particular temperature. ρ is called the **resistivity** of the material at that temperature and is defined by

$$\rho = \frac{RA}{L}$$

The resistance is in ohms, the cross-sectional area in metres squared and the length in metres, hence the unit of resistivity is the ohm metre (Ω m).

Remember that A is the cross-sectional area through which the current is passing, not the surface area.

We have already seen that the resistance of a wire depends on temperature. Thus, resistivity also depends on temperature. The resistivity of a metal increases with increasing temperature, and the resistivity of a semiconductor decreases very rapidly with increasing temperature.

The values of the resistivity of some materials at room temperature are given in Table 9.1. Note the enormous range of resistivity spanned by the materials in this list – a range of 23 orders of magnitude, from 10^{-8} Ω m to 10^{15} Ω m.

Note, too, that the resistivity is a property of a material, while the resistance is a property of a particular wire or device.

material	resistivity /Ω m
metals	
copper	1.7×10^{-8}
gold	2.4×10^{-8}
aluminium	2.6×10^{-8}
semiconductors	
germanium (pure)	0.6
silicon (pure)	2.3×10^{3}
insulators	
glass	about 10^{12}
perspex	about 10^{13}
polyethylene	about 10^{14}
sulfur	about 10^{15}

▲ **Table 9.1** Resistivity of some materials at room temperature

WORKED EXAMPLE 9F

Calculate the resistance per metre at room temperature of a constantan wire of diameter 1.25 mm. The resistivity of constantan at room temperature is $5.0 \times 10^{-7}\,\Omega\,m$.

Answer

The cross-sectional area of the wire is calculated using πr^2.

$$\text{Area} = \pi\left(\frac{1.25 \times 10^{-3}}{2}\right)^2 = 1.23 \times 10^{-6}\,m^2$$

(Don't forget to change the units from mm to m.)

The resistance per metre is given by R/L, and $R/L = \rho/A$. So

$$\text{resistance per metre} = \frac{5.0 \times 10^{-7}}{(1.23 \times 10^{-6})} = \mathbf{0.41\,\Omega\,m^{-1}}$$

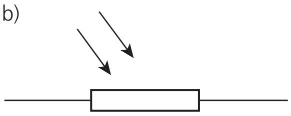
Questions

10 Find the length of copper wire, of diameter 0.63 mm, which has a resistance of 1.00 Ω. The resistivity of copper at room temperature is $1.7 \times 10^{-8}\,\Omega\,m$.

11 Find the diameter of a copper wire which has the same resistance as an aluminium wire of equal length and diameter 1.20 mm. The resistivities of copper and aluminium at room temperature are $1.7 \times 10^{-8}\,\Omega\,m$ and $2.6 \times 10^{-8}\,\Omega\,m$ respectively.

a)

b)

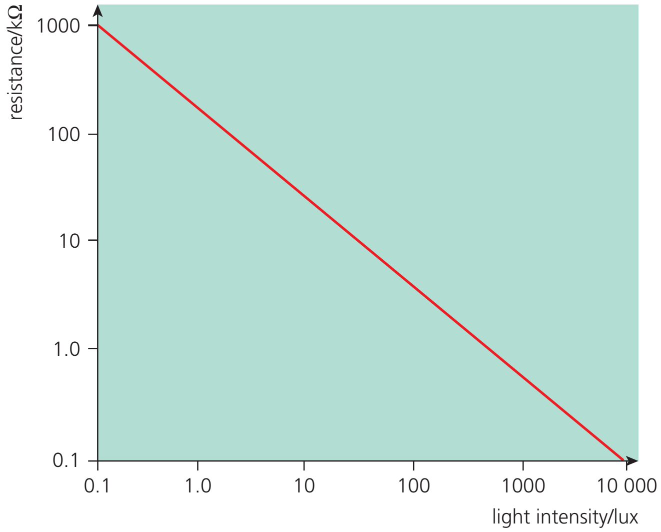


▲ **Figure 9.12** An LDR and its symbol

The light-dependent resistor (LDR)

A **light-dependent resistor (LDR)** consists of two metal grids that intersect each other. The space between the grids is filled with a semiconductor material, for example, cadmium sulfide doped with copper, as shown in Figure 9.12.

When light is incident on the semiconductor material, the number of electrons in the semiconductor that are free to conduct increases. The higher the intensity of light on the LDR, the greater the number of electrons that can move freely. Hence, as the light intensity increases, the resistance of the LDR decreases. Figure 9.13 shows the variation with light intensity of the resistance of a typical LDR.



▲ **Figure 9.13** Resistance versus light intensity for an LDR

Note: Both light intensity, measured in lux, and resistance, measured in ohms, are plotted on **logarithmic scales** in Figure 9.13. The graph is a straight line but this does not mean that resistance is inversely proportional to light intensity. Data relating to light intensity and resistance for a typical LDR is shown in Table 9.2.

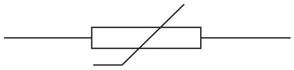
light level	illumination/lux	LDR resistance/Ω
moonlight	0.1	1×10^6
normal room lighting	450	900
sunlight	28 000	100

▲ **Table 9.2** Typical LDR data

The lux is a unit that is used to measure the light power incident per unit area of a surface.

a)

b)



▲ **Figure 9.14** Thermistors and their symbol

The thermistor

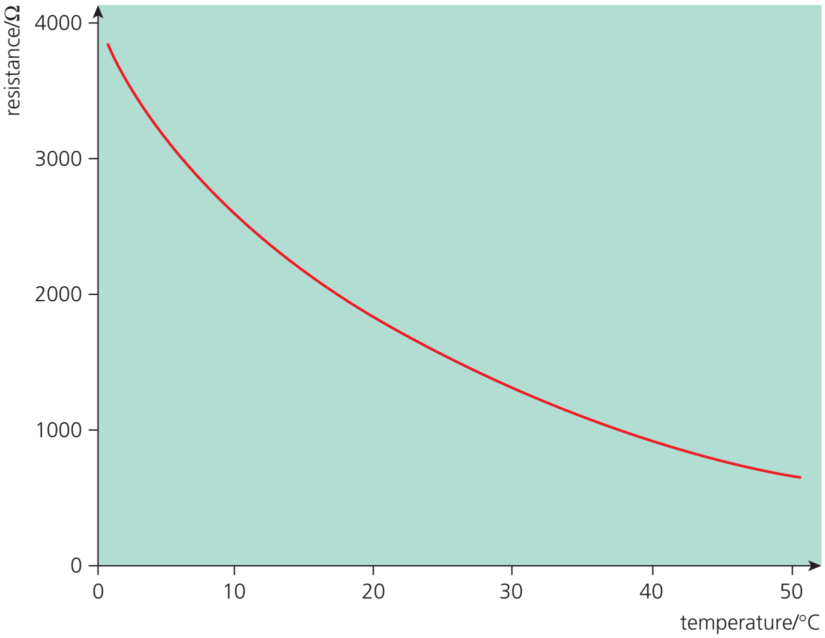
The resistance of most metals increases to a certain extent with rise in temperature. Negative temperature coefficient devices, often referred to as **thermistors**, are made from semiconductor material, usually the oxides of metals. The resistance of thermistors decreases significantly with rise in temperature. Thermistors are manufactured in various shapes and sizes, including rods, discs and beads. Figure 9.14a shows an example of a disc and bead thermistor.

Data relating to the temperature and resistance of a typical bead thermistor is shown in Table 9.3.

temperature/°C	thermistor resistance/Ω
1	3700
10	2500
20	1800
30	1300
40	900
50	660

▲ **Table 9.3** Typical thermistor data

The variation with temperature of a typical thermistor is shown in Figure 9.15. This variation is non-linear and is, in fact, approximately exponential over a limited range of temperature.



▲ **Figure 9.15** Resistance versus temperature for a thermistor

WORKED EXAMPLE 9G

Explain what is meant by a negative temperature coefficient thermistor.

Answer

It is an electrical device whose resistance decreases as its temperature increases.

Question

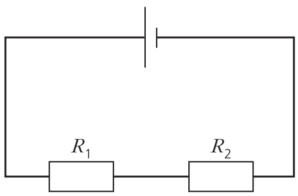
12 a Draw a sketch graph to show the variation with temperature θ of the resistance R of a thermistor. Mark typical values on the axes of your graph.

b Draw a sketch graph to show the variation with light intensity L of the resistance R of an LDR. Mark typical values on the axes of your graph.

SUMMARY

- Electric current is the rate of flow of charge: $I = Q/t$.
- Charge carriers can be positive or negative.
- Conventional current is defined as a flow of positive charge from positive to negative. In metals, the charge carriers are electrons, which travel from negative to positive.
- Charge on charge carriers is quantised in integer multiples of the fundamental charge on the electron, e, 1.6×10^{-19} C.
- The coulomb is the unit of charge and 1 coulomb is equivalent to an ampere second.
- Charge can be calculated using $Q = It$.
- For a current-carrying conductor $I = Anvq$, where n is the number density of charge carriers and v is the mean drift speed of charge carriers.
- Potential difference is defined as the energy transferred per unit of charge: $V = W/Q$.
- The volt is the unit of potential difference and 1 volt is equivalent to 1 joule per coulomb.
- Resistance R of a resistor is defined as: $R = V/I$.
- The ohm is the unit of resistance and is a volt per ampere.
- Electrical power $P = VI = I^2R = V^2/R$.
- Ohm's law: for a metallic conductor at constant temperature, the current in the conductor is proportional to the potential difference across it.
- The I–V characteristic of an electrical component is a graph of current against potential difference; the shape of the graph is characteristic for different components.
- For a metallic conductor at constant temperature the I–V graph is a straight line through the origin, showing it obeys Ohm's law.
- The I–V graph for a filament lamp has a constant gradient for low voltages and a decreasing gradient (showing an increase in resistance) for higher voltages, hence the filament does not obey Ohm's law.
- The I–V graph for a diode has a zero current for reverse bias (very high resistance). For low voltages in forward bias the current is still zero. As the voltage increases (above about 0.5V) the current in the diode increases. The graph is almost a straight line. However, the resistance is not constant but is decreasing showing that the diode does not obey Ohm's law.
- The resistance of a metallic conductor increases with increasing temperature; the resistance of a semiconductor decreases with increasing temperature.
- The resistance of a filament in a lamp increases with increasing current because higher currents cause the temperature of the filament to increase.
- A diode has a low resistance when connected in forward bias, and a very high resistance in reverse bias.
- Resistivity ρ of a conductor of length L and cross-sectional area A is given by the equation: $R = \rho L/A$.
- The resistance of an LDR decreases as the light intensity on it increases.
- The resistance of a negative temperature coefficient thermistor decreases with increasing temperature.

END OF TOPIC QUESTIONS



▲ **Figure 9.16**

1 Fig. 9.16 shows a circuit with two resistors, R_1 and R_2, in series connected to a cell. The resistors are metal wires made of the same material and are of the same length. The diameter of R_1 is twice the diameter of R_2. The drift velocity of the electrons in R_1 is v_1 and the drift velocity of the electrons in R_2 is v_2.

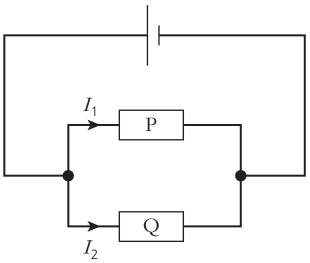
Which of the following answers gives the correct ratio v_1/v_2?

A 0.25 **B** 0.50 **C** 1.0 **D** 4.0

2 Fig. 9.17 shows a cell connected to two resistors P and Q that are connected in parallel. The resistors are metal wires made of the same material. The length of P is L and the length of Q is $2L$. The diameter of P is $d/2$ and the diameter of Q is d. The current through P is I_1 and the current through Q is I_2.

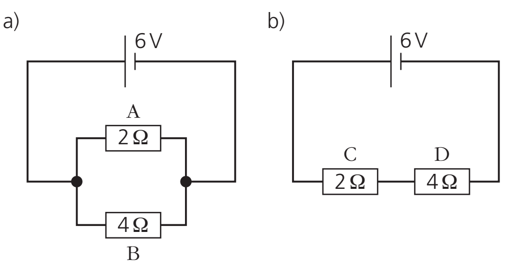
What is the ratio I_1/I_2?

A 0.13 **B** 0.25 **C** 4.0 **D** 8.0



▲ **Figure 9.17**

3 Fig. 9.18 show a cell of e.m.f. 6 V connected to two resistors of 2 Ω and 4 Ω. In a) the resistors are in parallel and b) the resistors are in series. In which of the resistor's positions A, B, C or D is the smallest power transferred?



▲ **Figure 9.18**

4 A 240 V heater takes a current of 4.2 A. Calculate:

a the charge that passes through the heater in 3 minutes,

b the rate at which heat energy is produced by the heater,

c the resistance of the heater.

5 A small torch has a 3.0 V battery connected to a bulb of resistance 15 Ω.

a Calculate:

i the current in the bulb,

ii the power delivered to the bulb.

b The battery supplies a constant current to the bulb for 2.5 hours. Calculate the total energy delivered to the bulb.

6 The capacity of storage batteries is rated in ampere-hours (A h). An 80 A h battery can supply a current of 80 A for 1 hour, or 40 A for 2 hours, and so on. Calculate the total energy, in J, stored in a 12 V, 80 Ah car battery.

7 An electric kettle is rated at 2.2 kW, 240 V. The supply voltage is reduced from 240 V to 230 V.

Calculate the new power of the kettle.

8 The element of an electric kettle has resistance 26 Ω at room temperature. The element is made of nichrome wire of diameter 0.60 mm and resistivity 1.1×10^{-6} Ω m at room temperature. Calculate the length of the wire.

9 The values of the current I through an electrical component for different potential differences V across it are shown in Fig. 9.19.

V/V	0	0.19	0.48	1.47	2.92	4.56	6.56	8.70
I/A	0	0.20	0.40	0.60	0.80	1.00	1.20	1.40

▲ **Figure 9.19**

a Draw a diagram of the circuit that could be used to obtain these values.
b Calculate the resistance of the component at each value of current.
c Plot a graph to show the variation with current of the resistance of the component.
d Suggest what the component is likely to be, giving a reason for your answer.

10 The current in a 2.50 m length of wire of diameter 1.5 mm is 0.65 A when a potential difference of 0.40 V is applied between its ends. Calculate:
a the resistance of the wire,
b the resistivity of the material of the wire.

11 a The output of a heater is 2.5 kW when connected to a 220 V supply.
i Calculate the resistance of the heater. [2]
ii The heater is made from a wire of cross-sectional area 2.0×10^{-7} m^2 and resistivity 1.1×10^{-6} Ω m.
Use your answer in **i** to calculate the length of the wire. [3]
b The supply voltage is changed to 110 V.
i Calculate the power output of the heater at this voltage, assuming there is no change in the resistance of the wire. [1]
ii State and explain quantitatively **one** way that the wire of the heater could be changed to give the same power as in **a**. [2]

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12 a Define *charge*. [1]
b A heater is made from a wire of resistance 18.0 Ω and is connected to a power supply of 240 V. The heater is switched on for 2.60 Ms.
Calculate:
i the power transformed in the heater, [2]
ii the current in the heater, [1]
iii the charge passing through the heater in this time, [2]
iv the number of electrons per second passing a given point in the heater. [2]

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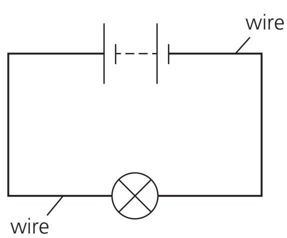
13 Wires are used to connect a battery of negligible internal resistance to a lamp, as shown in Fig. 9.20.



▲ **Figure 9.20**

The lamp is at its normal operating temperature. Some data for the filament wire of the lamp and for the connecting wires of the circuit are shown in Fig. 9.21.

	filament wire	connecting wires
diameter	d	$14d$
total length	L	$7.0L$
resistivity of metal (at normal operating temperature)	ρ	0.028ρ

▲ **Figure 9.21**

i Show that $\dfrac{\text{resistance of filament wire}}{\text{total resistance of connecting wires}} = 1000$ [2]

ii Use the information in **i** to explain qualitatively why the power dissipated in the filament wire of the lamp is greater than the total power dissipated in the connecting wires. **[1]**

iii The lamp is rated as 12 V, 6.0 W. Use the information in **i** to determine the total resistance of the connecting wires. **[3]**

iv The diameter of the connecting wires is decreased. The total length of the connecting wires and the resistivity of the metal of the connecting wires remain the same. State and explain the change, if any, that occurs to the resistance of the filament wire of the lamp. **[3]**

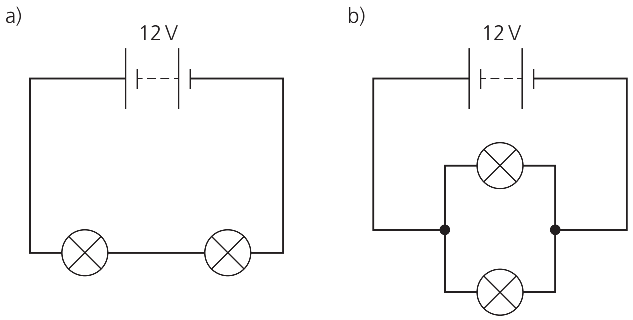
Cambridge International AS and A Level Physics (9702) Paper 21 Q7 part b only Oct/Nov 2017

14 a Describe the I–V characteristic of:

i a metallic conductor at constant temperature, **[1]**

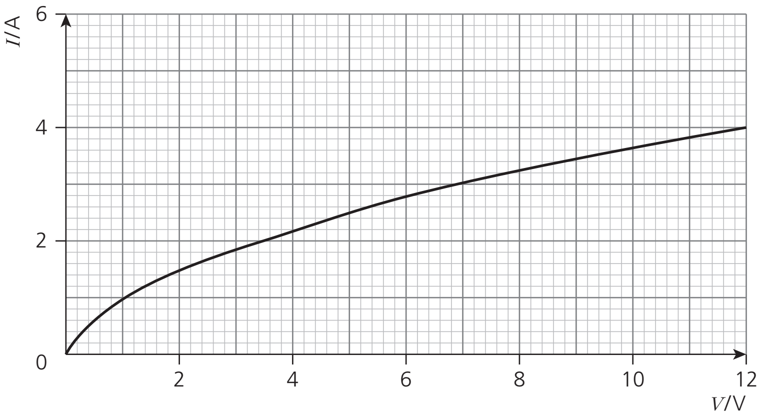
ii a semiconductor diode. **[2]**

b Two identical filament lamps are connected in series and then in parallel to a battery of electromotive force (e.m.f.) 12 V and negligible internal resistance, as shown in Fig. 9.22a and Fig. 9.22b.



▲ **Figure 9.22**

The I–V characteristic of each lamp is shown in Fig. 9.23.



▲ **Figure 9.23**

i Use the information shown in Fig. 9.23 to determine the current through the battery in:

1 the circuit of Fig. 9.22a,

2 the circuit of Fig. 9.22b. **[3]**

ii Calculate the total resistance in:

1 the circuit of Fig. 9.22a,

2 the circuit of Fic 9.22b. **[3]**

iii Calculate the ratio:

$$\frac{\text{power dissipated in a lamp in the circuit of Fig. 9.22a}}{\text{power dissipated in a lamp in the circuit of Fig. 9.22b}}$$ **[2]**

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10 D.C. circuits

Learning outcomes

By the end of this topic, you will be able to:

10.1 Practical circuits

1. recall and use the appropriate circuit symbols for this syllabus
2. draw and interpret circuit diagrams containing power sources, switches, resistors, ammeters and voltmeters and/or any other type of component referred to in the syllabus
3. define and use the electromotive force (e.m.f.) of a source as energy transferred per unit charge in driving charge around a complete circuit
4. distinguish between e.m.f. and potential difference (p.d.) in terms of energy considerations
5. understand the effects of the internal resistance of a source of e.m.f. on the terminal potential difference

10.2 Kirchhoff's laws

1. recall Kirchhoff's first law and understand that it is a consequence of conservation of charge
2. recall Kirchhoff's second law and understand that it is a consequence of conservation of energy
3. derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series
4. use the formula for the combined resistance of two or more resistors in series
5. derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in parallel
6. use the formula for the combined resistance of two or more resistors in parallel
7. use Kirchhoff's laws to solve simple circuit problems

10.3 Potential dividers

1. understand the principle of a potential divider circuit
2. recall and use the principle of the potentiometer as a means of comparing potential differences
3. understand the use of a galvanometer in null methods
4. explain the use of thermistors and light-dependent resistors in potential dividers to provide a potential difference that is dependent on temperature and light intensity

Starting points

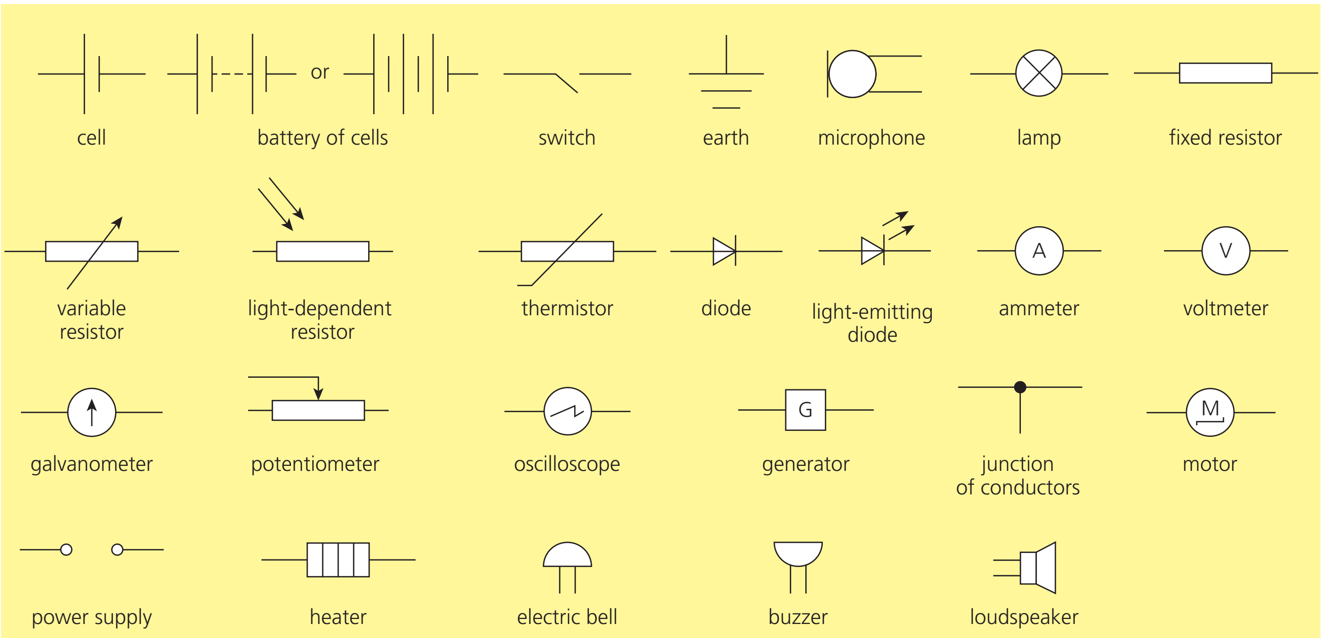
★ Basic knowledge of appropriate circuit symbols.
★ Methods for drawing and interpreting circuit diagrams.
★ Electric current is the rate of flow of charge.
★ A potential difference is required to provide energy to move charge through a device.
★ The p.d. V across a component and the current I through it are related by $V = IR$ where R is its resistance.

10.1 Practical circuits

When reporting an electrical experiment, or describing a circuit, it is essential to know exactly how the components are connected. This could be done by taking a photograph, but this technique has disadvantages. The photograph in Figure 9.4, for example, is not clear and does not show all the components. You could sketch a block diagram, in which the components are indicated as rectangular boxes labelled 'cell', 'ammeter', 'resistor', etc.

The blocks would then be connected with lines to indicate the wiring. This is also unsatisfactory; it takes a lot of time to label all the boxes. It is much better to draw the circuit diagram using a set of symbols that is recognised by everyone and which do not need to be labelled.

Figure 10.1 shows the symbols that you are likely to need in school and college work, and which you will meet in examination questions. (You will have met many of them already.) It is important that you learn these so that you can recognise them straight away. The only labels you are likely to see on them will be the values of the components, for example, 1.5 V for a cell or 22 Ω for a resistor.



▲ **Figure 10.1** Circuit symbols

Electromotive force and potential difference

When charge passes through a power supply such as a battery, there is a transfer of energy. The power supply is said to have an **electromotive force**, or **e.m.f.** for short. The e.m.f. is a property of the power supply, battery or cell. The electromotive force measures, in volts, the energy transferred per unit of charge that passes through the power supply. Note that, in spite of its name, the e.m.f. is not a force. The energy gained by the charge comes from the chemical energy of a battery.

$$\text{e.m.f.} = \frac{\text{energy transferred from other forms to electrical in driving charge around a complete circit}}{\text{charge}}$$

When a **potential difference (p.d.)** is applied across a resistor, charge passes through the resistor. The energy of the charge is converted to thermal energy in the resistor. As we saw in Topic 9.2, the potential difference measures, in volts, the energy transferred per unit of charge that passes through the resistor.

p.d. = energy transferred per unit charge

WORKED EXAMPLE 10A

Two lamps are connected in series to a battery. State the energy transformation that occurs in

a the battery
b the lamps.

Answers

a chemical to electrical
b electrical to thermal (heat) and light

Question

1 Each lamp in the example above has a resistance R and the e.m.f. of the battery is E. The current in the circuit is I. State the rate of energy transformation in:

a the battery
b a lamp.

Internal resistance

When a car engine is started with the headlights switched on, the headlights sometimes dim. This is because the car battery has resistance.

All power supplies have some resistance between their terminals, called **internal resistance**. When a power supply delivers a current the charge passing around the circuit dissipates some of its electrical energy as thermal energy in the power supply itself. The power supply becomes warm when it delivers a current.

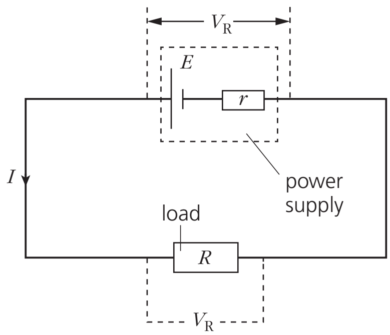


Figure 10.2

Figure 10.2 shows a power supply which has e.m.f. E and internal resistance r. It delivers a current I when connected to an external resistor of resistance R (called the load). V_R is the potential difference across the load, and V_r is the potential difference across the internal resistance. Using conservation of energy,

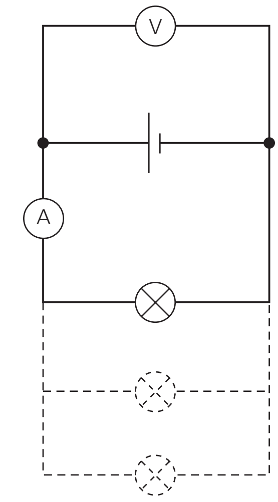
$$E = V_R + V_r$$

The potential difference V_R across the load is thus given by

$$V_R = E - V_r$$

V_R is also the **terminal potential difference** across the power supply.

The terminal potential difference is the p.d. between the terminals of a cell or power supply when a current is being delivered.



▲ **Figure 10.3** Effect of circuit current on terminal potential difference

The terminal potential difference is always less than the electromotive force when the power supply delivers a current. This is because of the potential difference across the internal resistance.

p.d. across the internal resistance = e.m.f. – terminal p.d.

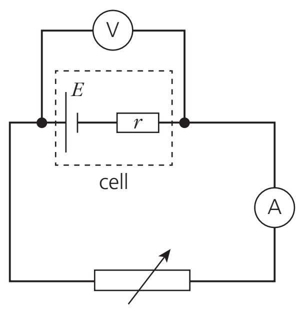
The electromotive force is the terminal potential difference when the power supply is on open circuit (when no current is delivered). The p.d. across the internal resistance V_r is zero ($V_r = Ir = 0$). The e.m.f. may be measured by connecting a very high resistance voltmeter across the terminals of the power supply.

You can use the circuit in Figure 10.3 to show that the greater the current delivered by the power supply, the lower its terminal potential difference. As more lamps are connected in parallel to the power supply, the current increases and the p.d. across the internal resistance, is given by

p.d. across the internal resistance = current × internal resistance

will increase. Thus the terminal potential difference decreases.

To return to the example of starting a car with its headlights switched on, a large current (perhaps 100 A) is supplied to the starter motor by the battery. There will then be a large potential difference across the internal resistance of the battery; that is, the lost voltage will be large. The terminal potential difference will drop and the lights will dim. They return to normal brightness once the engine starts. This is because the starter motor is automatically disconnected when the engine starts.

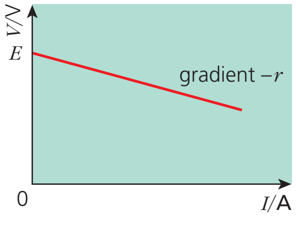


▲ **Figure 10.4** Circuit for measuring e.m.f. and internal resistance

In the terminology of Figure 10.2, $V_R = IR$ and $V_r = Ir$, so $E = V_R + V_r$ becomes

$$E = IR + Ir, \text{ or } E = I(R + r)$$

The e.m.f. E and internal resistance r of a cell or battery may be measured using the circuit shown in Figure 10.4. The high resistance voltmeter measures the p.d. V across the terminals of the battery. The current I in the circuit is varied using the variable resistor (rheostat). The terminal p.d. decreases as the current supplied by the battery increases and the p.d. across the internal resistance increases. A set of readings for V and I is measured. A graph of terminal p.d. V is plotted against the current I supplied by the battery (see Figure 10.5). The graph is a straight line with a negative gradient and a positive intercept on the V axis. The equation of the line is given by: terminal p.d. = e.m.f. – p.d. across the internal resistance, or $V = E - Ir$. This is an equation of a straight line of the form $y = mx + c$.



▲ **Figure 10.5** Graph of terminal p.d. against current

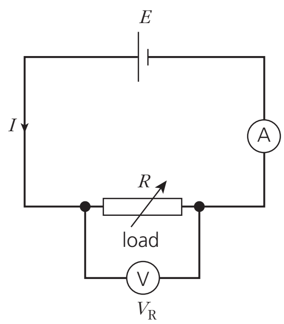
Hence the gradient is $-r$ and the intercept is E.

The maximum current that a power supply can deliver will be when its terminals are short-circuited by a wire of negligible resistance, so that $R = 0$. In this case, the potential difference across the internal resistance will equal the e.m.f. of the cell. The terminal p.d. is then zero. **Warning: do not try out this experiment, as the wire gets very hot; there is also a danger of the battery exploding.**

Quite often, in problems, the internal resistance of a supply is assumed to be negligible, so that the potential difference V_R across the load and the terminal p.d. are equal to the e.m.f. of the power supply.

Effect of internal resistance on power from a battery

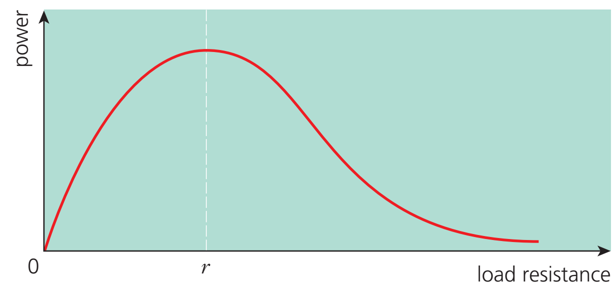
The power delivered by a battery to a variable external load resistance can be investigated using the circuit of Figure 10.6. Readings of current I and potential difference V_R across the load are taken for different values of the variable load resistor.



▲ **Figure 10.6** Circuit for investigating power transfer to an external load

The product $V_R I$ gives the power dissipated in the load, and the quotient V_R/I gives the load resistance R.

Figure 10.7 shows the variation with load resistance R of the power $V_R I$ dissipated. The graph indicates that there is a maximum power delivered by the battery at one value of the external resistance. This value is equal to the internal resistance r of the battery.



▲ **Figure 10.7** Graph of power delivered to external load against load resistance

A battery delivers maximum power to a circuit when the load resistance of the circuit is equal to the internal resistance of the battery.

WORKED EXAMPLE 10B

A high-resistance voltmeter reads 13.0 V when it is connected across the terminals of a battery. The voltmeter reading drops to 12.0 V when the battery delivers a current of 3.0 A to a lamp. State the e.m.f. of the battery. Calculate the potential difference across the internal resistance when the battery is connected to the lamp. Calculate the internal resistance of the battery.

Answer

The e.m.f. is **13.0 V**, since this is the voltmeter reading when the battery is delivering negligible current.

Using $V_r = E - V_R$, p.d. across the internal resistance = V_r = 13.0 – 12.0 = **1.0 V**.

Using $V_r = Ir$, r = 1.0/3.0 = **0.33 Ω**.

Questions

2 Three identical cells, each of e.m.f. 1.5 V, are connected in series to a 15 Ω lamp. The current in the circuit is 0.27 A. Calculate the internal resistance of each cell.

3 A cell of e.m.f. 1.5 V has an internal resistance of 0.50 Ω.

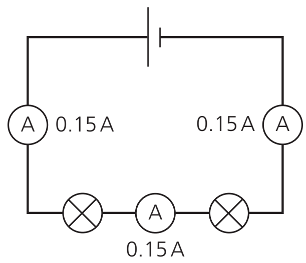
  a Calculate the maximum current it can deliver. Under what circumstances does it deliver this maximum current?

  b Calculate also the maximum power it can deliver to an external load. Under what circumstances does it deliver this maximum power?

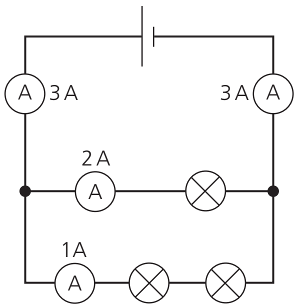
10.2 Kirchhoff's laws

Conservation of charge: Kirchhoff's first law

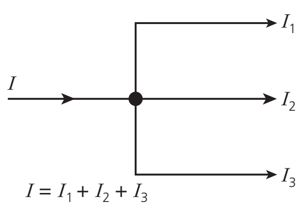
A **series circuit** is one in which the components are connected one after another, forming one complete loop. You have probably connected an ammeter at different



▲ **Figure 10.8** The current at each point in a series circuit is the same.



▲ **Figure 10.9** The current divides in a parallel circuit.



▲ **Figure 10.10**

points in a series circuit to show that it reads the same current at each point (see Figure 10.8).

A **parallel circuit** is one where the current can take alternative routes in different loops. In a parallel circuit, the current divides at a junction, but the current entering the junction is the same as the current leaving it (see Figure 10.9). The fact that the current does not get 'used up' at a junction is because current is the rate of flow of charge, and charges cannot accumulate or get 'used up' at a junction. The consequence of this conservation of electric charge is known as **Kirchhoff's first law**. This law is usually stated as follows.

The sum of the currents entering a junction in a circuit is always equal to the sum of the currents leaving the junction.

At the junction shown in Figure 10.10,

$$I = I_1 + I_2 + I_3$$

WORKED EXAMPLE 10C

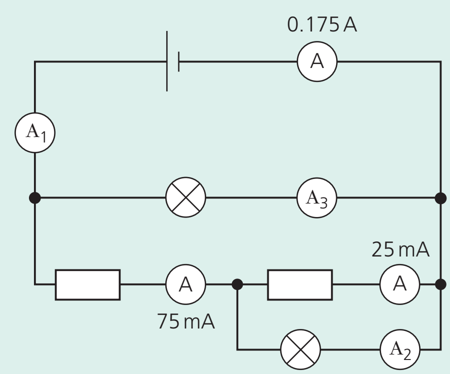
For the circuit of Figure 10.11, state the readings of the ammeters A_1, A_2 and A_3.

Answer

A_1 would read **175 mA**, as the current entering the power supply must be the same as the current leaving it.

A_2 would read 75 – 25 = **50 mA**, as the total current entering a junction is the same as the total current leaving it.

A_3 would read 175 – 75 = **100 mA**.

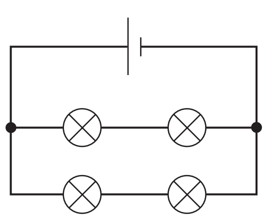


▲ **Figure 10.11**

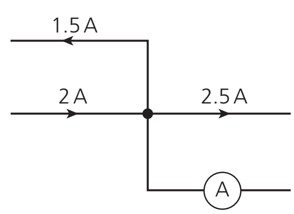
Questions

4 The lamps in Figure 10.12 are identical. There is a current of 0.50 A through the battery. What is the current in each lamp?

5 Figure 10.13 shows one junction in a circuit. Calculate the ammeter reading.



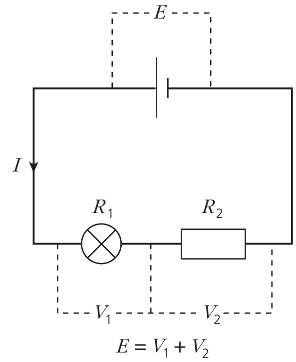
▲ **Figure 10.12**



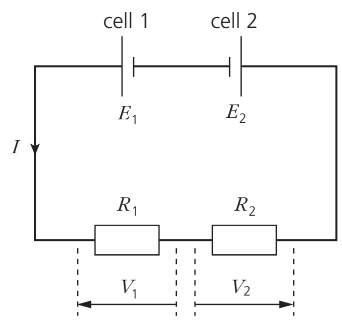
▲ **Figure 10.13**

Conservation of energy: Kirchhoff's second law

When a battery supplies a current to a circuit the charge flowing round the circuit gains electrical energy on passing through the battery and loses electrical energy on passing through the rest of the circuit. From the law of conservation of energy, we know that the total energy must remain the same. The consequence of this conservation of energy is known as **Kirchhoff's second law**. This law may be stated as follows.



▲ **Figure 10.14**



▲ **Figure 10.15**

The sum of the electromotive forces in a closed circuit is equal to the sum of the potential differences.

Figure 10.14 shows a circuit containing a battery, lamp and resistor in series. The battery has negligible internal resistance. Applying Kirchhoff's second law, the electromotive force in the circuit is the e.m.f. E of the battery. The sum of the potential differences is the p.d. V_1 across the lamp plus the p.d. V_2 across the resistor. Thus, $E = V_1 + V_2$. If the current in the circuit is I and the resistances of the lamp and resistor are R_1 and R_2 respectively, the p.d.s can be written as $V_1 = IR_1$ and $V_2 = IR_2$, so

$$E = IR_1 + IR_2$$

It should be remembered that both electromotive force and potential difference have direction. This must be considered when working out the equation for Kirchhoff's second law. For example, in the circuit of Figure 10.15, two cells have been connected in opposition. Both cells have negligible internal resistance.

Here the total electromotive force in the circuit is $E_1 - E_2$, and by Kirchhoff's second law

$$E_1 - E_2 = V_1 + V_2 = IR_1 + IR_2$$

In Figure 10.15 the direction of the current will depend on whether $E_1 > E_2$ or $E_1 < E_2$. If the normal direction of current through a cell is reversed, that cell is recharged rather than transferring energy to the charges that pass through it. The current through cell 1 is in the normal direction and that through cell 2 is reversed. Hence cell 2 is being charged by cell 1.

In problems, the e.m.f. of a cell or battery is taken to be constant. In reality the e.m.f. decreases with use and the battery is recharged if it rechargeable.

WORKED EXAMPLE 10D

For the circuit in Figure 10.15, cell 1 has e.m.f. 3.0 V and cell 2 e.m.f. 1.5 V. The resistance values of the resistors R_1 and R_2 are 2.0 Ω and 4.0 Ω respectively. Calculate the current in the circuit.

Answer

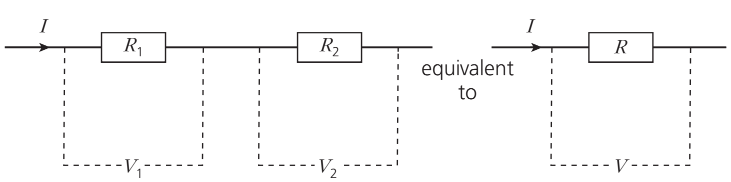
Using Kirchhoff's second law: $3.0 - 1.5 = (I \times 2) + (I \times 4)$ therefore, $I = 1.5/6 =$ **0.25 A**.

Question

6 For the circuit in Figure 10.15 the e.m.f .of cell 1 is 6.0 V and cell 2 has an e.m.f. E_2. The resistance values of the resistors R_1 and R_2 are 5.0 Ω and 3.0 Ω respectively. The current in the circuit is 0.50 A. Calculate the e.m.f. of cell 2.

Resistors in series

Figure 10.16 shows two resistors of resistances R_1 and R_2 connected in series, and a single resistor of resistance R equivalent to them. The current I in the resistors, and in their equivalent single resistor, is the same.



▲ **Figure 10.16** Resistors in series

The total potential difference V across the two resistors must be the same as that across the single resistor. If V_1 and V_2 are the potential differences across each resistor,

$$V = V_1 + V_2$$

But since potential difference is given by multiplying the current by the resistance,

$$IR = IR_1 + IR_2$$

Dividing by the current I,

$$R = R_1 + R_2$$

This equation can be extended so that the equivalent resistance R of several resistors connected in series is given by the expression

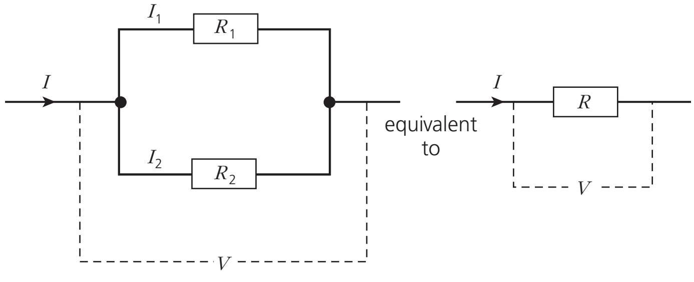
$$R = R_1 + R_2 + R_3 + \ldots$$

Thus

The combined resistance of resistors in series is the sum of all the individual resistances.

Resistors in parallel

Now consider two resistors of resistance R_1 and R_2 connected in parallel, as shown in Figure 10.17. The current through each will be different, but they will each have the same potential difference. The equivalent single resistor of resistance R will have the same potential difference across it, but the current will be the total current through the separate resistors.



▲ **Figure 10.17** Resistors in parallel

By Kirchhoff's first law,

$$I = I_1 + I_2$$

and using resistance = p.d./current, so $I = V/R$

$$V/R = V/R_1 + V/R_2$$

Dividing by the potential difference V,

$$1/R = 1/R_1 + 1/R_2$$

This equation can be extended so that the equivalent resistance R of several resistors connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Thus

The **reciprocal** of the combined resistance of resistors in parallel is the sum of the **reciprocals** of all the individual resistances.

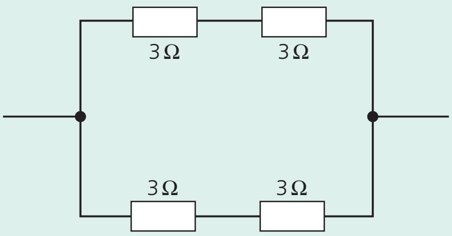
Note that:

1 For two identical resistors in parallel, the combined resistance is equal to half of the value of each one.

2 For resistors in parallel, the combined resistance is always less than the value of the smallest individual resistance.

WORKED EXAMPLE 10E

Calculate the equivalent resistance of the arrangement of resistors in Figure 10.18.



▲ **Figure 10.18**

Answer

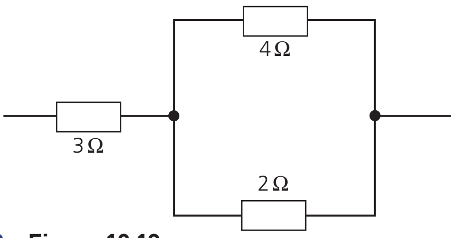
The arrangement is equivalent to two 6 Ω resistors in parallel, so the combined resistance R is given by $1/R = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$. (Don't forget to find the reciprocal of this value.)

Thus $R = 3\,\Omega$.

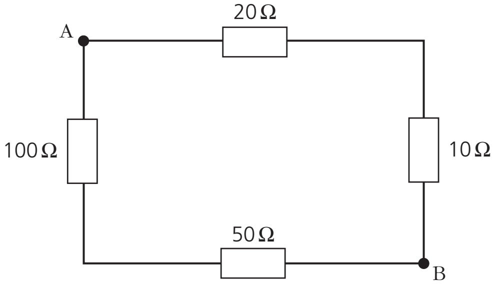
Questions

7 Calculate the equivalent resistance of the arrangement of resistors in Figure 10.19. *Hint*: First find the resistance of the parallel combination.

8 Calculate the effective resistance between the points A and B in the network in Figure 10.20.



▲ **Figure 10.19**

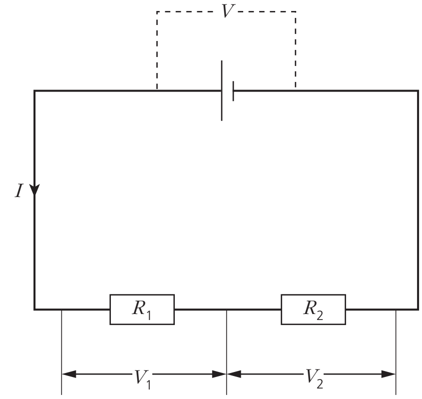


▲ **Figure 10.20**

10.3 Potential dividers

Two resistors connected in series with a cell each have a potential difference. They may be used to divide the e.m.f. of the cell. This is illustrated in Figure 10.21 on the next page.

The current in each resistor is the same, because they are in series. Thus $V_1 = IR_1$ and $V_2 = IR_2$. Dividing the first equation by the second gives $V_1/V_2 = R_1/R_2$. The ratio of the voltages across the two resistors is the same as the ratio of their resistances. If the potential difference across the combination were 12 V and R_1 were equal to R_2, then each resistor would have 6 V across it. If R_1 were twice the magnitude of R_2, then V_1 would be 8 V and V_2 would be 4 V.



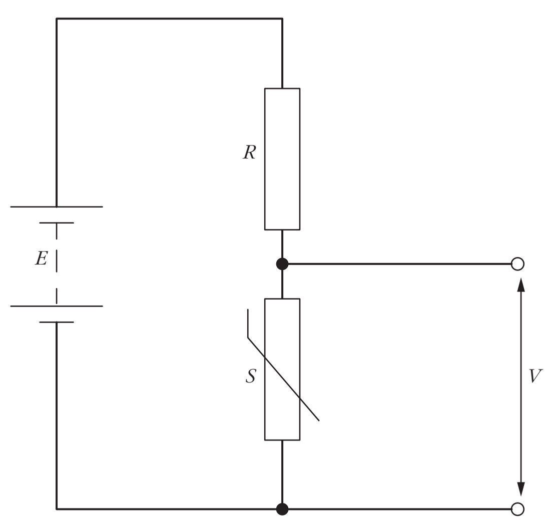
▲ **Figure 10.21** The potential divider

The use of potential dividers

If a component with a variable resistance is connected in series with a fixed resistor, and the combination is connected to a cell or battery to make a potential divider, then we have the situation of a potential divider that is variable between certain limits. The component of variable resistance could be, for example, a light-dependent resistor or a thermistor (see Topic 9.3). Changes in the light intensity or the temperature cause a change in the resistance of one component of the potential divider, so that the potential difference across this component changes. The change in the potential difference can be used to operate control circuitry if, for example, the light intensity becomes too low or too high, or the temperature falls outside certain limits.

The voltage can then be used to control an output device.

In Figure 10.22, a thermistor of varying resistance S is connected in series with a resistor of constant resistance R.



▲ **Figure 10.22** Potential divider circuit

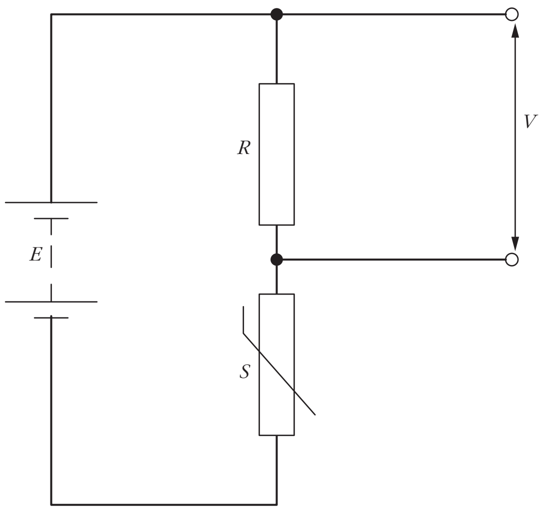
The battery of e.m.f. E is assumed to have negligible internal resistance. The output voltage V across the thermistor is given by the expression

$$V = \frac{SE}{(S+R)}$$

The magnitude V of the potential difference (voltage) at any particular resistance value S of the thermistor is dependent on the relative values of S and R. Note that, as the resistance S of the thermistor increases, the output voltage V also increases.

By monitoring the potential difference (voltage) across the resistor of fixed resistance R, as shown in Figure 10.23, the output voltage will be given by

$$V = \frac{RE}{(S+R)}$$



▲ **Figure 10.23** Alternative connections for a potential divider

The output voltage V will then decrease as the thermistor resistance S increases (temperature of the thermistor decreases).

If the thermistor in the circuits shown in Figures 10.22 and 10.23 is replaced with a light-dependent resistor (LDR) the output voltage will be controlled by the resistance of the LDR. Hence the output voltage depends on the light intensity on the LDR. For example, in the circuit shown in Figure 10.22 the output voltage will increase as the resistance of the LDR increases (light intensity decreases).

WORKED EXAMPLE 10F

A potential divider consists of a battery of e.m.f. 6.00 V and negligible internal resistance connected in series with a resistor of resistance 120 Ω and a variable resistor of resistance 0 → 200 Ω. Determine the range of potential difference that can be obtained across the fixed resistor.

Answer

When the variable resistor is at 0 Ω, the p.d. across the fixed resistor = 6.0 V.
When the variable resistor is at 200 Ω,

$$\text{p.d. across fixed resistor} = 120/(120 + 200) \times 6.00 = 2.25\,\text{V}$$

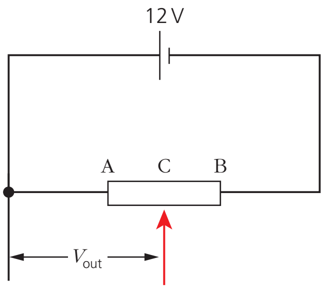
The range is **2.25 V → 6.00 V**.

Question

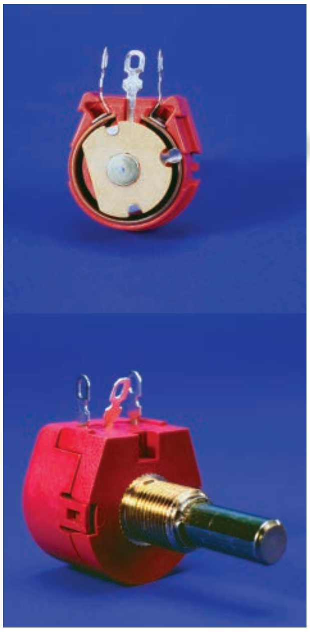
9 A potential divider consists of a battery of e.m.f. 7.5 V and negligible internal resistance connected in series with a resistor of resistance R and a variable resistor of resistance 0 → 500 Ω. Deduce how the potential divider may be arranged so as to provide a potential difference that may be varied between 0 and 3.0 V.

Potentiometers

A **potentiometer** is a continuously variable potential divider. In Topic 9, a variable voltage supply was used to vary the voltage across different circuit components. A **variable resistor**, or **rheostat**, may be used to produce a continuously variable voltage.



▲ **Figure 10.24** Potentiometer circuit



▲ **Figure 10.25** Internal and external views of a potentiometer

Such a variable resistor is shown in Figure 10.24. The fixed ends AB are connected across the battery so that there is the full battery voltage across the whole resistor. As with the potential divider, the ratio of the voltages across AC and CB will be the same as the ratio of the resistances of AC and CB. When the sliding contact C is at the end B, the output voltage V_{out} will be 12 V. When the sliding contact is at end A, then the output voltage will be zero. So, as the sliding contact is moved from A to B, the output voltage varies continuously from zero up to the battery voltage. In terms of the terminal p.d. V of the cell, the output V_{out} of the potential divider is given by

$$V_{out} = \frac{VR_1}{(R_1 + R_2)}$$

where R_1 is the resistance of AC and R_2 is the resistance of CB.

A variable resistor connected in this way is called a potentiometer. A type of potentiometer is shown in Figure 10.25. Note the three connections.

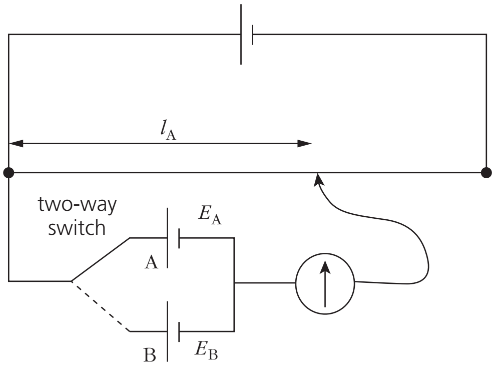
Using a potentiometer to compare potential differences or e.m.f.s of cells

A potentiometer can also be used as a means of comparing potential differences. The circuit of Figure 10.26 illustrates the principle. In this circuit the variable potentiometer resistor consists of a length of uniform resistance wire, stretched along a metre rule. Contact can be made to any point on this wire using a sliding contact. A **centre-zero galvanometer** is used to detect the current through cell A. A galvanometer is a sensitive current-measuring analogue meter. A galvanometer with a centre-zero scale shows negative currents when the needle is to the left-hand side of the zero mark and positive currents when it is to the right. In this circuit it is used as a null indicator; that is, to detect when the current through cell A is zero.

Suppose that the cell A has a known e.m.f. E_A. This cell is switched into the circuit using the two-way switch. The sliding contact is then moved along the wire until the centre-zero galvanometer reads zero. This position of the sliding contact on the wire is called the **balance point**. The current through cell A is zero and the p.d. across the length of wire l_A is 'balanced' with the p.d. across cell A. Since the current through the cell is zero then the p.d. across the cell is equal to its e.m.f. The length l_A of the wire from the common zero end to the sliding contact is noted. Cell B has an unknown e.m.f. E_B. This cell is then switched into the circuit and the balancing process repeated. Suppose that the position at which the galvanometer reads zero is then a distance l_B from the common zero to the sliding contact. The ratio of the e.m.f.s is the ratio of the balance lengths; that is,

$$E_B/E_A = l_B/l_A$$

and E_B can be determined in terms of the known e.m.f. E_A.



▲ **Figure 10.26** Potentiometer used to compare cell e.m.f.s

WORKED EXAMPLE 10G

1 A light-emitting diode (LED) is connected in series with a resistor to a 5.0 V supply.
  - **a** Calculate the resistance of the series resistor required to give a current in the LED of 12 mA, with a voltage across it of 2.0 V.
  - **b** Calculate the potential difference across the LED when the series resistor has resistance 500 Ω. Assume the resistance of the LED remains constant.

2 The e.m.f.s of two cells are compared using the slide-wire circuit of Figure 10.26. Cell A has a known e.m.f. of 1.02 V; using this cell, a balance point is obtained when the slider is 37.6 cm from the zero of the scale. Using cell B, the balance point is at 55.3 cm.
  - **a** Calculate the e.m.f. of cell B.
  - **b** State the advantage of using this null method to compare the e.m.f.s.

Answers

1 a If the supply voltage is 5.0 V and the p.d. across the LED is 2.0 V, the p.d. across the resistor must be 5.0 – 2.0 = 3.0 V. The current through the resistor is 12 mA as it is in series with the LED. Using $R = V/I$, the resistance of the resistor is $3.0/12 \times 10^{-3}$ = **250 Ω**.

b The resistance of the LED is given by $R = V/I = 2.0/12 \times 10^{-3}$ = 167 Ω. If this resistance is in series with a 500 Ω resistor and a 5.0 V supply, the p.d. across the LED is 5.0 × 167/(167 + 500) = **1.25 V**.

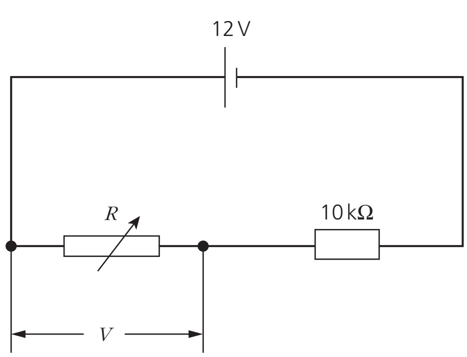
2 a This is a straightforward application of the formula for the potentiometer, $E_B/E_A = l_B/l_A$. Substituting the values, E_B = **1.50 V**.

b When comparing the e.m.f.s of cells, it is necessary to arrange for the cells to be on open circuit so that there is no drop in terminal potential difference because of a current passing through the internal resistance. When the potentiometer is balanced, there is no current from the cell under test, which is exactly what is required.

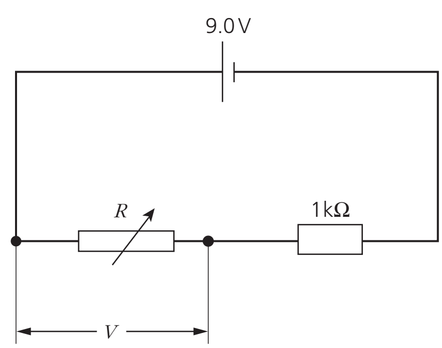
Questions

10 Figure 10.27 shows a variable resistor R connected in series with a 10 kΩ resistor and a 12 V supply. Calculate:
  - **a** the p.d. V across R when it has resistance 8.0 MΩ
  - **b** the p.d. V across R when it is has resistance 500 Ω
  - **c** the resistance of R which makes the p.d. V across R equal to 4.0 V.

11 The resistor R in the potential divider circuit of Figure 10.28 has a resistance which varies between 100 Ω and 6.0 kΩ. Calculate the potential difference V across R when its resistance is:
  - **a** 100 Ω
  - **b** 6.0 kΩ.



▲ **Figure 10.27**

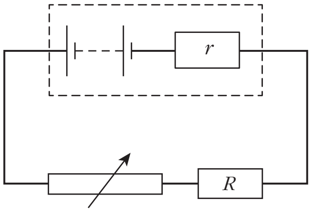


▲ **Figure 10.28**

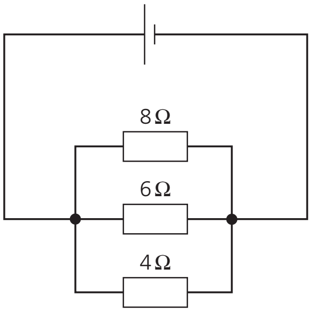
SUMMARY

- The electromotive force (e.m.f.) of a power supply is equal to the energy transferred from other forms to electrical per unit charge in driving charge around a complete circuit.
- The potential difference (p.d.) across a component is equal to the energy transferred per unit of charge passing through the component. For a resistor the energy is transferred to thermal and light energies.
- The voltage across the terminals of a supply (the terminal p.d.) is always less than the e.m.f. of the supply when the supply is delivering a current, because of the p.d. across the internal resistance.
- For a supply of e.m.f. E which has internal resistance r, $E = I(R+r)$ where R is the external circuit resistance and I is the current in the supply.
- A supply delivers maximum power to a load when the load resistance is equal to the internal resistance of the supply.
- At any junction in a circuit, the total current entering the junction is equal to the total current leaving it. This is Kirchhoff's first law, and is a consequence of the law of conservation of charge.
- In any closed loop of a circuit, the sum of the electromotive forces is equal to the sum of the potential differences. This is Kirchhoff's second law, and is a consequence of the law of conservation of energy.
- The equivalent resistance R of resistors connected in series is given by:
  $R = R_1 + R_2 + R_3 + \dots$
- The equivalent resistance R of resistors connected in parallel is given by:
  $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \dots$
- Two resistors in series act as a potential divider, where $V_1/V_2 = R_1/R_2$. If V is the supply voltage and V_{out} is the voltage across R_1 then:
  $V_{out} = VR_1/(R_1 + R_2)$.
- A potentiometer is a variable resistor connected as a potential divider to give a continuously variable output voltage.
- Thermistors and light dependent resistors may be used in potential divider circuits to provide a p.d. that is dependent on temperature and light intensity respectively.
- A potentiometer may be used to compare e.m.f.s of cells or potential differences.
- When using a potentiometer to compare e.m.f.s of cells or potential differences, a galvanometer with a centre-zero scale is used to detect when the current through the cell is zero.

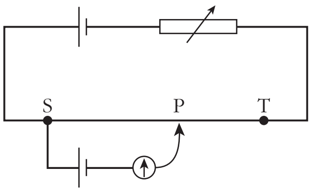
END OF TOPIC QUESTIONS



▲ **Figure 10.29**



▲ **Figure 10.30**



▲ **Figure 10.31**

1 Fig 10.29 shows a battery with an internal resistance r connected to a fixed resistor R and a variable resistor. The resistance of the variable resistor is increased. Which of the following statements is correct?

A The potential difference across the battery decreases.
B The current through the battery increases.
C The potential difference across the battery increases.
D The potential difference across r increases.

2 Fig. 10.30 shows a cell connected to three resistors of resistance, 4 Ω, 6 Ω and 8 Ω, in parallel. Which statement is correct?

A The current through the 8 Ω resistor is greater than the current through the 4 Ω resistor.
B The potential difference (p.d.) across the 8 Ω resistor is greater than the p.d. across the 4 Ω resistor.
C The power transferred in each resistor is greatest in the 4 Ω resistor.
D The total resistance in the circuit is greater than 8 Ω.

3 Fig 10.31 shows a potentiometer circuit used to compare electromotive force (e.m.f.). The galvanometer reads zero when the connection is at point P on the wire ST. The resistance of the rheostat is increased. Which statement is correct?

A The potential difference (p.d.) across ST increases.
B The current in ST increases.
C The balance point is now closer to S.
D The balance point is closer to T.

4 The internal resistance of a dry cell increases gradually with age, even if the cell is not being used. However, the e.m.f. remains approximately constant. You can check the age of a cell by connecting a low-resistance ammeter across the cell and measuring the current. For a new 1.5 V cell of a certain type, the short-circuit current should be about 30 A.

a Calculate the internal resistance of a new cell.

b A student carries out this test on an older cell, and finds the short-circuit current to be only 5 A. Calculate the internal resistance of this cell.

5 A torch bulb has a power supply of two 1.5 V cells connected in series. The potential difference across the bulb is 2.2 V, and it dissipates energy at the rate of 550 mW. Calculate:

a the current through the bulb,

b the internal resistance of each cell,

c the heat energy dissipated in each cell in 2 minutes.

6 Two identical light bulbs are connected first in series, and then in parallel, across the same battery (assumed to have negligible internal resistance). Use Kirchhoff's laws to decide which of these connections will give the greater total light output.

7 You are given three resistors of resistance 22 Ω, 47 Ω and 100 Ω. Calculate:

a the maximum possible resistance,

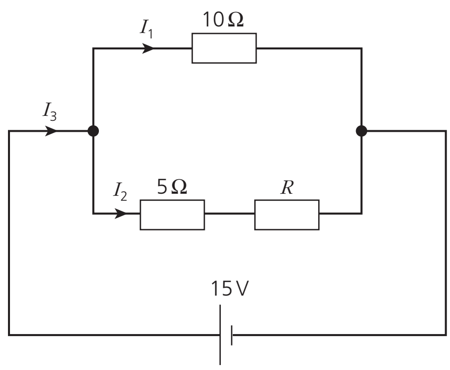
b the minimum possible resistance,

that can be obtained by combining any or all of these resistors.

8 In the circuit of Fig. 10.32, the currents I_1 and I_2 are equal. Calculate:

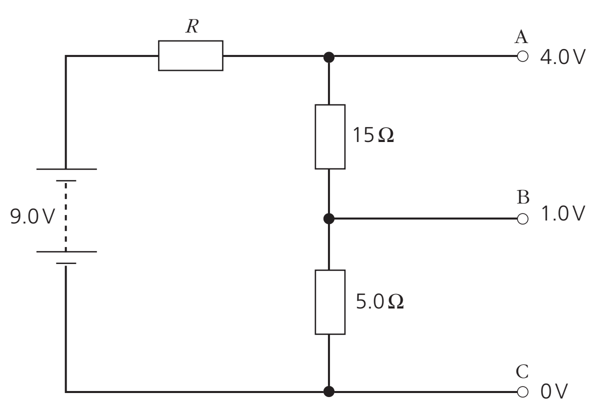
a the resistance R of the unknown resistor,

b the total current I_3.



▲ **Figure 10.32**

9 Fig. 10.33 shows a potential divider circuit, designed to provide p.d.s of 1.0 V and 4.0 V from a battery of e.m.f. 9.0 V and negligible internal resistance.

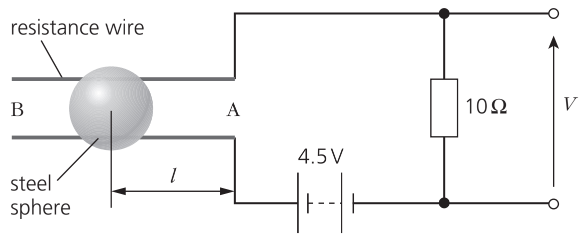


▲ **Figure 10.33**

a Calculate the value of resistance R.

b State and explain what happens to the voltage at terminal A when an additional 1.0 Ω resistor is connected between terminals B and C in parallel with the 5.0 Ω resistor. No calculations are required.

10 A student designs an electrical method to monitor the position of a steel sphere rolling on two parallel rails. Each rail is made from bare wire of length 30 cm and resistance 20 Ω. The position-sensing circuit is shown in Fig. 10.34. The resistance of the steel sphere and the internal resistance of the battery are negligible.



▲ **Figure 10.34**

a State the voltage across the 10 Ω resistor when the sphere is at A, where $l = 0$.

b With the sphere at end B of the rails, calculate:

i the total resistance of the circuit,

ii the current in the 10 Ω resistor,

iii the output voltage V.

11 Two equations for the power P dissipated in a resistor are $P = I^2R$ and $P = V^2/R$. The first suggests that the greater the resistance R of the resistor, the more power is dissipated. The second suggests the opposite: the greater the resistance, the less the power. Explain this inconsistency.

12 State the minimum number of resistors, each of the same resistance and power rating of 0.5 W, which must be used to produce an equivalent 1.2 kΩ, 5 W resistor. Calculate the resistance of each, and state how they should be connected.

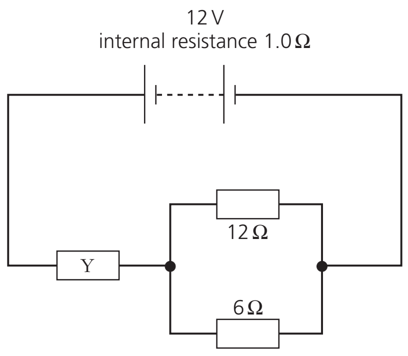
13 In the circuit shown in Fig. 10.35 the current in the battery is 1.5 A. The battery has internal resistance 1.0 Ω. Calculate:

a the combined resistance of the resistors that are connected in parallel in the circuit of Fig. 10.35,

b the total resistance of the circuit,

c the resistance of resistor Y,

d the current through the 6 Ω resistor.



▲ **Figure 10.35**

14 The current in the starter motor of a car is 160 A when starting the engine. The connecting cable has total length 1.3 m, and consists of 15 strands of wire, each of diameter 1.2 mm. The resistivity of the metal of the strands is 1.4×10^{-8} Ω m.

a Calculate:
  i the resistance of each strand,
  ii the total resistance of the cable,
  iii the power loss in the cable.

b When the starter motor is used to start the car, 700 C of charge pass through a given cross-section of the cable.
  i Assuming that the current is constant at 160 A, calculate for how long the charge flows.
  ii Calculate the number of electrons which pass a given cross-section of the cable in this time. The electron charge e is -1.6×10^{-19} C.

c The e.m.f. of the battery is 13.6 V and its internal resistance is 0.012 Ω. Calculate:
  i the potential difference across the battery terminals when the current in the battery is 160 A,
  ii the rate of production of heat energy in the battery.

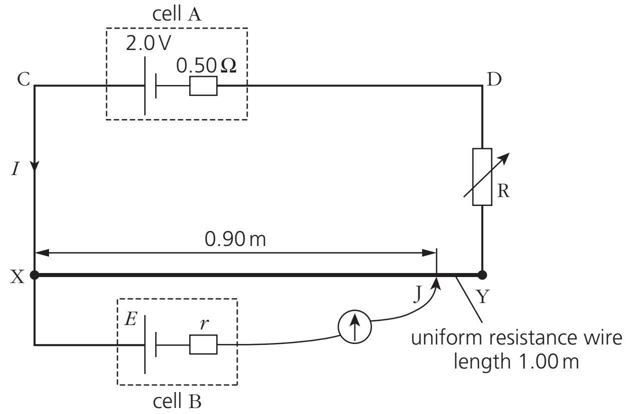
15 A copper wire of length 16 m has a resistance of 0.85 Ω. The wire is connected across the terminals of a battery of e.m.f. 1.5 V and internal resistance 0.40 Ω.

a Calculate the potential difference across the wire and the power dissipated in it.

b In an experiment, the length of this wire connected across the terminals of the battery is gradually reduced.
  i Sketch a graph to show how the power dissipated in the wire varies with the connected length.
  ii Calculate the length of the wire when the power dissipated in the wire is a maximum.
  iii Calculate the maximum power dissipated in the wire.

16 a i State Kirchhoff's second law. **[1]**
  ii Kirchhoff's second law is linked to the conservation of a certain quantity. State this quantity. **[1]**

b The circuit shown in Fig. 10.36 is used to compare potential differences.



▲ **Figure 10.36**

The uniform resistance wire XY has length 1.00 m and resistance 4.0 Ω. Cell A has e.m.f. 2.0 V and internal resistance 0.50 Ω. The current through cell A is I. Cell B has e.m.f. E and internal resistance r.

The current through cell B is made zero when the movable connection J is adjusted so that the length of XJ is 0.90 m. The variable resistor R has resistance 2.5 Ω.

  i Apply Kirchhoff's second law to the circuit CXYDC to determine the current I. **[2]**
  ii Calculate the potential difference across the length of wire XJ. **[2]**
  iii Use your answer in **ii** to state the value of E. **[1]**
  iv State why the value of the internal resistance of cell B is not required for the determination of E. **[1]**

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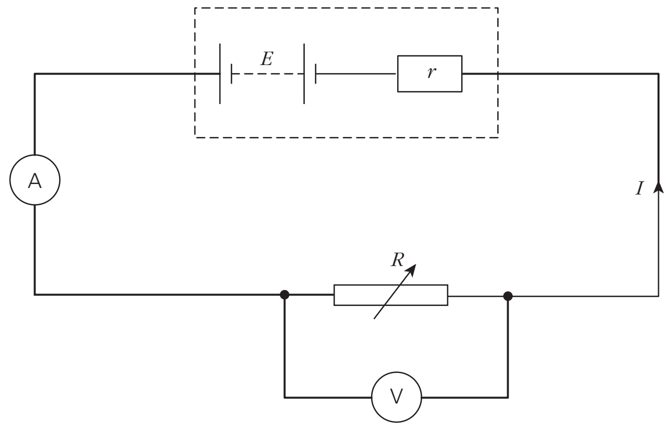
17 A circuit used to measure the power transfer from a battery is shown in Fig. 10.37. The power is transferred to a variable resistor of resistance R.



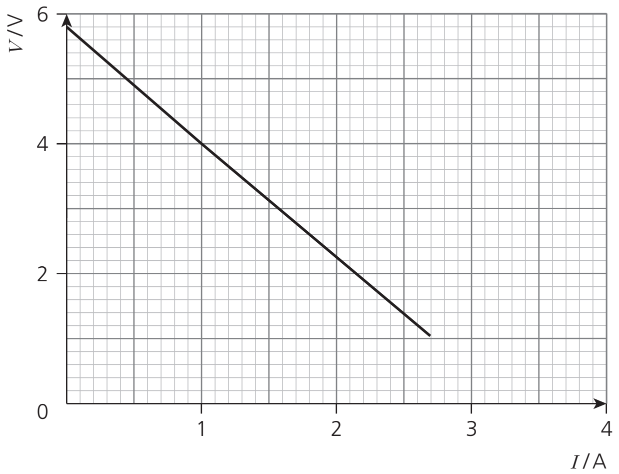
▲ **Figure 10.37**

The battery has an electromotive force (e.m.f.) E and an internal resistance r. There is a potential difference (p.d.) V across R. The current in the circuit is I.

a By reference to the circuit shown in Fig. 10.37, distinguish between the definitions of e.m.f. and p.d. **[3]**

b Using Kirchoff's second law, determine an expression for the current I in the circuit. **[1]**

c The variation with current I of the p.d. V across R is shown in Fig. 10.38.



▲ **Figure 10.38**

Use Fig. 10.38 to determine:

i the e.m.f. E, **[1]**

ii the internal resistance r. **[2]**

d **i** Using the data from Fig. 10.38, calculate the power transferred to R for a current of 1.6 A. **[2]**

ii Use your answers from **c i** and **d i** to calculate the efficiency of the battery for a current of 1.6 A. **[2]**

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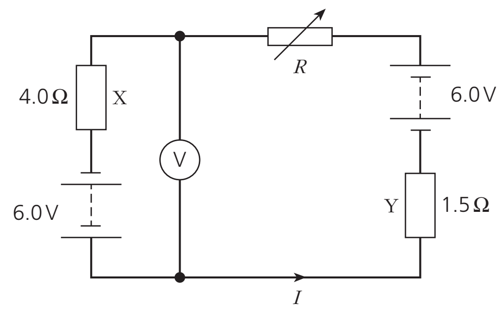
18 **a** State Kirchoff's second law. **[2]**

b Two batteries, each of electromotive force (e.m.f.) 6.0 V and negligible internal resistance, are connected in series with three resistors, as shown in Fig. 10.39. Resistor X has resistance 4.0 Ω and resistor Y has resistance 1.5 Ω.



▲ **Figure 10.39**

i The resistance R of the variable resistor is changed until the voltmeter in the circuit reads zero.
Calculate:
1 the current I in the circuit, [1]
2 the resistance R. [2]

ii Resistors X and Y are wires made from the same material. The diameter of the wire of X is twice the diameter of the wire of Y.
Determine the ratio:

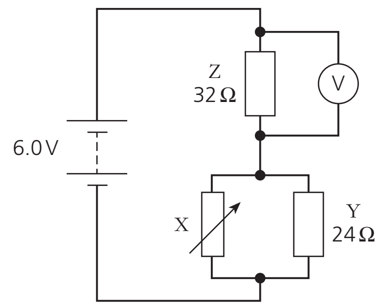
$$\frac{\text{average drift speed of free electrons in X}}{\text{average drift speed of free electrons in Y}}$$ [2]

iii The resistance R of the variable resistor is now increased. State and explain the effect of the increase in R on the power transformed by each of the batteries. [3]

Cambridge International AS and A Level Physics (9702) Paper 22 Q5 parts a, bi, bii, biii March 2018

19 a Using energy transformations, describe the *electromotive force* (*e.m.f.*) of a battery and the *potential difference* (*p.d.*) across a resistor. [2]

b A battery of e.m.f. 6.0 V and negligible internal resistance is connected to a network of resistors and a voltmeter, as shown in Fig 10.40. Resistor Y has a resistance of 24 Ω and resistor Z has a resistance of 32 Ω.



▲ **Figure 10.40**

i The resistance R_X of the variable resistor X is adjusted until the voltmeter reads 4.8 V. Calculate:
1 the current in resistor Z, [1]
2 the total power provided by the battery, [2]
3 the number of conduction electrons that move through the battery in a time interval of 25 s, [2]
4 the total resistance of X and Y connected in parallel, [2]
5 the resistance R_X. [2]

ii The resistance R_X is now decreased. State and explain the change, if any, to the reading on the voltmeter. [2]

Cambridge International AS and A Level Physics (9702) Paper 22 Q6 March 2019

11 Particle physics

Learning outcomes

By the end of this topic, you will be able to:

11.1 Atoms, nuclei and radiation

1. infer from the results of the α-particle scattering experiment the existence and small size of the nucleus
2. describe a simple model for the nuclear atom to include protons, neutrons and orbital electrons
3. distinguish between nucleon number and proton number
4. understand that isotopes are forms of the same element with different number of neutrons in their nuclei
5. understand and use the notation $^{A}_{Z}X$ for the representation of nuclides
6. understand that nucleon number and charge are conserved in nuclear processes
7. describe the composition, mass and charge of α-, β- and γ-radiations. Both β^- (electrons) and β^+ (positrons) are included
8. understand that an antiparticle has the same mass but opposite charge to the corresponding particle and that a positron is the antiparticle of an electron
9. state that (electron) antineutrinos are produced during β^- decay and (electron) neutrinos are produced during β^+ decay
10. understand that α-particles have discrete energies but that β-particles have a continuous range of energies because (anti) neutrinos are emitted in β-decay
11. represent α- and β-decay by a radioactive decay equation of the form $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha$
12. use the unified atomic mass unit (u) as a unit of mass

11.2 Fundamental particles

1. understand that a quark is a fundamental particle and that there are six flavours (types) of quark: up, down, strange, charm, top and bottom
2. recall and use the charge of each flavour of quark and understand that its respective antiquark has the opposite charge (no knowledge of any other properties of quarks is required)
3. recall that protons and neutrons are not fundamental particles and describe protons and neutrons in terms of their quark composition
4. understand that a hadron may be either a baryon (consisting of three quarks) or a meson (consisting of one quark and one antiquark)
5. describe the changes to quark composition that take place during β^- and β^+ decay
6. recall that electrons and neutrinos are fundamental particles called leptons

Starting points

- ★ The atom consists of a very small nucleus containing protons and neutrons, surrounded by orbiting electrons.
- ★ The decay of unstable nuclei leads to radioactive emissions.
- ★ Protons and neutrons are not fundamental particles.

11.1 Atoms, nuclei and radiation

The atoms of all elements are made up of three particles called **protons**, **neutrons** and **electrons**. The protons and neutrons are at the centre or **nucleus** of the atom. The electrons travel around (orbit) the nucleus.

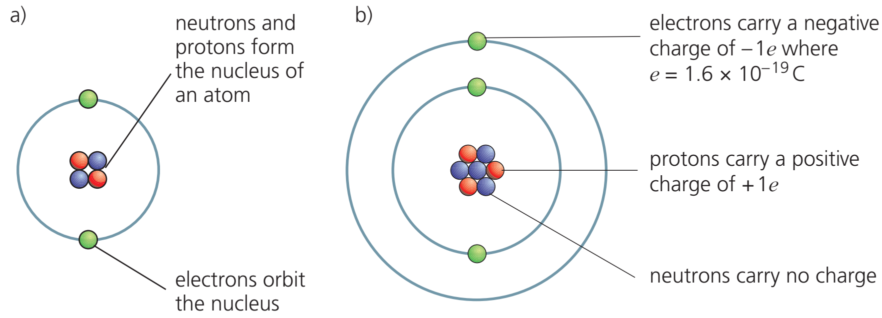
We shall see later in this Topic that the diameter of the nucleus is only about 1/10 000 of the diameter of an atom. We shall also discover how the evidence for the existence and small size of the nucleus was obtained.

Figure 11.1 illustrates very simple models (not to scale) of a helium atom and a lithium atom.

The mass of atoms and their constituent particles is more conveniently expressed in **atomic mass units (u)** than the SI unit of mass the kilogram. ($1u = 1.66 \times 10^{-27}$ kg).

Protons and neutrons both have a mass of about one atomic mass unit u. By comparison, the mass of an electron is very small, about 1/2000 of 1u. The vast majority of the mass of the atom is, therefore, in the nucleus.

The basic properties of the proton, neutron and electron are summarised in Table 11.1.



▲ **Figure 11.1 Structures of a) a helium atom and b) a lithium atom**

	mass/u	charge	position
proton	1.0073	$+e$	in nucleus
neutron	1.0087	0	in nucleus
electron	0.00055	$-e$	orbiting nucleus

$e = 1.60 \times 10^{-19}$ C

▲ **Table 11.1**

Atoms and ions

Atoms are uncharged (neutral) because they contain equal numbers of protons and electrons and the charge on an electron is equal and opposite to the charge on a proton. If an atom loses one or more electrons, so that it does not contain an equal number of protons and electrons, it becomes charged and is called an **ion**.

For example, if a sodium atom loses one of its electrons, it becomes a positive sodium ion.

$$\underset{\text{sodium atom}}{\text{Na}} \rightarrow \underset{\text{sodium ion}}{\text{Na}^+} + \underset{\text{electron}}{\text{e}^-}$$

If an atom gains an electron, it becomes a negative ion.

Proton number and nucleon number

The number of protons in the nucleus of an atom is called the **proton number** (or **atomic number**) Z.

The number of protons together with the number of neutrons in the nucleus is called the **nucleon number** (or **mass number**) A.

A **nucleon** is the name given to either a proton or a neutron in the nucleus.

The difference between the nucleon number (*A*) and the proton number (*Z*) gives the number of neutrons in the nucleus.

Representation of nuclides

If the chemical symbol of an element is X, a particular atom of this element, a nuclide, is represented by the notation

$$^{\text{nucleon number}}_{\text{proton number}}\text{X} = {}^{A}_{Z}\text{X}$$

The element changes for every *Z* number (proton number) and the symbol X changes. A nuclide is the name given to a class of atoms whose nuclei contain a specified number of protons and a specified number of neutrons. The nucleus of one form of sodium contains 11 protons and 12 neutrons. Therefore, its proton number *Z* is 11 and the nucleon number *A* is 11 + 12 = 23. This nuclide can be shown as ${}^{23}_{11}\text{Na}$. All atoms with nuclei that contain 11 protons and 12 neutrons belong to this class and are the same nuclide.

WORKED EXAMPLE 11A

An oxygen nucleus is represented by ${}^{16}_{8}\text{O}$. Describe its atomic structure.

Answer

The nucleus has a proton number of 8 and a nucleon number of 16. Thus, its nucleus contains **8 protons** and 16 − 8 = **8 neutrons**. There are also **8 electrons** (equal to the number of protons) orbiting the nucleus.

Question

1 Write down the proton number and the nucleon number for the potassium nucleus ${}^{40}_{19}\text{K}$. Deduce the number of neutrons in the nucleus.

Isotopes

Sometimes atoms of the same element have different numbers of neutrons in their nuclei. If two different nuclides have nuclei with the same atomic number *Z* but a different mass number *A* they are called **isotopes** of the same element.

The most abundant form of chlorine contains 17 protons and 18 neutrons in its nucleus, giving it a nucleon number of *A* = 17 + 18 = 35. This form of chlorine is often called chlorine-35. Another form of chlorine contains 17 protons and 20 neutrons in the nucleus, giving it a nucleon number of 37. This is chlorine-37. Chlorine-35 and chlorine-37 are said to be isotopes of chlorine.

Isotopes are different forms of the same element which have the same number of protons but different numbers of neutrons in their nuclei.

Some elements have many isotopes, but others have very few. For hydrogen, the most common isotope is hydrogen-1. Its nucleus is a single proton. Hydrogen-2 is called deuterium; its nucleus contains one proton and one neutron. Hydrogen-3, with one proton and two neutrons, is called tritium.

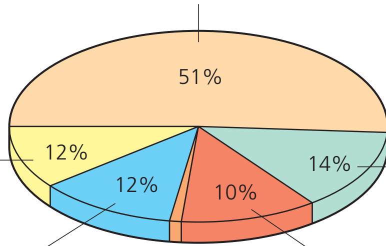
Note that the term isotope is used to describe nuclei with the same proton number (that is, nuclei of the same element) but with different nucleon numbers.

Note that the term **nuclide** is used to describe a particular class of nuclei:

A nuclide is a class of nuclei that have a particular nucleon number and a particular proton number.

Background radiation

Radioactivity is a natural phenomenon. Rocks such as granite contain small amounts of radioactive nuclides, some foods we eat emit radiation, and even our bodies are naturally radioactive. Although the atmosphere provides life on Earth with some shielding, there is, nevertheless, some radiation from outer space (cosmic radiation). In addition to this natural radioactivity, we are exposed to radiation from human-made sources. These are found in medicine, in fallout from nuclear explosions, and in leaks from nuclear power stations. The sum of all this radiation is known as **background radiation**. Figure 11.2 indicates the relative proportions of background radiation coming from various sources.



▲ **Figure 11.2** Sources of background radiation

α-particles, β-particles and γ-radiation

Some elements have nuclei which are unstable. That is, the combination of protons and neutrons in the nucleus is such that the forces acting on the nucleons do not balance. In order to become more stable, they emit particles and/or electromagnetic radiation. The nuclei are said to be **radioactive**, and the emission is called **radioactivity**. The emissions are invisible to the eye, but their tracks were first made visible in a device called a cloud chamber. The photograph in Figure 11.3 shows tracks created by one type of emission, α-particles.

▲ **Figure 11.3** Tracks of α-particles from a radioactive source

Investigations of the nature and properties of the emitted particles or radiation show that there are three different types of emission. The three types are α-particles (alpha-particles), β-particles (beta-particles) and γ-radiation (gamma radiation). All three emissions originate from the nucleus.

α-particles

An α-particle consists of two protons and two neutrons and hence has a charge of $+2e$. Therefore, an α-particle is like a helium nucleus. α-particles are emitted from a heavy nucleus (for example, nuclides with an atomic number between bismuth 83 and uranium 92) with high speeds of up to about $10^7\,\mathrm{m\,s^{-1}}$ (about 5% of the speed of light). α-particle emission is the least penetrating of the three types of emission. It can pass through very thin paper, but is unable to penetrate thin card. Its range in air is a few centimetres.

An α-particle is identical to the nucleus of a helium atom.

In terms of symbols:

An α-particle is written as $^4_2\mathrm{He}$.

As α-particles travel through matter, they interact with nearby atoms, causing them to lose one or more electrons. The ionised atom and the dislodged electron are called an ion pair. The production of an ion pair requires the separation of unlike charges, and this process requires energy. α-particles have a relatively large mass and charge, and consequently they are efficient ionisers. They may produce as many as 10^5 ion pairs for every centimetre of air through which they travel. Thus, they lose energy relatively quickly, and have low penetrating power.

When the nucleus of an atom emits an α-particle, it is said to undergo α-decay. The nucleus loses two protons and two neutrons in this emission.

In α-decay, the proton number of the nucleus decreases by two, and the nucleon number decreases by four.

Each element has a particular proton number and, therefore, α-decay causes one element to change into another. (This process is sometimes called **transmutation**.) The original nuclide is called the **parent nuclide**, and the new one the **daughter nuclide**.

For example, uranium-234 (the parent nuclide) may emit an α-particle. The daughter nuclide is thorium-230. In addition, energy is released, which shows up as the kinetic energy of the alpha particle and the daughter nuclide. This emission is represented by the nuclear equation

$$^{234}_{92}\text{U} \rightarrow {}^{230}_{90}\text{Th} + {}^{4}_{2}\text{He} + \text{energy}$$

Note that in all radioactive decay processes (and, in fact, in all processes of nuclear reactions) nucleon number and proton number (and hence charge) are *conserved* (stay the same). Hence, for all equations representing nuclear reactions, the sum of the numbers at the top of the symbols on the left-hand side of the equation (the sum of the nucleon numbers) is equal to the sum of the nucleon numbers on the right-hand side. Similarly, the sum of the numbers at the bottom of the symbols on the left-hand side (the sum of the proton numbers) is equal to the sum of the proton numbers on the right-hand side.

In the equation above:

- The number of protons in the parent nuclide ($Z = 92$) is equal to the sum of the number of protons in the daughter nuclide ($Z = 90$) plus the number of protons in the alpha particle ($Z = 2$), so charge is conserved.
- The number of nucleons in the parent nuclide ($A = 234$) is equal to the sum of the number of nucleons in the daughter nuclide ($A = 230$) plus the number of nucleons in the alpha particle ($A = 4$).

Energy and mass, taken together, are also conserved in all nuclear processes, as is momentum.

β-particles

A radioactive nucleus that decays by β decay may emit a negative (β^-) or positive (β^+) electron. The positive electron (β^+) is also known as a **positron** or an antielectron. The positron is the **antiparticle** of an electron and, therefore, has the same mass but opposite charge to an electron.

β-particles are fast moving electrons, β^-, or positrons, β^+.

β-particles have a range of speeds that may reach in excess of 99% of the speed of light. These particles have half the charge and very much less mass than α-particles. Consequently, they are much less efficient than α-particles in producing ion pairs as they pass through matter. They are, thus, far more penetrating than α-particles, being able to travel up to about a metre in air. They can penetrate card and sheets of aluminium up to a few millimetres thick.

A β^- particle may be emitted from a lead-214 nucleus (the parent nuclide). The daughter nuclide is bismuth-214 and, in addition, energy is released. The β^- emission is represented by the nuclear equation

$$^{214}_{82}Pb \rightarrow {}^{214}_{83}Bi + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \text{energy}$$

A β^+ particle may be emitted from a phosphorus-30 nucleus (the parent nuclide). The daughter nuclide is silicon-30 and energy is also released. The β^+ emission is represented by the nuclear equation:

$$^{30}_{15}P \rightarrow {}^{30}_{14}Si + {}^{0}_{1}e + {}^{0}_{0}\nu + \text{energy}$$

In the nuclear equations above, the symbols ${}^{0}_{-1}e$ and ${}^{0}_{1}e$ represent an electron and positron respectively. β^+ emission is also known as positron emission, We will look at this in more detail in the A Level Topic 24.3 in the context of positron emission tomography (PET scanning).

The symbols ${}^{0}_{0}\nu$ and ${}^{0}_{0}\bar{\nu}$ represent a neutrino and an antineutrino respectively. The antineutrino is the antimatter equivalent of the neutrino. Neutrinos have no electrical charge and little or no mass and are emitted from the nucleus at the same time as the β-particle (the electron or the positron).

Since the neutrino and antineutrino have no charge we can show that in both types of β decay, charge is conserved and the total number of nucleons is conserved.

The changes that take place in the nucleus

It was stated earlier that the nucleus contains protons and neutrons. What, then, is the origin of β^- particle emission? Each β-particle certainly comes from a nucleus, not from the electrons outside the nucleus. The process for this type of decay is that, just prior to β^- emission, a neutron in the nucleus transforms into a proton, a negative electron and an antineutrino. We shall learn more about how this change happens in Topic 11.2. The ratio of protons to neutrons in the nucleus is changed and this makes the daughter nucleus more stable.

In fact, *free* neutrons (not inside the nucleus) are also known to decay the same way:

$$^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \text{energy}$$

A similar process happens in the nucleus. In β^- decay, a negative electron and antineutrino $\bar{\nu}$ are emitted from the nucleus. This leaves the nucleus with the same number of nucleons as before, but with one extra proton and one fewer neutron.

In β^+ emission, a proton in the nucleus forms a neutron, a positive electron and a neutrino. This process again changes the ratio of protons to neutrons in the nucleus and makes the daughter nucleus more stable.

In β^+ decay the proton is considered to transform itself as follows:

$$^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}e + {}^{0}_{0}\nu + \text{energy}$$

In β^+ decay, the positive electron and a neutrino are emitted from the nucleus. This leaves the nucleus with the same number of nucleons as before, but with one extra neutron and one fewer proton.

In β^- decay (negative electron emitted), a daughter nuclide is formed with the proton number increased by one, but with the same nucleon number.

In β^+ decay (positive electron emitted), a daughter nuclide is formed with the proton number decreased by one, but with the same nucleon number.

The antimatter particle, the positron (positive electron), very quickly meets its equivalent matter particle, the negative electron. The two particles annihilate each other to produce γ-radiation. This makes the positive electron difficult to detect.

WORKED EXAMPLE 11B

A strontium-90 atom (the parent nuclide) may decay with the emission of a β-particle to form the daughter nuclide yttrium-90. The decay is represented by the nuclear equation

$$^{90}_{38}\mathrm{Sr} \quad ^{90}_{39}\mathrm{Y} + \beta + x + \text{energy}$$

State and explain whether the β-particle is a negative or positive electron. State the type of particle represented by *x*.

Answer

The proton number has increased by one, hence a **negative electron** is emitted. The *x* is an **antineutrino** as this particle is emitted with a negative electron.

Questions

2 Write down a nuclear equation to represent the α decay of a thorium nucleus ($^{232}_{90}\mathrm{Th}$) to a radium nucleus (Ra).

3 Write down a nuclear equation to represent the β^- decay of a radium nucleus ($^{228}_{88}\mathrm{Ra}$) to a actinium nucleus (Ac).

Kinetic energy of emitted alpha and beta particles

The same amount of energy is released in the decay of each nucleus and depends on the parent nuclide and the type of decay. For example, the same amount of energy is released in the decay by α-emission of each nucleus of $^{234}_{92}\mathrm{U}$.

The α-particles emitted from a particular radioactive nuclide all have the same kinetic energy. The β-particles from a particular nuclide, by contrast, have a continuous range of energies, from zero to the maximum energy available.

Neutrinos or antineutrinos are emitted from the nucleus at the same time as the β-particle (the electron or the positron) in β-decay. Thus the energy released in β-decay is shared between the kinetic energy of the β-particle and the recoiling daughter nucleus and the energy of the neutrino or antineutrino. The same amount of energy is released in the decay of each particular parent nucleus. However, the β-particles (electrons or positrons) emitted from a particular radioactive nuclide have varying amounts of kinetic energy. The amount depends on the way the total energy available is shared between the β-particle and the neutrino or antineutrino. The sum of the positron's energy and the neutrino's energy is constant for the decay of a particular nuclide. The sum of the electron's energy and the antineutrino's energy is constant for the decay of a particular nuclide.

The α-particles emitted from a particular radioactive nuclide all have the *same kinetic energy*.

The β-particles emitted from a particular radioactive nuclide have a *continuous range of kinetic energies* because neutrinos or antineutrinos are emitted in β-decay.

WORKED EXAMPLE 11C

A stationary polonium nucleus ($^{212}_{84}\mathrm{Po}$) of mass 212 u spontaneously emits an α-particle. The α-particle is emitted with an energy of 1.4×10^{-12} J and the reaction gives rise to a nucleus of lead (Pb).

a Write down a nuclear equation to represent the α decay of the polonium nucleus.

b Calculate the speed of the α-particle.

Answers

a $(^{212}_{84}\mathrm{Po}) \rightarrow (^{208}_{82}\mathrm{Pb}) + {}^{4}_{2}\mathrm{He} + \text{energy}$

b $1/2mv^2$ = energy of α-particle

$v = [(2 \times 1.4 \times 10^{-12})/(4 \times 1.66 \times 10^{-27})]^{1/2}$

$= \mathbf{2.1 \times 10^{7}\ m\ s^{-1}}$

4 The kinetic energy of a β-particle is 3.2×10^{-16} J. Calculate the speed of this β-particle.

5 Use the conservation of momentum to calculate the speed of the lead nucleus in Worked Example 11C.

γ-radiation

γ-radiation is part of the electromagnetic spectrum with wavelengths between 10^{-11} m and 10^{-13} m.

Since γ-radiation has no charge, its ionising power is much less than that of either α- or β-particles. γ-radiation penetrates almost unlimited thicknesses of air, several metres of concrete or several centimetres of lead.

α- and β-particles are emitted by unstable nuclei which have excess energy. The emission of these particles results in changes in the ratio of protons to neutrons, but the daughter nuclei may still have excess energy. The nucleus may return to its unexcited (or ground) state by emitting energy in the form of γ-radiation.

In γ-emission, no particles are emitted and there is, therefore, no change to the proton number or nucleon number of the parent nuclide.

For example, when uranium-238 decays by emitting an α-particle, the resulting nucleus of thorium-234 contains excess energy (it is in an excited state) and emits a photon of γ-radiation to return to the ground state. This process is represented by the nuclear equation

$$^{234}_{90}\text{Th}^{*} \rightarrow {}^{234}_{90}\text{Th} + \gamma$$

The* next to the symbol Th on the left-hand side of the equation shows that the thorium nucleus is in an excited state.

Summary of the composition of radioactive emissions

Table 11.2 summarises the composition, mass and charge of α-particles, β-particles and γ-radiation.

property	α-particle	β-minus particle	β-plus particle	γ-radiation
mass	4u	about u/2000	about u/2000	0
charge	+2*e*	−*e*	+*e*	0
nature	helium nucleus (2 protons + 2 neutrons)	electron	positron	short-wavelength electromagnetic waves
speed	up to 0.05*c*	up to 0.99*c*	up to 0.99*c*	*c*
affects photographic film?	yes	yes	yes	yes

▲ **Table 11.2**

Radioactive decay series

The daughter nuclide of a radioactive decay may, itself, be unstable and so may emit radiation to give another different nuclide. This sequence of radioactive decay from parent nuclide through succeeding daughter nuclides is called a **radioactive decay series**. The series ends when a stable nuclide is reached.

Part of such a radioactive decay series, the uranium series, is shown in Table 11.3.

decay	radiation emitted
${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + \gamma$	α, γ
${}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e + \gamma$	β^-, γ
${}^{234}_{91}Pa \rightarrow {}^{234}_{92}U + {}^{0}_{-1}e + \gamma$	β^-, γ
${}^{234}_{92}U \rightarrow {}^{230}_{90}Th + {}^{4}_{2}He + \gamma$	α, γ
${}^{230}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He + \gamma$	α, γ
${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$	α
${}^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}He$	α

▲ **Table 11.3** Part of the decay series of uranium-238

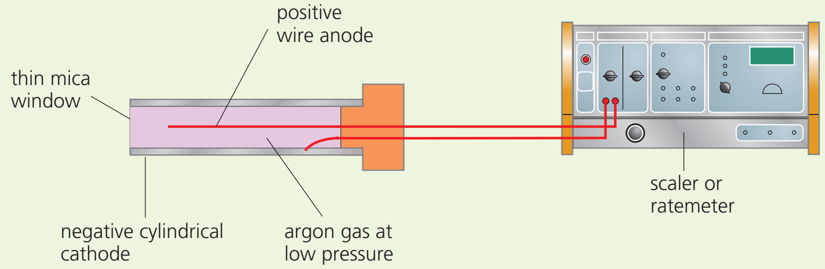
EXTENSION

Detecting radioactivity

Some of the methods used to detect radioactive emissions are based on the ionising properties of the particles or radiation.

The Geiger counter

Figure 11.4 illustrates a Geiger-Müller tube with a scaler connected to it. When radiation enters the window, it creates ion pairs in the gas in the tube. These charged particles, and particularly the electrons, are accelerated by the potential difference between the central wire anode and the cylindrical cathode. These accelerated particles then cause further ionisation. The result of this continuous process is described as an **avalanche effect**. That is, the entry of one particle into the tube and the production of one ion pair results in very large numbers of electrons and ions arriving at the anode and cathode respectively. This gives a pulse of charge which is amplified and counted by the scaler or ratemeter. (A scaler measures the total count of pulses in the tube during the time that the scaler is operating. A ratemeter continuously monitors the number of counts per second.) Once the pulse has been registered, the charges are removed from the gas in readiness for further radiation entering the tube.



▲ **Figure 11.4** Geiger-Müller tube and scaler

Photographic plates

When a radioactive emission strikes a photographic film, the film reacts as if it had been exposed to a small amount of visible light. When the film is developed, fogging or blackening is seen. This fogging can be used to detect, not only the presence of radioactivity, but also the dose of the radiation.

Figure 11.5 shows a film badge dosimeter. It contains a piece of photographic film which becomes fogged when exposed to radiation. Workers who are at risk from radiation wear such badges to gauge the type and dose of radiation to which they have been exposed. The radiation passes through different filters before reaching the film. Consequently, the type of radiation, as well as the quantity, can be assessed.

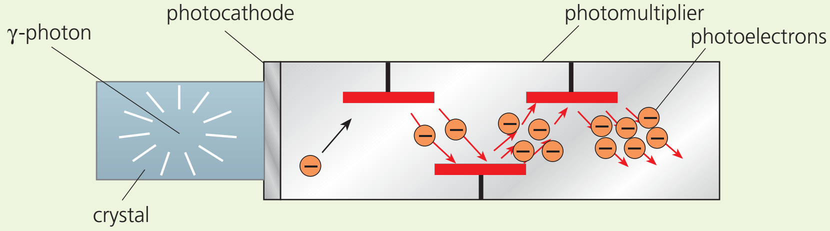
▲ **Figure 11.5** Film badge dosimeter

The scintillation counter

Early workers with radioactive materials used glass screens coated with zinc sulfide to detect radiation. When radiation is incident on the zinc sulfide, it emits a tiny pulse of light called a **scintillation**. The rate at which these pulses are emitted indicates the intensity of the radiation.

The early researchers worked in darkened rooms, observing the zinc sulfide screen by eye through a microscope and counting the number of flashes of light occurring in a certain time. Now a scintillation counter is used (see Figure 11.6).

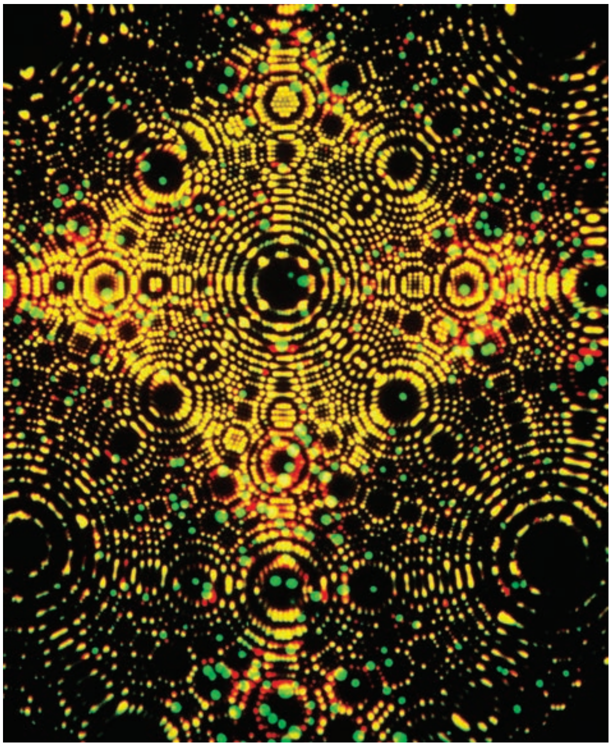
Often a scintillator crystal is used instead of a zinc sulfide screen. The crystal is mounted close to a device known as a photomultiplier, a vacuum-tube device which uses the principle of photoelectric emission (see Topic 22). Flashes of light cause the emission of photoelectrons from the negative electrode of the photomultiplier. The photoelectric current is amplified inside the tube. The output electrode is connected to a scaler or ratemeter, as with the Geiger-Müller tube.



▲ **Figure 11.6** Scintillation counter

The α-particle scattering experiment

Figure 11.7 shows a photograph taken with an ion microscope, a device which makes use of the de Broglie wavelength of gas ions (see Topic 22). It shows a sample of iridium at a magnification of about five million. The positions of individual iridium atoms can be seen.

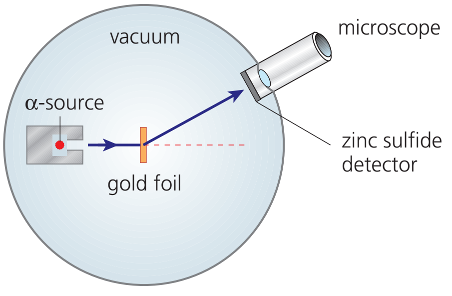


▲ **Figure 11.7** Ion microscope photograph of iridium

Photographs like this reinforce the idea that all matter is made of very small particles that we call atoms. Experiments performed at the end of the nineteenth and the beginning of the twentieth century led most physicists to believe that atoms themselves are made from even smaller particles, some of which have positive or negative charges. Unfortunately, even the most powerful microscopes cannot show us the internal structure of the atom. Many theories were put forward about the structure of the atom, but it was a series of experiments carried out by Ernest Rutherford and his colleagues around 1910 that led to the birth of the model we now know as the **nuclear atom**.

The α-particle scattering experiment

In 1911, Rutherford and two of his associates, Hans Geiger and Ernest Marsden, fired a beam of α-particles at a very thin piece of gold foil. A zinc sulfide detector was moved around the foil to detect the directions in which the α-particles travelled after striking the foil (Figure 11.8).



▲ **Figure 11.8** α-scattering experiment

They discovered that:

- the vast majority of the α-particles passed through the foil with very little or no deviation from their original path
- a small number of particles were deviated through an angle of more than about 10°
- an extremely small number of particles (one in ten thousand) were deflected through an angle greater than 90°.

From these observations, the following conclusions could be drawn.

- The majority of the mass of an atom is concentrated in a very small volume at the centre of the atom. Most α-particles would, therefore, pass through the foil undeviated.
- The centre (or nucleus) of an atom is charged. α-particles, which are also charged, passing close to the nucleus will experience a force causing them to deviate.
- Only α-particles that pass very close to the nucleus, almost striking it head-on, will experience large enough forces to cause them to deviate through angles greater than 90°. The fact that so few particles did so confirms that the nucleus is very small, and that most of the atom is empty space.

Figure 11.9 shows some of the possible trajectories of the α-particles. Using the nuclear model of the atom and equations to describe the force between charged particles, Rutherford calculated the fraction of α-particles that he would expect to be deviated through various angles. The calculations agreed with the results from the experiment. This confirmed the nuclear model of the atom. Rutherford calculated that the diameter of the nucleus is about 10^{-15} m, and the diameter of the whole atom about 10^{-10} m. Figure 11.10 shows the features of the nuclear model of a nitrogen atom.

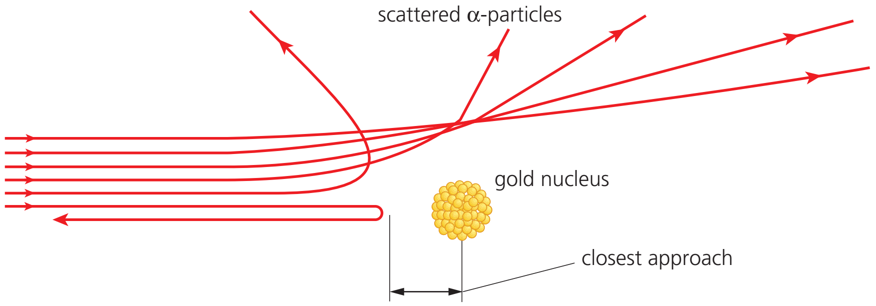


Figure 11.9 Possible trajectories of alpha particles

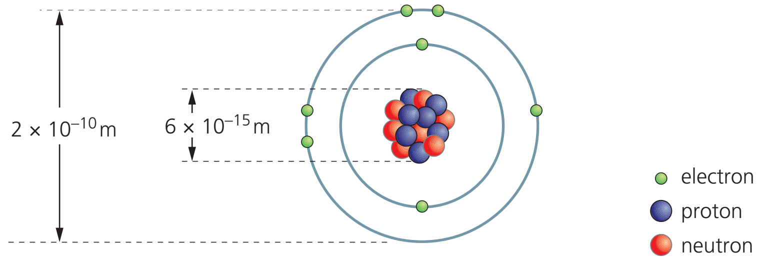


Figure 11.10 The diameter of a nitrogen atom is more than 30 000 times bigger than the diameter of its nucleus.

Some years later, the α-particle scattering experiment was repeated using α-particles with higher energies. Some discrepancies between the experimental results and Rutherford's scattering formula were observed. These seemed to be occurring because the high-energy α-particles were passing very close to the nucleus, and were experiencing, not only the repulsive electrostatic force, but also a strong attractive force. The force does not seem to have any effect outside the nucleus and is, therefore, considered to be very short range (a little more than the diameter of nuclei, 10^{-14} m).

This became known as the **strong nuclear force**. This is the force that holds the nucleons in the nucleus together. The strong force acts on protons and neutrons but not on electrons.

11.2 Fundamental particles

In the nineteenth century, the atom was considered to be the **fundamental particle** from which all matter was composed. A fundamental particle is not formed from other particles. The idea of atoms was used to explain the basic structure of all elements. Experiments performed at the end of the nineteenth century and beginning of the twentieth century, such as the alpha particle scattering experiment, provided evidence for the structure of an atom. The conclusions were that all atoms have a nucleus containing protons which is surrounded by electrons and that the nucleus is very small compared with the size of the atom. A neutral particle was then proposed to explain the discrepancy between the mass of the atom and the mass from the number of protons (number of positive charges). In 1932 James Chadwick discovered the neutron and the fundamental particles were then considered to be the proton, the neutron and the electron. The structure of the atom was then considered to be similar to that shown earlier in Figure 11.1.

The particles in an atom must experience forces in order to maintain its structure. The forces were the gravitational force that acts between all masses (see Topic 13) and the electrostatic force that acts between charged objects (see Topic 18). The electrostatic force of repulsion is approximately 10^{36} times greater than the gravitational force of attraction between protons. Another attractive force must keep the protons together in the nucleus. This force is known as the **strong force** and acts between nucleons. The force does not seem to have any effect outside the nucleus and is, therefore, considered to be very short range (a little more than the diameter of nuclei, 10^{-14} m). There appears to be a limiting spacing between nucleons which is similar in different nuclei and this suggests that the force is repulsive as soon as the nucleons come close together. The strong force does not act on electrons.

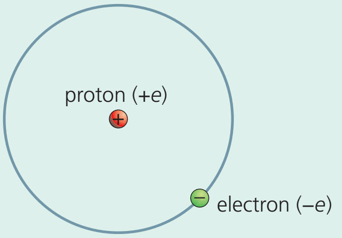
The strong force acts on protons and neutrons but *not* on electrons.

WORKED EXAMPLE 11D

Figure 11.11 illustrates a hydrogen atom with an electron orbiting the nucleus.

a State, for the forces acting on the electron and the proton:
  - **i** their nature
  - **ii** their direction.

b Explain why a strong force does not act on the electron or proton.



▲ **Figure 11.11** Hydrogen atom

Answers

a **i** Gravitational force (due to the mass of the electron and proton), electrostatic force (due to the charge on the electron and proton)

ii Both forces are attractive and, therefore, directed from the one particle towards the other particle.

b The electron is not a nucleon and, hence, is not affected by the strong force. There is only one nucleon and the strong force acts between nucleons.

Question

6 State the forces acting on the nucleons of a helium nucleus.

The discovery of antimatter in cosmic radiation supported the theory developed from the special theory of relativity and quantum theory that all fundamental particles have a corresponding antimatter particle. Each matter and its antimatter particle have the same mass but opposite charge. The following particles were required to support the theory: the antiproton, the antineutron and the antielectron. The antiparticle may be represented by the corresponding symbol for the particle with the opposite sign for the charge or with a bar above it. Therefore, the symbols used for the antiproton are p^- or $\bar{p}$ and for the antineutron and $\bar{n}^0$ or $\bar{n}$.

The antielectron or positive electron (positron) was introduced in β-particle decay in Topic 11.1. An electron is represented by e^- or e and an antielectron (a positron) is represented by e^+ or $\bar{e}$.

Many other particles and their antiparticles were discovered in cosmic radiation throughout the twentieth country. These results gave support for the idea that the proton and neutron were not fundamental particles.

Hadrons and leptons

The numerous types of subatomic particles are placed into two main categories depending on their properties. Those affected by the strong force are called **hadrons**, for example protons and neutrons, and those not affected by the strong force are called **leptons**, for example electrons and positrons.

The many different particles discovered in cosmic radiation have been reproduced in high-energy collisions of atomic nuclei using particle accelerators such as those at Stanford in California and CERN in Switzerland. A vast number of collisions have been carried out and a large number of hadrons have been produced. Two of the conclusions to these reactions were:

- the total electrical charge remains constant
- the total number of nucleons generally remains constant.

Many hadrons were detected with masses and properties different from those of protons and neutrons. These results gave support for the idea that the proton and neutron were not fundamental particles, but were made up of smaller (fundamental) particles.

The quark model of hadrons

The problem of what were considered to be fundamental particles was resolved by the **quark model** for hadrons. In the quark model, the hadrons are made up of fundamental particles called **quarks**. Three types of quark, called flavours of quark, were initially introduced: **up (u), down (d)** and **strange (s)**.

The quark model was developed as more particles were discovered. The total number of types of quark or 'flavours' is considered to be six. The additional flavours are **charm (c), bottom (b)** and **top (t)**.

The quark flavours have charge and **strangeness** as shown in Table 11.4.

flavour of quark	charge	strangeness
up (u)	$+\frac{2}{3}e$	0
down (d)	$-\frac{1}{3}e$	0
strange (s)	$-\frac{1}{3}e$	–1
charm (c)	$+\frac{2}{3}e$	0
bottom (b)	$-\frac{1}{3}e$	0
top (t)	$+\frac{2}{3}e$	0

▲ **Table 11.4** Charge values for the six quarks

There are also six **antiquarks**, $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, $\bar{b}$ and $\bar{t}$ and these have the opposite values of charge.

The different possible combinations of quarks and antiquarks explain the many different hadron particles. The fractional charge carried by quarks means that only certain combinations of quarks occur, to produce hadrons with whole or zero charge.

There are two types of hadron: **baryons** and **mesons**.

Protons and neutrons are baryons. Their corresponding antiparticles ($\bar{p}$ and $\bar{n}$) have opposite electrical charge.

Pions (π) and Kappas (K) are examples of mesons.

A baryon is made up of three quarks or three antiquarks. Hence protons and neutrons consist of three quarks.

proton:		u	u	d	**neutron:**		u	d	d
charge	$+1e$	$+\frac{2}{3}e$	$+\frac{2}{3}e$	$-\frac{1}{3}e$	charge	0	$+\frac{2}{3}e$	$-\frac{1}{3}e$	$-\frac{1}{3}e$

A meson is made up of a quark and an antiquark. For example, a K^+ meson is formed with an up quark and an antistrange quark. A π^+ meson is formed by an up quark and an antidown quark.

EXTENSION

In strong interactions, the quark flavour is conserved.

WORKED EXAMPLE 11E

State the values of charge for the antiquarks $\bar{u}$ and $\bar{d}$.

Answer

$\bar{u}$ charge $-\frac{2}{3}e$

and $\bar{d}$ charge $+\frac{1}{3}e$

Questions

7 Show that the charge on a) a π^+ meson is $+e$ and b) a K^- is $-e$.

8 Show whether the following reaction can occur.
$p + p \rightarrow p + \bar{p} + n$

Leptons

Leptons are particles that are not affected by the strong force. The electron and neutrino and their antimatter partners, the positron and antineutrino, are examples of leptons. These types of particle do not appear to be composed of any smaller particles and, therefore, leptons are considered to be fundamental particles.

Understanding beta decay in terms of quarks and leptons

The emission of electrons or positrons from nuclei was discussed earlier in this topic. During the decay of a neutron in the nucleus, a proton is formed and an electron and antineutrino emitted. This is β^- decay. In terms of the fundamental particles, quarks, the reaction can be shown as follows:

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$$

u u

d u

d d

Note that there are no quarks shown for the electron and antineutrino because these are leptons and leptons are not made of quarks.

EXTENSION

The quark flavour is not conserved as a down quark has changed to an up quark. The reaction cannot be due to the strong force. The β-decay must be due to another force. This force is called the **weak force** or **weak interaction**.

The total lepton number before a reaction is equal to the total lepton number after the reaction.

The lepton number is +1 for the particle and –1 for the antiparticle.

The total lepton number before the reaction is zero in the β^--decay above. The lepton numbers for the particles after the reaction are +1 for the electron and –1 for the antineutrino, giving a total of zero.

WORKED EXAMPLE 11F

Give the equation for the reaction where a proton in the nucleus decays into a neutron and emits a β-particle. Describe the reaction in terms of the quark model.

Answer

This is β+ decay, with the emission of a positron.

$${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + {}^{0}_{0}\nu$$

u → u

u → d

d → d

During the decay of a proton in the nucleus, a neutron is formed and a positron and neutrino emitted. In terms of quarks, an up quark changes into a down quark.

Question

9 What is the difference between a hadron and a lepton?

SUMMARY

- An atom consists of a nucleus containing protons and neutrons surrounded by orbiting electrons.
- Most of the mass of an atom is contained in its nucleus.
- An atom is neutral as it contains an equal number of protons and electrons.
- The unified atomic mass unit (u) is used as a unit of mass for subatomic particles.
- Atoms which have gained or lost electrons are charged, and are called ions.
- The nucleon number A of a nucleus is the number of nucleons (protons and neutrons) in the nucleus.
- The proton number Z of a nucleus is the number of protons in the nucleus; hence the number of neutrons in the nucleus is $A - Z$.
- A nucleus with chemical symbol X may be represented by: $^{\text{nucleon number}}_{\text{proton number}}\mathrm{X}$
- Isotopes are different forms of the same element, that is, nuclei with the same proton number but with different nucleon numbers.
- The nucleon number and the charge (number of protons, Z) are conserved in nuclear processes
- An α-particle is a helium nucleus (two protons and two neutrons).
- A β^--particle is a fast-moving electron, a β^+-particle is a fast moving positron.
- γ-radiation consists of short-wavelength electromagnetic waves.
- In nuclear notation the emissions are represented as: α-particle $^4_2\mathrm{He}$; β^--particle $^{0}_{-1}\mathrm{e}$ or β^+-particle $^{0}_{+1}\mathrm{e}$; γ-radiation $^0_0\gamma$.
- α-emission reduces the nucleon number of the parent nucleus by 4, and reduces the proton number by 2.
- β-emission causes no change to the nucleon number of the parent nucleus, and increases or decreases the proton number by 1.
- γ-emission causes no change to nucleon number or proton number of the parent nucleus.
- For every subatomic particle there is an antiparticle that has the same mass but opposite charge to the corresponding particle.
- The positron is the antiparticle of the electron and, therefore, has the same mass as an electron but a charge of $+e$.
- α-particles emitted from a particular nuclide have discrete energies.
- β-particles emitted from a particular nuclide have a continuous range of energies due to the additional emission of neutrinos or antineutrinos in β-decay.
- The Rutherford α-particle scattering experiment confirmed the nuclear model of the atom: the atom consists of a small, positively charged nucleus, surrounded by negatively charged electrons in orbit about the nucleus and that the vast majority of the mass of the atom is in the nucleus.
- The diameter of the nucleus is about 10^{-15} m; the diameter of the atom is about 10^{-10} m.
- Protons and neutrons are hadrons and are affected by the strong force.
- The quark model has six flavours of quark (up, down, strange, charm, bottom and top) together with their antiquarks.
- Hadrons are made up of quarks.
- Quarks have fractional charge such as $\pm\frac{2}{3}e$ or $\pm\frac{1}{3}e$.
- Hadrons combine quarks or antiquarks to give a whole or zero charge.
- Baryons contain three quarks (or three antiquarks) and mesons contain a quark and an antiquark.
- Protons are composed of quarks up, up and down, and neutrons of quarks up, down and down.
- Electrons and neutrinos are leptons which are fundamental particles.
- During β^- decay: $^1_0\mathrm{n} \rightarrow {}^1_1\mathrm{p} + {}^{0}_{-1}\mathrm{e}$ (electron) $+ {}^0_0\bar{\nu}$ (antineutrino).
- During β^+ decay: $^1_1\mathrm{p} \rightarrow {}^1_0\mathrm{n} + {}^0_1\mathrm{e}$ (positron) $+ {}^0_0\nu$ (neutrino).
- During β^- decay: a down quark changes to an up quark.
- During β^+ decay: an up quark changes to a down quark.

END OF TOPIC QUESTIONS

1 Which of the following is a fundamental particle?

A atom **B** electron **C** neutron **D** proton

2 What is the charge on a strange quark?

A $-\frac{1}{3}e$ **B** $+\frac{1}{3}e$ **C** $-\frac{2}{3}e$ **D** $+\frac{2}{3}e$

3 What is the speed of an α-particle with a kinetic energy of 72 pJ?

A $15\,\mathrm{km\,s^{-1}}$ **B** $21\,\mathrm{km\,s^{-1}}$ **C** $15\,\mathrm{Mm\,s^{-1}}$ **D** $21\,\mathrm{Mm\,s^{-1}}$

4 State **i** the number of protons and **ii** the number of neutrons in the nucleus of the following nuclei:

a $^{54}_{26}\mathrm{Fe}$ **b** $^{109}_{47}\mathrm{Ag}$ **c** $^{196}_{79}\mathrm{Au}$ **d** $^{232}_{94}\mathrm{Pu}$

5 Explain the changes that take place to the nucleus of an atom when it emits:

a an α-particle,

b a β-particle,

c γ-radiation.

6 Complete the following radioactive series.

$^{235}_{92}\mathrm{U} \rightarrow {}^{?}_{?}\mathrm{Y} + {}^{?}_{?}\mathrm{He}$

${}^{?}_{?}\mathrm{Y} \rightarrow {}^{?}_{?}\mathrm{Z} + {}^{0}_{-1}\mathrm{e}$

${}^{?}_{?}\mathrm{Z} \rightarrow {}^{?}_{?}\mathrm{Z} + {}^{?}_{?}?$

7 Calculate the speed of:

a an electron with kinetic energy of 2.4×10^{-16} J,

b an α-particle with kinetic energy of 2.4×10^{-16} J.

8 A stationary radium nucleus ($^{224}_{88}\mathrm{Ra}$) of mass 224 u spontaneously emits an α-particle. The α-particle is emitted with an energy of 9.2×10^{-13} J, and the reaction gives rise to a nucleus of radon (Rn).

a Write down a nuclear equation to represent α-decay of the radium nucleus.

b Show that the speed with which the α-particle is ejected from the radium nucleus is $1.7 \times 10^{7}\,\mathrm{m\,s^{-1}}$.

c Calculate the speed of the radon nucleus on emission of the α-particle. Explain how the principle of conservation of momentum is applied in your calculation.

9 **a** β-radiation is emitted during the spontaneous radioactive decay of an unstable nucleus.

i State the nature of a β-particle. [1]

ii State two properties of β-radiation. [2]

b The following equation represents the decay of a nucleus of hydrogen-3 by the emission of a β-particle. Copy and complete the equation.

$^{3}_{1}\mathrm{H} \rightarrow {}^{\cdots}_{\cdots}\mathrm{He} + {}^{\cdots}_{\cdots}\beta$ [2]

c The β-particle is emitted with an energy of 9.1×10^{-16} J. Calculate the speed of the β-particle. [3]

d A different isotope of hydrogen is hydrogen-2 (deuterium). Describe the similarities and differences between the atoms of hydrogen-2 and hydrogen-3. [2]

Cambridge International AS and A Level Physics (9702) Paper 23 Q6 parts a i and ii, b, c, d Oct/Nov 2012

10 **a** Give one example of:

i a hadron,

ii a lepton. [1]

b Describe, in terms of the simple quark model:

i a proton, [1]

ii a neutron. [1]

c Beta particles may be emitted during the decay of an unstable nucleus of an atom. The emission of a beta particle is due to the decay of a neutron.

i Complete the following word equation for the particles produced in this reaction. neutron → ... + ... + ... **[1]**

ii State the change in quark composition of the particles during this reaction. **[1]**

Cambridge International AS and A Level Physics (9702) Paper 21 Q7 May/June 2016

11 a State **one** difference between a hadron and a lepton. **[1]**

b A proton within a nucleus decays to form a neutron and two other particles. A partial equation to represent this decay is

$${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{....}_{....}... + {}^{....}_{....}...$$

i Copy and complete the equation. **[2]**

ii State the name of the interaction or force that gives rise to this decay. **[1]**

iii State three quantities that are conserved in the decay. **[3]**

c Use the quark composition of a proton to show that it has a charge of +*e*, where *e* is the elementary charge. Explain your working. **[3]**

Cambridge International AS and A Level Physics (9702) Paper 22 Q6 Oct/Nov 2016

12 a The following particles are used to describe the structure of an atom.

electron neutron proton quark

State the fundamental particles in the above list. **[1]**

b The following equation represents the decay of a nucleus of ${}^{60}_{27}Co$ to form nucleus Q by β^- emission.

$${}^{60}_{27}Co \rightarrow {}^{A}_{B}Q + \beta^- + x$$

i Copy and complete Fig. 11.12. **[1]**

	value
A	
B	

▲ **Figure 11.12**

ii State the name of the particle *x*. **[1]**

Cambridge International AS and A Level Physics (9702) Paper 23 Q7 May/June 2017

13 A neutron within a nucleus decays to produce a proton, a β^- particle and an (electron) antineutrino.

$$n \rightarrow p + \beta^- + \bar{\nu}$$

a Use the quark composition of the neutron to show that the neutron has no charge. **[3]**

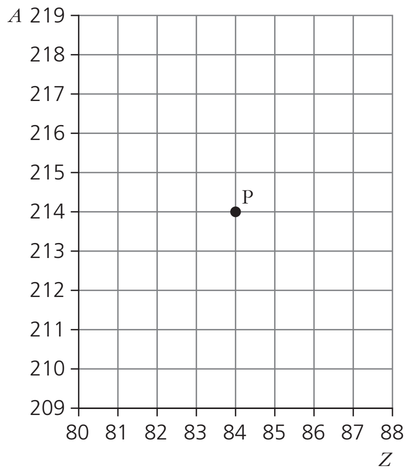
b Copy and complete Fig. 11.13 by giving appropriate values of the charge and the mass of the proton, the β^- particle and the (electron) antineutrino. **[2]**

	proton	β^- particle	antineutrino
charge			
mass			

▲ **Figure 11.13**

Cambridge International AS and A Level Physics (9702) Paper 21 Q8 Oct/Nov 2017

14 A graph of nucleon number *A* against proton number *Z* is shown in Fig. 11.14.



▲ **Figure 11.14**

The graph shows a cross (labelled P) that represents a nucleus P. Nucleus P decays emitting an α particle to form a nucleus Q. Nucleus Q then decays by emitting a β^- particle to form a nucleus R.

a On a copy of Fig. 11.14 use a cross to represent:

i nucleus Q (label this cross Q), [1]

ii nucleus R (label this cross R). [1]

b State the name of the class (group) of particles that includes the β^- particle. [1]

c The quark composition of one nucleon in Q is changed during the emission of the β^- particle. Describe this change to the quark composition. [1]

Cambridge International AS and A Level Physics (9702) Paper 23 Q7 May/June 2018

15 a The names of four particles are listed below.

alpha beta-plus neutron proton

State the name(s) of the particle(s) in this list that:

i are not fundamental, [1]

ii do not experience an electric force when situated in an electric field, [1]

iii has the largest charge to mass ratio. [2]

b A hadron has a charge of +*e* where *e* is the elementary charge. The hadron is composed of only two quarks. One of these quarks is an antidown ($\bar{d}$) quark. By considering charge, state and explain the name (flavour) of the other quark. [3]

Cambridge International AS and A Level Physics (9702) Paper 22 Q7 a i & ii, b, March 2019

16 a One of the results of the α-particle scattering experiment is that a very small minority of the α-particles are scattered through angles greater than 90°.

State what may be inferred about the structure of the atom from this result. [2]

b A hadron has an overall charge of +*e*, where *e* is the elementary charge.

The hadron contains three quarks. One of the quarks is a strange (s) quark.

i State the charge, in terms of *e*, of the strange (s) quark. [1]

ii The other two quarks in the hadron have the same charge as each other.

By considering charge, determine a possible type (flavour) of the other two quarks.

Explain your working. [2]

Cambridge International AS and A Level Physics (9702) Paper 21 Q7 May/June 2019

17 a Evidence from the nuclear atom was provided by the α-particle scattering experiment. State the results of this experiment. [2]

b Give estimates for the diameter of:

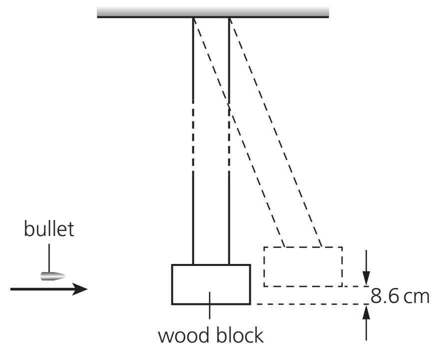
i an atom, [1]

ii a nucleus. [1]

Cambridge International AS and A Level Physics (9702) Paper 02 Q7 Oct/Nov 2007

AS Level review exercise

1 A bullet of mass 2.0 g is fired horizontally into a block of wood of mass 600 g. The block is suspended from strings so that it is free to move in a vertical plane. The bullet buries itself in the block. The block and bullet rise together through a vertical distance of 8.6 cm, as shown in Figure 1.



▲ **Figure 1**

a **i** Calculate the change in gravitational potential energy of the block and bullet. **[2]**

ii Show that the initial speed of the block and the bullet, after they began to move off together, was $1.3\,\mathrm{m\,s^{-1}}$. **[1]**

b Using the information in **aii** and the principle of conservation of momentum, determine the speed of the bullet before the impact with the block. **[2]**

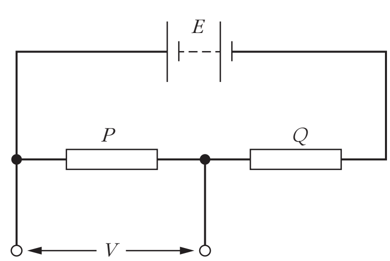
c **i** Calculate the kinetic energy of the bullet just before impact. **[2]**

ii State and explain what can be deduced from your answers to **ci** and **ai** about the type of collision between the bullet and the block. **[2]**

[Total: 9]

Cambridge International AS and A Level Physics (9702) Paper 02 Q3 May/June 2005

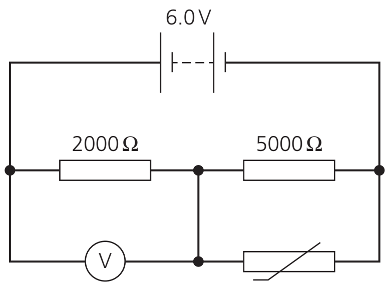
2 A potential divider circuit consists of two resistors of resistances P and Q, as shown in Figure 2. The battery has e.m.f. E and negligible internal resistance.



▲ **Figure 2**

a Deduce that the potential difference V across the resistor of resistance P is given by the expression $V = \frac{P}{P+Q}E$. **[2]**

b The resistances *P* and *Q* are 2000 Ω and 5000 Ω respectively. A voltmeter is connected in parallel with the 2000 Ω resistor and a thermistor is connected in parallel with the 5000 Ω resistor, as shown in Figure 3. The battery has e.m.f. 6.0 V. The voltmeter has infinite resistance.



▲ **Figure 3**

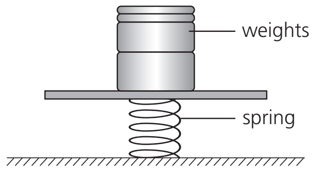
i State and explain qualitatively the change in the reading of the voltmeter as the temperature of the thermistor is raised. **[3]**

ii The voltmeter reads 3.6 V when the temperature of the thermistor is 19°C. Calculate the resistance of the thermistor at 19°C. **[4]**

[Total: 9]

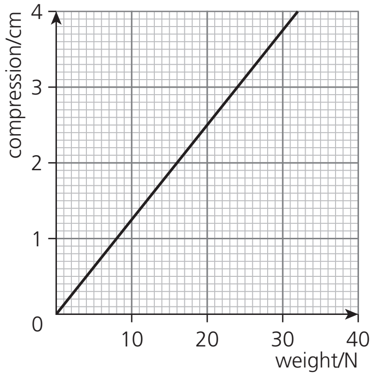
Cambridge International AS and A Level Physics (9702) Paper 02 Q7 Oct/Nov 2008

3 A spring is placed on a flat surface and different weights are placed on it, as shown in Figure 4.



▲ **Figure 4**

The variation with weight of the compression of the spring is shown in Figure 5.



▲ **Figure 5**

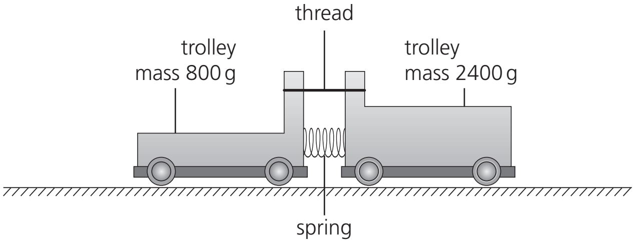
The elastic limit of the spring has not been exceeded.

a **i** Determine the spring constant *k* of the spring. **[2]**

ii Deduce that the strain energy stored in the spring is 0.49 J for a compression of 3.5 cm. **[2]**

b Two trolleys of masses 800 g and 2400 g are free to move on a horizontal table. The spring in **a** is placed between the trolleys and the trolleys are tied together using thread so that the compression of the spring is 3.5 cm, as shown in

Figure 6. Initially, the trolleys are not moving. The thread is then cut and the trolleys move apart.



▲ **Figure 6**

i Deduce that the ratio

$$\frac{\text{speed of trolley of mass 800 g}}{\text{speed of trolley of mass 2400 g}}$$

is equal to 3.0. **[2]**

ii Use the answers in **a ii** and **b i** to calculate the speed of the trolley of mass 800 g. **[3]**

[Total: 9]

Cambridge International AS and A Level Physics (9702) Paper 02 Q2 May/June 2008

4 A shopping trolley and its contents have a total mass of 42 kg. The trolley is being pushed along a horizontal surface at a speed of $1.2\,\text{m}\,\text{s}^{-1}$. When the trolley is released, it travels a distance of 1.9 m before coming to rest.

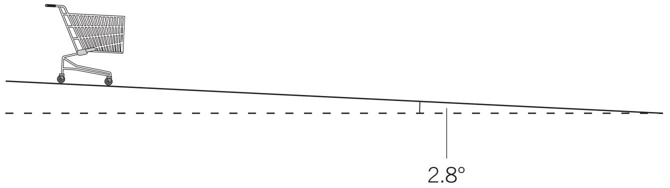
a Assuming that the total force opposing the motion of the trolley is constant:

i calculate the deceleration of the trolley, **[2]**

ii show that the total force opposing the motion of the trolley is 16 N. **[1]**

b Using the answer to **a ii**, calculate the power required to overcome the total force opposing the motion of the trolley at a speed of $1.2\,\text{m}\,\text{s}^{-1}$. **[2]**

c The trolley now moves down a straight slope that is inclined at an angle of 2.8° to the horizontal, as shown in Figure 7.



▲ **Figure 7**

The constant force that opposes the motion of the trolley is 16 N. Calculate, for the trolley moving down the slope:

i the component down the slope of the trolley's weight, **[2]**

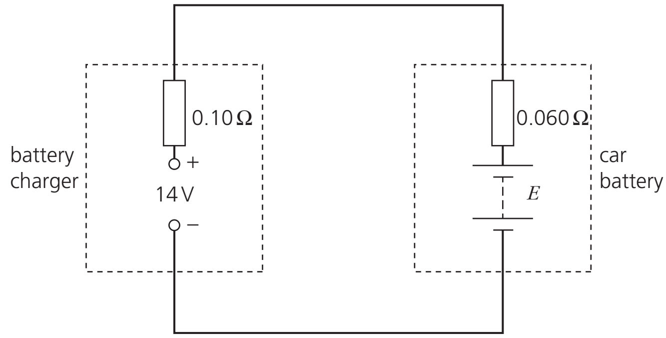
ii the time for the trolley to travel from rest a distance of 3.5 m along the length of the slope. **[4]**

d Use you answer to **c ii** to explain why, for safety reasons, the slope is not made any steeper. **[1]**

[Total: 12]

Cambridge International AS and A Level Physics (9702) Paper 02 Q3 May/June 2008

5 A car battery has an internal resistance of 0.060 Ω. It is recharged using a battery charger having an e.m.f. of 14 V and an internal resistance of 0.10 Ω, as shown in Figure 8.



▲ **Figure 8**

a At the beginning of the recharging process, the current in the circuit is 42 A and the e.m.f. of the battery is E (measured in volts).

i For the circuit in Figure 8, state:

1 the magnitude of the total resistance,

2 the total e.m.f. in the circuit. Give your answer in terms of E. **[2]**

ii Use your answers to **i** and data from the question to determine the e.m.f. of the car battery at the beginning of the recharging process. **[2]**

b For the majority of the charging time of the car battery, the e.m.f. of the car battery is 12 V and the charging current is 12.5 A. The battery is charged at this current for 4 hours. Calculate, for this charging time:

i the charge that passes through the battery, **[2]**

ii the energy supplied from the battery charger, **[2]**

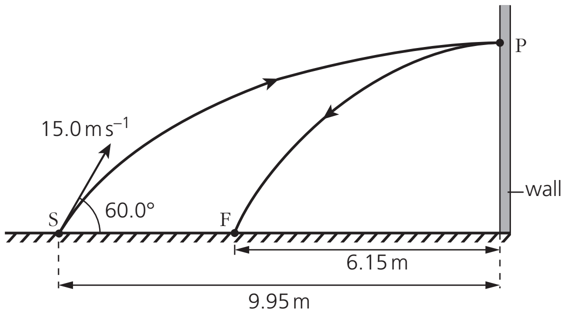
iii the total energy dissipated in the internal resistance of the battery charger and the car battery. **[2]**

c Use your answers to **b** to calculate the percentage efficiency of transfer of energy from the battery charger to stored energy in the car battery. **[2]**

[Total: 12]

Cambridge International AS and A Level Physics (9702) Paper 02 Q6 May/June 2007

6 A ball is thrown against a vertical wall. The path of the ball is shown in Figure 9. The ball is thrown from S with an initial velocity of 15.0 m s^{-1} at 60.0° to the horizontal. Assume that air resistance is negligible.



▲ **Figure 9 (not to scale)**

a For the ball at S, calculate:

i its horizontal component of velocity, **[1]**

ii its vertical component of velocity. **[1]**

b The horizontal distance from S to the wall is 9.95 m. The ball hits the wall at P with a velocity that is at right angles to the wall. The ball rebounds to a point F that is 6.15 m from the wall.

Using your answers to **a**:

i calculate the vertical height gained by the ball when it travels from S to P, **[1]**

ii show that the time taken for the ball to travel from S to P is 1.33 s, **[1]**

iii show that the velocity of the ball immediately after rebounding from the wall is about 4.6 m s^{-1}. **[1]**

c The mass of the ball is 60×10^{-3} kg.

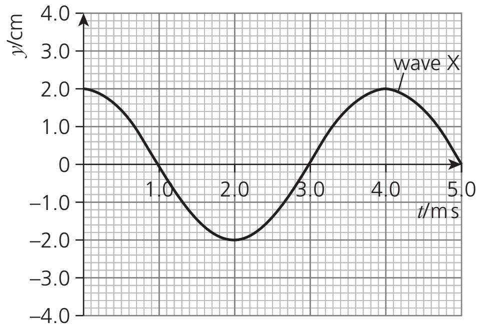
i Calculate the change in momentum of the ball as it rebounds from the wall. **[2]**

ii State and explain whether the collision is elastic or inelastic. **[1]**

[Total: 8]

Cambridge International AS and A Level Physics (9702) Paper 21 Q3 Oct/Nov 2011

7 The variation with time t of the displacement y of a wave X, as it passes a point P, is shown in Figure 10. The intensity of wave X is I.

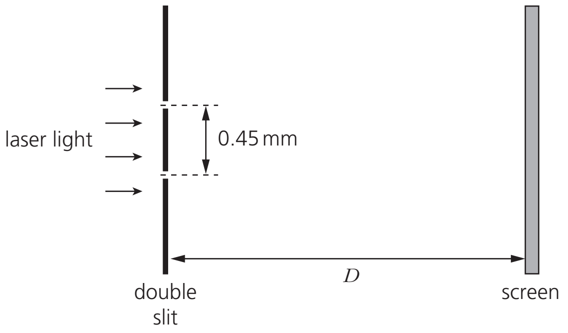


▲ **Figure 10**

a Use Figure 10 to determine the frequency of wave X. **[2]**

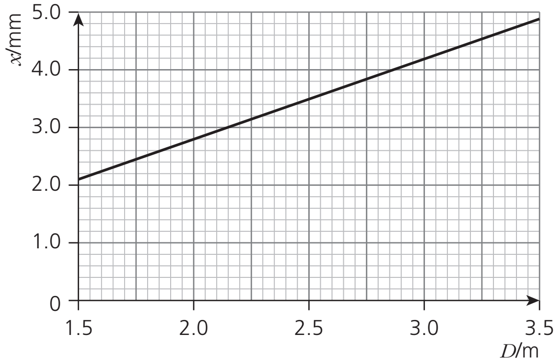
b A second wave Z with the same frequency as wave X also passes point P. Wave Z has intensity $2I$. The phase difference between the two waves is 90°. On a copy of Figure 10, sketch the variation with time t of the displacement y of wave Z. Show your working. **[3]**

c A double-slit interference experiment is used to determine the wavelength of light emitted from a laser, as shown in Figure 11.



▲ **Figure 11 (not to scale)**

The separation of the slits is 0.45 mm. The fringes are viewed on a screen at a distance D from the double slit. The fringe width x is measured for different distances D. The variation with D of x is shown in Figure 12.



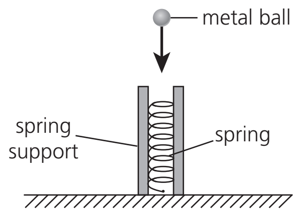
▲ **Figure 12**

i Use the gradient of the line in Figure 12 to determine the wavelength, in nm, of the laser light. **[4]**

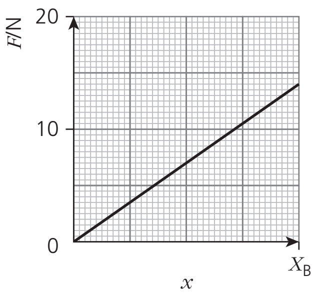
ii The separation of the slits is increased. State and explain the effects, if any, on the graph of Figure 12. **[2]**

[Total: 11]

Cambridge International AS and A Level Physics (9702) Paper 21 Q5 May/June 2016



▲ **Figure 13 (not to scale)**



▲ **Figure 14**

8 A metal ball of mass 40 g falls vertically onto a spring, as shown in Figure 13. The spring is supported and stands vertically. The ball has a speed of 2.8 m s^{-1} as it makes contact with the spring. The ball is brought to rest as the spring is compressed.

a Show that the kinetic energy of the ball as it makes contact with the spring is 0.16 J. **[2]**

b The variation of the force F acting on the spring with the compression x of the spring is shown in Figure 14.

The ball produces a maximum compression X_B when it comes to rest. The spring has a spring constant of 800 N m^{-1}. Use Figure 14 to:

i calculate the compression X_B, **[2]**

ii show that not all the kinetic energy in **a** is converted into elastic potential energy in the spring. **[2]**

[Total: 6]

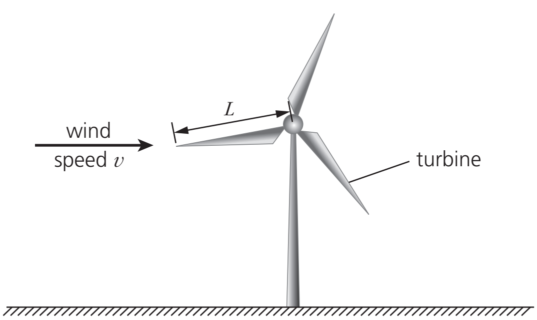
Cambridge International AS and A Level Physics (9702) Paper 21 Q4 May/June 2014

9 **a** Determine the SI base units of power. **[3]**

b Figure 15 shows a turbine that is used to generate electrical power from the wind. The power P available from the wind is given by

$$P = CL^2\rho v^3$$

where L is the length of each blade of the turbine, ρ is the density of air, v is the wind speed and C is a constant.



▲ **Figure 15**

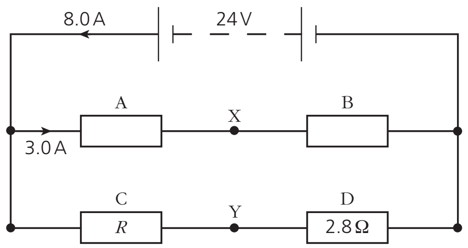
i Show that C has no units. **[3]**

ii The length L of each blade of the turbine is 25.0 m and the density ρ of air is 1.30 in SI units. The constant C is 0.931. The efficiency of the turbine is 55% and the electric power output P is 3.50×10^5 W. Calculate the wind speed. **[3]**

iii Suggest two reasons why the electrical power output of the turbine is less than the power available from the wind. **[2]**

[Total: 11]

Cambridge International AS and A Level Physics (9702) Paper 22 Q1 May/June 2013



▲ Figure 16

10 a Define *potential difference*.

A battery of electromotive force (e.m.f.) 24 V and negligible internal resistance is connected to a network of resistors A, B, C and D, as shown in Figure 16.

The resistors A and B in the circuit have equal resistances. The two resistors C and D have resistances *R* and 2.8 Ω. The resistors A and B are connected at junction X and the resistors C and D are connected at junction Y. The current in the battery is 8.0 A and the current in the resistors A and B is 3.0 A.

b Calculate:
 i the resistance of resistor A,
 ii resistance *R*.

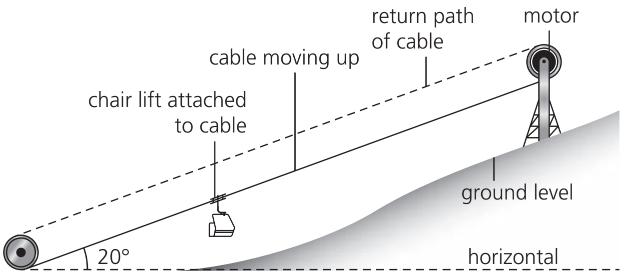
c Determine the potential difference between points X and Y.

d Calculate:
 i the total power dissipated in the resistors A and B,
 ii the total power produced by the battery.

e The resistor of resistance *R* is replaced with a resistor of greater resistance. State and explain the effect, if any, of this change on:
 i the total power dissipated by the two resistors A and B,
 ii the total power produced by the battery.

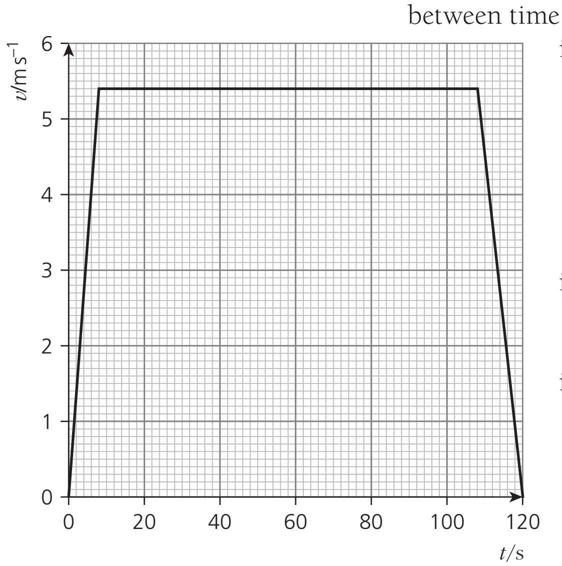
11 a Define *power*.

b A chair lift and passengers have a total weight of 3.5 kN. The lift is attached to a cable that is connected to a motor, as shown in Figure 17.



▲ Figure 17

The cable is supported so that the lift is above ground level as it is pulled up a mountain by the motor. The cable is at an angle of 20° to the horizontal. The variation with time *t* of the speed *v* of the lift is shown in Figure 18. A constant frictional force of 0.40 kN acts against the lift when it is moving at constant speed between time *t* = 8.0 s and *t* = 108 s.



▲ Figure 18

i Use Figure 18:
 1 to determine the acceleration of the lift between time *t* = 0 and *t* = 8.0 s,
 2 to show that the tension in the cable is 1.6 kN between time *t* = 8.0 s and *t* = 108 s,
 3 to calculate the work done by the motor to raise the lift between time *t* = 8.0 s and *t* = 108 s.

ii The motor has an efficiency of 75%. The tension in the cable is 1.7 kN at time *t* = 6.0 s.
Determine the input power to the motor at this time.

iii State and explain whether the increase in gravitational potential energy of the lift between *t* = 8.0 and *t* = 108 s is less than, the same as, or greater than the work done by the motor. A calculation is not required.

12 Motion in a circle

Learning outcomes

By the end of this topic, you will be able to:

12.1 Kinematics of uniform circular motion

1. define the radian and express angular displacement in radians
2. understand and use the concept of angular speed
3. recall and use $\omega = 2\pi/T$ and $v = r\omega$

12.2 Centripetal acceleration

1. understand that a force of constant magnitude that is always perpendicular to the direction of motion causes centripetal acceleration
2. understand that centripetal acceleration causes circular motion with a constant angular speed
3. recall and use $a = r\omega^2$ and $a = v^2/r$
4. recall and use $F = mr\omega^2$ and $F = mv^2/r$

Starting points

- Velocity is instantaneous speed in a given direction.
- An acceleration is change in velocity brought about by a resultant force.
- Newton's laws of motion.

12.1 Radian measure and angular displacement

In circular motion, it is convenient to measure angles in **radians** rather than degrees. One degree is, by tradition, equal to the angle of a complete circle divided by 360.

One radian (rad) is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

Thus, to obtain an angle in radians, we divide the length of the arc by the radius of the circle (see Figure 12.1).

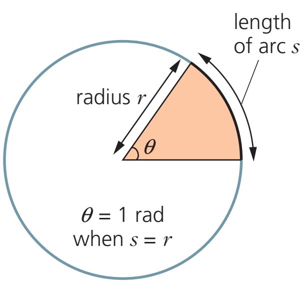


Figure 12.1 θ in radians = arc/radius

$$\theta = \frac{\text{length of arc}}{\text{radius of circle}} \left(= \frac{s}{r}\right)$$

The angle in radians in a complete circle would be

$$\theta = \frac{\text{circumference of the circle}}{\text{radius of circle}} = \frac{2\pi r}{r} = 2\pi$$

Since the angle of a complete circle is 360°, then

$$2\pi \text{ rad} = 360°$$

or

$$1 \text{ rad} = 57.3°$$

Angular speed

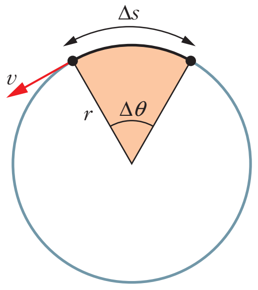
For an object moving in a circle:

The **angular speed** is defined as the angle swept out by the radius of the circle per unit time.

The **angular velocity** is the angular speed in a given direction (for example, clockwise). The unit of angular speed and angular velocity is the radian per second ($rad\,s^{-1}$).

$$\text{angular speed } \omega = \frac{\Delta\theta}{\Delta t}$$

Figure 12.2 shows an object travelling at constant speed v in a circle of radius r.



▲ **Figure 12.2** Angular velocity $\omega = v/r$

In a time Δt the object moves along an arc of length Δs and sweeps out an angle $\Delta\theta$. From the definition of the radian,

$\Delta\theta = \Delta s/r$ or $\Delta s = r\Delta\theta$

Dividing both sides of this equation by Δt,

$\Delta s/\Delta t = r\Delta\theta/\Delta t$

By definition, for small angles, $\Delta s/\Delta t$ is the linear speed v of the object, and $\Delta\theta/\Delta t$ is the angular speed ω. Hence,

$$v = r\omega$$

Also, if an object makes one complete revolution of a circle in time T, the object will have rotated through 2π rad. So, its angular speed ω will be given by

$$\omega = \frac{2\pi}{T}$$

WORKED EXAMPLE 12A

A rocket makes a turn in a horizontal circle of radius 150 m. It is travelling at a speed of $240\,m\,s^{-1}$.

Calculate the angular speed of the aircraft.

Answer

From $v = r\omega$, the angular speed ω is 240/150 = $\mathbf{1.6\,rad\,s^{-1}}$.

Questions

1 A car is travelling along a circular path with linear speed $18\,m\,s^{-1}$ and angular speed $0.30\,rad\,s^{-1}$. What is the radius of curvature of the track?

2 A ball on a track travels round a complete loop in a time of 1.4 s. Calculate the average angular speed of the ball.

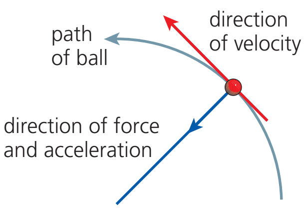
12.2 Centripetal acceleration and centripetal force

Newton's first law of motion (see Topic 3.1) tells us that an object with a resultant force of zero acting on it will either not be moving at all, or it will be moving in a straight line at constant speed (that is, its velocity does not change). The object is said to be in equilibrium. (The full conditions of equilibrium require there to be no resultant force and no resultant moment acting on the object.)

An object travelling in a circle may have a constant speed, but it is not travelling in a straight line. The velocity is changing as velocity is a vector (has magnitude and direction) and its direction is changing. A change in velocity means the object is accelerating.

This acceleration is towards the centre of the circle. It is called the **centripetal acceleration**. In order to make an object accelerate, there must be a resultant force acting on it. This resultant force is called the **centripetal force**. The centripetal force acts towards the centre of the circle, in the same direction as the acceleration. This means that the resultant force, the centripetal force, has constant magnitude and always acts at right angles to the instantaneous velocity of the object.

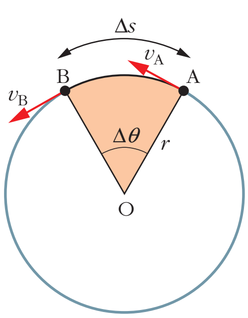
Consider a ball on a string which is being swung in a horizontal circle. The tension in the string provides the centripetal force.



▲ **Figure 12.3** A ball swung in a circle on the end of a string

At any instant, the direction of the ball's velocity is along the tangent to the circle, as shown in Figure 12.3. If the string breaks or is released, there is no longer any tension in the string and hence no centripetal force. The ball will travel in the direction of the tangent to the circle at the moment of release.

Figure 12.4 shows an object which has travelled at constant speed v in a circular path from A to B in time Δt. At A, its velocity is v_A, and at B the velocity is v_B. Both v_A and v_B are vectors.



▲ **Figure 12.4** Diagram for proof of $a = v^2/r$

The change in velocity Δv may be seen in the vector diagram of Figure 12.5. A vector Δv must be added to v_A in order to give the new velocity v_B.

The angle between the two radii OA and OB is $\Delta\theta$. This angle is also equal to the angle between the vectors v_A and v_B, because triangles OAB and CDE are similar. Consider angle $\Delta\theta$ to be so small that the arc AB may be approximated to a straight line. Then, using similar triangles, DE/CD = AB/OA, and $\Delta v/v_A = \Delta s/r$ or

$$\Delta v = \Delta s(v_A/r)$$

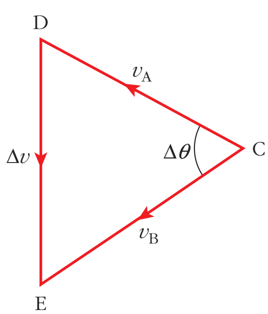
The time to travel either the distance Δs or the angle $\Delta\theta$ is Δt. Dividing both sides of the equation by Δt,

$$\Delta v/\Delta t = (\Delta s/\Delta t)(v_A/r)$$

and from the definitions of acceleration ($a = \Delta v/\Delta t$) and speed ($v = \Delta s/\Delta t = v_A = v_B$) we have $a = v(v/r)$ or $a = v^2/r$.

This expression can be written in terms of angular speed ω. Since $v = r\omega$, then

$$\text{centripetal acceleration} = \frac{v^2}{r} = r\omega^2$$



▲ **Figure 12.5** Vector diagram for proof of $a = v^2/r$

Now, force F is related to acceleration a by the expression $F = ma$, where m is the mass of the object.

$$\text{centripetal force} = \frac{mv^2}{r} = mr\omega^2$$

WORKED EXAMPLE 12B

The drum of a spin dryer has a radius of 50 cm and rotates at 720 revolutions per minute.

a Show that the angular speed of the drum is about 75 rad s^{-1}.

b Calculate, for a point on the edge of the drum:

- **i** its linear speed
- **ii** its acceleration towards the centre of the drum.

Answers

a 720 revolutions per minute is 12 revolutions per second. Each revolution is 2π rad, so the angular speed $\omega = 2\pi \times 12 =$ **75 rad s^{-1}**.

b **i** Using $v = r\omega$, $v = 0.50 \times 75 =$ **38 m s^{-1}** (**37.7 m s^{-1}**).

ii Using $a = v^2/r$, $a = (37.7)^2/0.50 =$ **2800 m s^{-2}** (**2840 m s^{-2}**).

Questions

3 A toy train moves round a circular track of diameter 0.76 m, completing one revolution in 12 seconds. Calculate, for this train:

- **a** the linear speed
- **b** the angular speed
- **c** the centripetal acceleration.

4 A stone attached to a string is moving in a horizontal circle of radius 96 cm. The stone has mass 64 g and completes one revolution in 0.72 s. Calculate the tension in the string, keeping the stone in its circular path.

Examples of circular motion

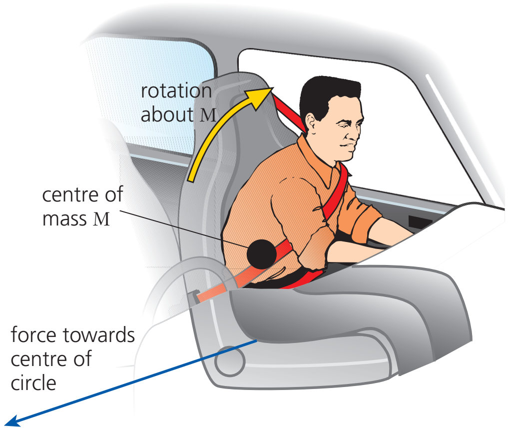
When a ball is whirled round on the end of a string, you can see clearly that the tension in the string is making the ball accelerate towards the centre of the circle. However, in other examples it is not always so easy to see what force is providing the centripetal acceleration.

A satellite in Earth orbit experiences gravitational attraction towards the centre of the Earth. This attractive force provides the centripetal force and causes the satellite to accelerate towards the centre of the Earth, and so it moves in a circle. We shall return to this in detail in Topic 13.2.

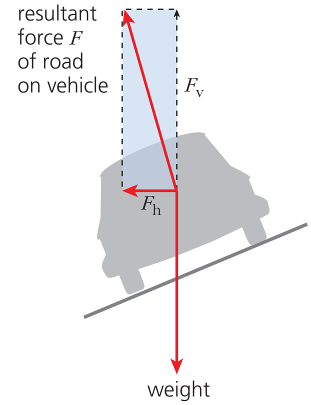
A charged particle moving at right angles to a magnetic field experiences a constant force at right angles to its direction of motion and, therefore, moves in the arc of a circle. This will be considered in more detail in Topic 20.3.

For a car travelling in a curved path, the frictional force between the tyres and the road surface provides the centripetal force. If this frictional force is not large enough, for example if the road is oily or slippery, then the car carries on moving in a straight line – it skids.

A passenger in a car that is cornering appears to be flung away from the centre of the circle. The centripetal force required to maintain the passenger in circular motion is provided through the seat of the car. This force is below the centre of mass M of the passenger, causing rotation about the centre of mass (see Figure 12.6). The effect is that the upper part of the passenger moves outwards unless another force acts on the upper part of the body, preventing rotation.



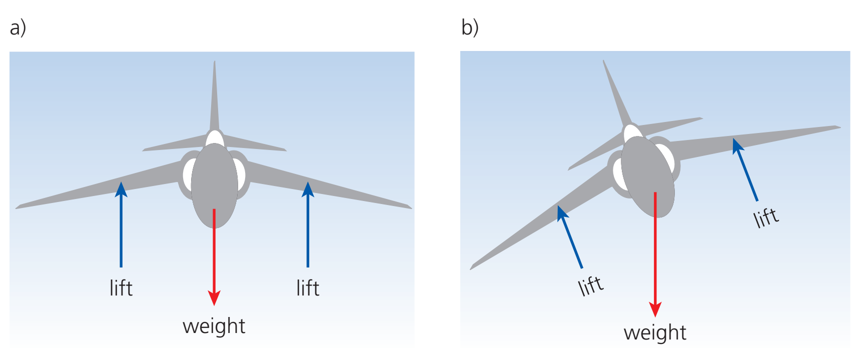
▲ **Figure 12.6** Passenger in a car rounding a corner



▲ **Figure 12.7** Cornering on a banked track

For cornering which does not rely only on friction, the road can be banked (Figure 12.7). The road provides a resultant force normal to its surface through contact between the tyres and the road. This resultant force F is at an angle to the vertical, and can be resolved into a vertical component F_v and a horizontal component F_h, as shown in Figure 12.7. F_v is equal to the weight of the vehicle, thus maintaining equilibrium in the vertical direction. The horizontal component F_h provides the centripetal force towards the centre of the circle. Many roads are banked for greater road safety, so as to reduce the chance of loss of control of vehicles due to skidding outwards on the corner, and for greater passenger comfort.

An aircraft has a lift force caused by the different rates of flow of air above and below the wings. The lift force balances the weight of the aircraft when it flies on a straight, level path (Figure 12.8a). In order to change direction, the aircraft is banked so that the wings are at an angle to the horizontal (Figure 12.8b). The lift force now has a horizontal component which provides a centripetal force to change the aircraft's direction.



▲ **Figure 12.8** An aircraft a) in straight, level flight and b) banking

A centrifuge (Figure 12.9) is a device that is used to spin objects at high speed about an axis. It is used to separate particles in mixtures. More massive particles require larger centripetal forces in order to maintain circular motion than do less massive ones. As a result, the more massive particles tend to separate from less massive particles, collecting further away from the axis of rotation. Space research centres, such as NASA, use centrifuges which are large enough to rotate a person (Figure 12.10, overleaf). Their purpose is to investigate the effects of large accelerations on the human body.

▲ **Figure 12.9** Separation of a solid from a liquid in a laboratory centrifuge

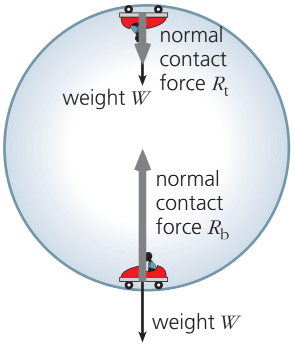
▲ **Figure 12.10** Centrifuge testing the effect of acceleration on the human body

Motion in a vertical circle

Some theme park rides involve rotation in a vertical circle (Figure 12.11). A person on such a ride must have a resultant force acting towards the centre of the circle.

The forces acting on the person are the person's weight, which always acts vertically downwards, and the normal contact force from the seat, which acts at right angles to the seat.

▲ **Figure 12.11** A big wheel at a theme park



▲ **Figure 12.12** Forces on a person on a circular ride

Consider a person moving round a vertical circle at speed v.

At the bottom of the ride, the normal contact force R_b from the seat must provide the centripetal force, as well as overcoming the weight W of the person. Figure 12.12 illustrates the situation.

The centripetal force is given by

$$mv^2/r = R_b - W$$

At the top of the ride, the weight W and the normal contact force R_t both act downwards towards the centre of the circle. The centripetal force is now given by

$$mv^2/r = R_t + W$$

This means that the force R_t from the seat at the top of the ride is less than the force R_b at the bottom. If the speed v is not large, then at the top of the circle the weight may be greater than the centripetal force. The person would lose contact with the seat and fall inwards.

WORKED EXAMPLE 12C

A rope is tied to a bucket filled with water, and the bucket is swung in a vertical circle of radius 1.2 m. What must be the minimum speed of the bucket at the highest point of the circle if the water is to stay in the bucket throughout the motion?

Answer

This example is similar to the problem of the theme park ride. Water will fall out of the bucket if its weight is greater than the centripetal force. The critical speed v is given by $mv^2/r = mg$ or $v^2 = gr$.

Here, $v =$ **3.4 m s^{-1}**.

Question

5 At an air show, an aircraft diving at a speed of 180 m s^{-1} pulls out of the dive by moving in the arc of a circle at the bottom of the dive.

a Calculate the minimum radius of this circle if the centripetal acceleration of the aircraft is not to exceed five times the acceleration of free fall.

b The pilot has mass 85 kg. What is the resultant force upwards on him at the instant when the aircraft is at its lowest point?

SUMMARY

- Angles may be measured in radians (rad). One radian is the angle subtended at the centre of a circle by an arc of the circle equal in length to its radius.
- Angular speed ω is the angle swept out per unit time by a line rotating about a point.
- A particle moving along a circle of radius r with linear speed v has angular speed ω given by $v = r\omega$.
- A particle that takes time T to complete one revolution of a circle has angular speed $\omega = 2\pi/T$.
- A resultant force acting towards the centre of the circle, called the centripetal force, is required to make an object move in a circle.
- An object moving along a circle of radius r with linear speed v and angular speed ω has an acceleration a towards the centre (the centripetal acceleration) given by $a = v^2/r = r\omega^2$.
- For an object of mass m moving along a circle of radius r with linear speed v and angular speed ω, the centripetal force F is given by $F = mv^2/r = mr\omega^2$.

END OF TOPIC QUESTIONS

1 State how the centripetal force is provided in the following examples:

- **a** a planet orbiting the Sun,
- **b** a child on a playground roundabout,
- **c** a train on a curved track,
- **d** a passenger in a car going round a corner.

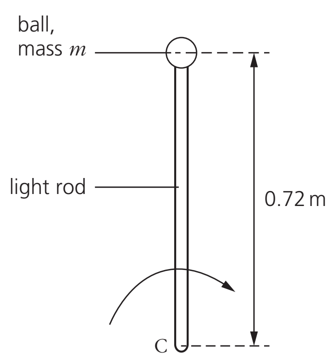
2 NASA's 20-G centrifuge is used for testing space equipment and the effect of acceleration on humans. The centrifuge consists of an arm of length 17.8 m, rotating at constant speed and producing an acceleration equal to 20 times the acceleration of free fall. Calculate:

- **a** the angular speed required to produce a centripetal acceleration of $20g$,
- **b** the rate of rotation of the arm ($g = 9.81\,\text{m s}^{-2}$).

3 A satellite orbits the Earth 200 km above its surface. The acceleration towards the centre of the Earth is $9.2\,\text{m s}^{-2}$. The radius of the Earth is 6400 km. Calculate:

- **a** the speed of the satellite,
- **b** the time to complete one orbit.

4 A light rigid rod is pivoted at one end C so that the rod rotates in a vertical circle at constant speed as shown in Fig. 12.13.



▲ **Figure 12.13**

A small ball of mass m is fixed to the free end of the rod so that the ball moves in a vertical circle. When the ball is vertically above point C, the tension T in the rod is given by

$T = 2mg$

where g is the acceleration of free fall.

a **i** Explain why the centripetal force on the ball is **not** equal to $2mg$. **[2]**

ii State, in terms of mg:

1 the magnitude of the centripetal force, **[1]**

2 the tension in the rod when the ball is vertically below point C. **[1]**

b The distance from point C to the centre of the ball is 0.72 m.
Determine, for the ball:

i the angular speed, **[3]**

ii the linear speed. **[2]**

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13 Gravitational fields

Learning outcomes

By the end of this topic, you will be able to:

13.1 Gravitational field

1. understand that a gravitational field is an example of a field of force and define gravitational field as force per unit mass
2. represent a gravitational field by means of field lines

13.2 Gravitational force between point masses

1. understand that, for a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre
2. recall and use Newton's law of gravitation $F = Gm_1m_2/r^2$ for the force between two point masses
3. analyse circular orbits in gravitational fields by relating the gravitational force to the centripetal acceleration it causes
4. understand that a satellite in a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator

13.3 Gravitational field of a point mass

1. derive, from Newton's law of gravitation and the definition of gravitational field, the equation $g = GM/r^2$ for the gravitational field strength of a point mass
2. recall and use $g = GM/r^2$
3. understand why g is approximately constant for small changes in height near the Earth's surface

13.4 Gravitational potential

1. define gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to the point
2. use $\Phi = -GM/r$ for the gravitational potential in the field due to a point mass
3. understand how the concept of gravitational potential leads to the gravitational potential energy of two point masses and use $E_p = -GMm/r$

Starting points

★ There is a force of attraction between all masses. On Earth, the force attracting objects to the Earth is referred to as 'force due to gravity' and is called weight.

★ At the surface of the Earth, all objects have the same acceleration when falling freely (no air resistance).

★ A resultant force acting towards the centre of the circle, called the centripetal force, is required to make an object move in a circle.

★ For an object moving in a circle, $v = r\omega$.

13.1 Gravitational field

We are familiar with the fact that the Earth's force of gravity is responsible for our weight, the force which pulls us towards the Earth. Isaac Newton concluded that the Earth's force of gravity is also responsible for keeping the Moon in orbit.

We now know that every mass attracts every other mass. This force is known as the force due to gravity – the gravitational force. The region around a mass where this gravitational force is felt is known as a **gravitational field**.

A gravitational field is a region of space where a mass experiences a force.

Gravitational field strength

The size of the force due to gravity may change. For example, as an object moves from the Earth's surface into space, so the gravitational force on it decreases. The **gravitational field strength** is said to change.

The gravitational field strength at a point is defined as the force per unit mass acting on a small mass placed at that point.

The force per unit mass is also a measure of the acceleration of free fall (from Newton's second law of motion $F = ma$, see Topic 3.1). Thus, the gravitational field strength is given by $F/m = g$, where F is in newtons and m is in kilograms. This means that the gravitational field strength at the Earth's surface is about $9.81\,\text{N kg}^{-1}$ since the acceleration of free fall is $9.81\,\text{m s}^{-2}$. Note that the unit N kg^{-1} is equivalent to the unit of acceleration, m s^{-2}. As you will see for electric fields (see Topic 18) we also have two equivalent units for electric field strength, N C^{-1} and V m^{-1}. Although there is a clear *analogy* between N kg^{-1} and N C^{-1}, there is no *direct link* between m s^{-2} and V m^{-1}.

Gravitational field lines

A gravitational field may be represented as a series of **gravitational field lines**.

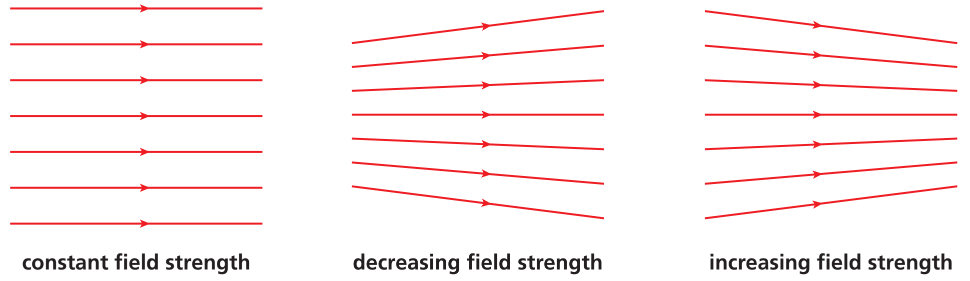
A gravitational field line is the direction of the gravitational force acting on a point mass.

For any gravitational field:

- the arrow on each line shows the direction of the gravitational force at that point situated on the line
- the gravitational field lines are smooth curves which never touch or cross
- the strength of the gravitational field is indicated by the closeness of the lines: the closer the lines, the stronger the gravitational field.

Note that gravitational field strength is a vector quantity.

Some aspects of gravitational fields are illustrated in Figure 13.1.

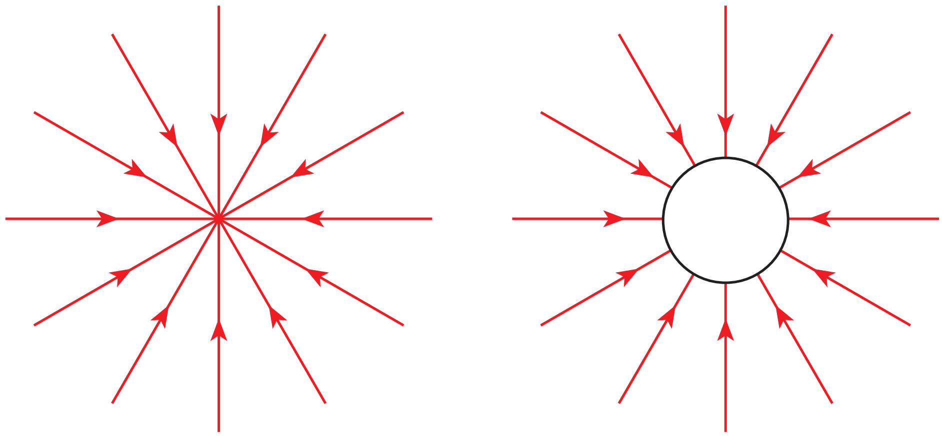


▲ **Figure 13.1** Gravitational fields

As we shall see in Topic 18, there is similarity between aspects of the representation of gravitational fields and electric fields.

13.2 Gravitational force between point masses

Figure 13.2 shows the gravitational fields in the regions around a point mass and a uniform sphere.



▲ **Figure 13.2 The gravitational field of a uniform sphere is similar to that of a point mass.**

The gravitational field outside the spherical uniform mass is radial. That is, all the lines of force appear to converge towards the centre of the uniform sphere. This means that, from outside the sphere, the sphere acts as a point mass that is situated at its centre.

For a point outside a sphere whose mass is uniformly distributed, the sphere behaves as a point mass with its mass concentrated at its centre.

Isaac Newton showed that the Earth's force of gravity extends into space, but weakens with distance according to an inverse square law. That is, the Earth's force of gravity varies inversely with the square of the distance from the centre of the Earth. If you go one Earth radius above the Earth's surface, the force is a quarter of the force on the Earth's surface. This is part of **Newton's law of gravitation**.

Newton's law of gravitation states that two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation.

Hence, if F is the force of attraction between two point masses of masses m_1 and m_2 with distance r between them, then

$$F \propto m_1 m_2 / r^2$$

or

$$F = G m_1 m_2 / r^2$$

where the constant of proportionality G is called the **gravitational constant**.

The value of G is $6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$.

Note: Newton's law of gravitation specifies that the two masses are point masses. However, the law still holds where the diameter/size of the masses is small compared to their separation. For example, the Sun and the Earth are not point masses. However, they may be considered to be point masses because their separation (1.5×10^8 km) is very large in comparison to their diameters (Earth, 1.3×10^4 km; Sun, 1.4×10^6 km).

It is possible to measure the gravitational constant G in a school laboratory, but the force of gravity between laboratory-sized masses is so small that it is not easy to obtain a reliable result.

WORKED EXAMPLE 13A

The masses of the Sun and the Earth are 2.0×10^{30} kg and 6.0×10^{24} kg respectively. The separation of their centres is 1.5×10^{8} km.

Calculate the force of attraction between the Sun and the Earth.

Answer

The separation is large in comparison to their radii so, using Newton's law of gravitation,

$$F = Gm_1m_2/r^2$$

$$F = (6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times 6.0 \times 10^{24})/(1.5 \times 10^{8} \times 10^{3})^2$$

$$= \mathbf{3.6 \times 10^{22}\ N}$$

Question

1 The mass of the dwarf planet Pluto is 1.2×10^{22} kg. Calculate the force of attraction between Pluto and the Sun, mass 2.0×10^{30} kg, when their separation is 5.9×10^{9} km.

Circular orbits

Most planets in the Solar System have orbits which are nearly circular. We now bring together the idea of a gravitational force and that of a centripetal force (see Topic 12.2) to derive a relation between the period and the radius of the orbit of a planet describing a circular path about the Sun, or a satellite moving round the Earth or another planet.

Consider a planet of mass m in circular orbit about the Sun, of mass M, as shown in Figure 13.3.

If the radius of the orbit is r, the gravitational force F_{grav} between the Sun and the planet is, by Newton's law of gravitation,

$$F_{grav} = GMm/r^2$$

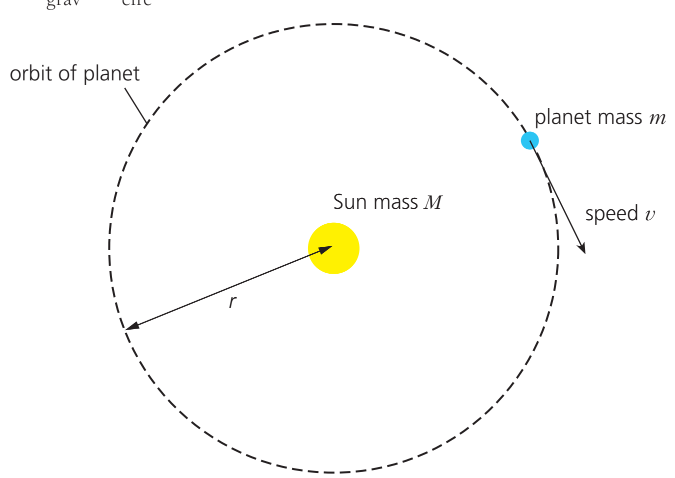
It is this force that provides the centripetal force as the planet moves in its orbit. Note that the planet is changing direction continuously and is, therefore, not in equilibrium. The gravitational force provides the accelerating force – the centripetal force.

The centripetal force F_{circ} is given by

$$F_{circ} = mv^2/r$$

where v is the linear speed of the planet. As has just been stated,

$$F_{grav} = F_{circ}$$



▲ **Figure 13.3** Circular orbit of a planet about the Sun

Thus

$$GMm/r^2 = mv^2/r$$

The period T of the planet in its orbit is the time required for the planet to travel a distance $2\pi r$. It is moving at speed v, so

$$v = 2\pi r/T$$

Putting this into the equation above, we have

$$GMm/r^2 = m(4\pi^2 r^2/T^2)/r$$

or, simplifying,

$$T^2 = (4\pi^2/GM)r^3$$

Another way of writing this is

$$T^2/r^3 = 4\pi^2/GM$$

Look at the right-hand side of this equation. The quantities π and G are constants. If we are considering the relation between T and r for planets in the Solar System, then M is the same for each planet because it is the mass of the Sun.

This equation shows that for planets or satellites describing circular orbits about the same central body, the square of the period is proportional to the cube of the radius of the orbit.

This relation is known as **Kepler's third law of planetary motion**. Johannes Kepler (1571–1630) analysed data collected by Tycho Brahe (1546–1601) on planetary observations. He showed that the observations fitted a law of the form $T^2 \propto r^3$. Fifty years later Newton showed that an inverse square law of gravitation together with the idea of centripetal acceleration and the second law of motion, gave an expression of exactly the same form. Newton cited Kepler's law in support of his law of gravitation. In fact, the orbits of the planets are not circular, but elliptical, a fact recognised by Kepler. The derivation is simpler for the case of a circular orbit.

Table 13.1 gives information about T and r for planets of the Solar System. The last column shows that the value of T^2/r^3 is indeed a constant. Moreover, the value of T^2/r^3 agrees very well with the value of $4\pi^2/GM$ which is $2.97 \times 10^{-25}\,\text{yr}^2\,\text{m}^{-3}$.

planet	T/(Earth years)	r/km	T^2/r^3(yr^2 km^{-3})
Mercury	0.241	57.9×10^6	2.99×10^{-25}
Venus	0.615	108.0×10^6	3.00×10^{-25}
Earth	1.00	150.0×10^6	2.96×10^{-25}
Mars	1.88	228.0×10^6	2.98×10^{-25}
Jupiter	11.9	778.0×10^6	3.01×10^{-25}
Saturn	29.5	1.43×10^9	2.98×10^{-25}
Uranus	84.0	2.87×10^9	2.98×10^{-25}
Neptune	165	4.50×10^9	2.99×10^{-25}
Pluto*	248	5.90×10^9	2.99×10^{-25}
			(average 2.99×10^{-25})

*Since 2006, Pluto has been classified as a 'dwarf planet'.

▲ **Table 13.1**

Satellites are widely used in telecommunication. Many communication satellites are placed in what is called a **geostationary orbit**. That is, they are in equatorial orbits with exactly the same period of rotation as the Earth (24 hours), and move in the same direction as the Earth (west to east) so that they are always above the same point on the Equator. Such satellites are called **geostationary satellites**. Details of the orbit of such a satellite are worked out in the example which follows.

▲ **Figure 13.4** Communication satellite

WORKED EXAMPLE 13B

For a geostationary satellite, calculate:

a the height above the Earth's surface,

b the speed in orbit.

(Radius of Earth = 6.38×10^6 m; mass of Earth = 5.98×10^{24} kg.)

Answers

a The period of the satellite is 24 hours = 8.64×10^4 s.

Equating the force of gravity to centripetal force, $GMm/r^2 = mv^2/r$, which rearranges to give $rv^2 = GM$ where r is the distance from the Earth's centre to the satellite.

Since $v = 2\pi r/T$, $r(2\pi r/T)^2 = GM$ and $r^3 = GMT^2/4\pi^2$

$r^3 = 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (8.64 \times 10^4)^2/4\pi^2$, giving $r^3 = 7.54 \times 10^{22}\,\text{m}^3$.

Taking the cube root, the radius r of the orbit is 4.23×10^7 m. The distance above the Earth's surface is $(4.23 \times 10^7 - 6.38 \times 10^6) = \mathbf{3.59 \times 10^7\,m}$.

b Since $v = 2\pi r/T$, the speed is given by

$v = 2\pi \times 4.23 \times 10^7/8.64 \times 10^4 = \mathbf{3070\,m\,s^{-1}}$.

Question

2 The radius of the Moon's orbit about the Earth is 3.84×10^8 m, and its period is 27.4 days. Calculate the period of the orbit of a satellite orbiting the Earth just above the Earth's surface (radius of Earth = 6.38×10^6 m).

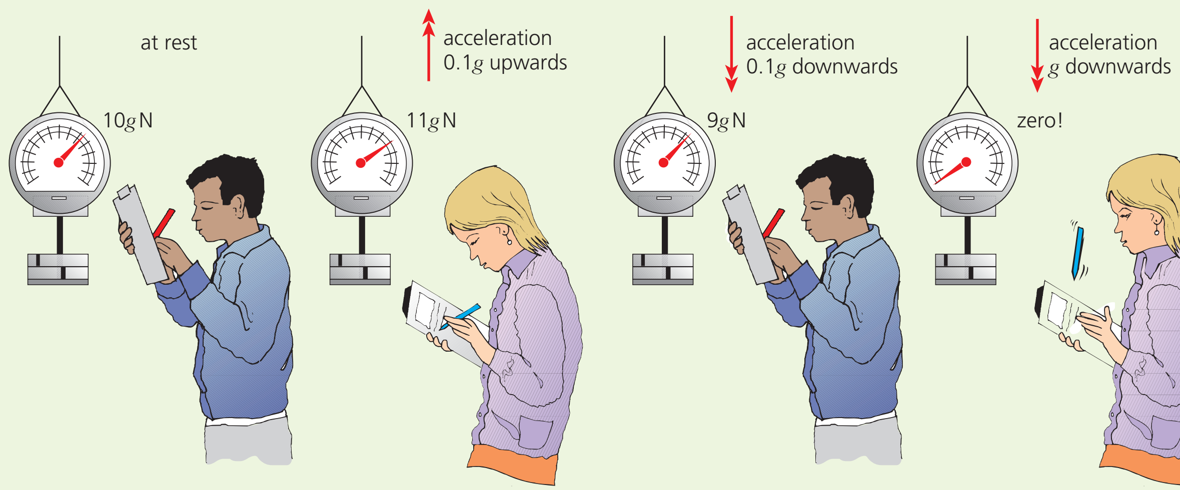
EXTENSION

Weightlessness

Suppose that you are carrying out an experiment involving the use of a newton balance in a lift. When the lift is stationary, an object of mass 10 kg, suspended from the balance, will give a weight reading of $10g$ N. If the lift accelerates upwards with acceleration 0.1g, the reading on the balance increases to $11g$ N. If the lift accelerates downwards with acceleration 0.1g, the apparent weight of the object decreases to $9g$ N. If, by an unfortunate accident, the lift cable were to break and there were no safety restraints, the lift would accelerate downwards with acceleration g. The reading on the newton balance would be zero. If, during the fall, you were to drop the pencil with which you are recording the balance readings, it would not fall to the floor of the lift but would remain stationary with respect to you. Both you and the pencil are in free fall. You are experiencing **weightlessness**.

Figure 13.5 illustrates your predicament. It might be more correct to refer to this situation as *apparent* weightlessness, as you can only be truly weightless in the absence of a gravitational field. That is, at an infinite distance from the Earth or any other attracting object.

A similar situation arises in a satellite orbiting the Earth. The force of gravity, which provides the centripetal force, is causing it to fall out of its expected straight-line path. People and objects inside the satellite are experiencing a free fall situation and apparent weightlessness.



▲ **Figure 13.5** Lift experiment on weightlessness

13.3 Gravitational field of a point mass

Of course, all the masses we come across in the laboratory have a finite size. But for calculations involving gravitational forces, it is fortunate that a spherical mass behaves as if it were a point mass at the centre of the sphere, with all the mass of the sphere concentrated at that point.

From Newton's law of gravitation, the attractive force on a mass m caused by another mass M, with a distance of r between their centres, is given by

$$F = GMm/r^2$$

This means that the force per unit mass or gravitational field strength g is given by

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

The field strengths due to masses that you find in a laboratory are tiny. For example, the field strength one metre away from an isolated mass of one kilogram is only 7×10^{-11} N kg^{-1}. However, field strengths due to the masses of objects such as the Earth or Moon are much larger. We already know that the field strength due to the Earth at the surface of the Earth is about 10 N kg^{-1}. We can use this information to deduce information about the Earth, for example, the mass of the Earth. Look at the example that follows.

WORKED EXAMPLE 13C

The radius of the Earth is 6.38×10^6 m and the gravitational field strength at its surface is 9.81 N kg^{-1}.

a Assuming that the field is radial, calculate the mass of the Earth.

b The radius of the Moon's orbit about the Earth is 3.84×10^8 m. Calculate the strength of the Earth's gravitational field at this distance.

c The mass of the Moon is 7.40×10^{22} kg. Calculate the gravitational attraction between the Earth and the Moon.
(Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

Answers

a Using $g = GM/r^2$, we have
$M = gr^2/G = 9.81 \times (6.38 \times 10^6)^2/6.67 \times 10^{-11} = \mathbf{5.99 \times 10^{24}\ kg}$

b Using $g = GM/r^2$, we have
$g = 6.67 \times 10^{-11} \times 5.99 \times 10^{24}/(3.84 \times 10^8)^2 = \mathbf{2.71 \times 10^{-3}\ N\ kg^{-1}}$

c Using $F = GMm/r^2$, we have
$F = 6.67 \times 10^{-11} \times 5.99 \times 10^{24} \times 7.40 \times 10^{22}/(3.84 \times 10^8)^2 = \mathbf{2.00 \times 10^{20}\ N}$

Question

3 The mass of Jupiter is 1.9×10^{27} kg and its radius is 7.1×10^7 m. Calculate the gravitational field strength at the surface of Jupiter.
(Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

Acceleration of free fall

We have already seen that gravitational field strength is defined as force per unit mass and that the gravitational field strength at the Earth's surface is also a measure of the acceleration of free fall. Thus, at a distance r from the centre of a uniform sphere of mass M

$$g = F/m = GM/r^2$$

If we assume that the Earth is a uniform sphere with approximately uniform density, we can apply the equation at and beyond the Earth's surface.

The radius r of Earth is approximately 6.4×10^3 km. If we move a few kilometres h above the Earth's surface, then the acceleration of free fall becomes

$$g_h = GM/(r + h)^2$$

Now, h is much smaller than r and so

$$r^2 \approx (r + h)^2$$

and

$$g \approx g_h$$

For small distances above the Earth's surface, g is approximately constant and is called the **acceleration of free fall**.

Mass and weight

In Topic 3, mass was said to be a measure of the inertia of an object to changes in velocity. Unless the object is travelling at speeds close to that of light, its mass is constant.

In a gravitational field, by definition, there is a force acting on the mass equal to the product of mass and gravitational field strength. This force is called the **weight**.

For an object of mass m in a gravitational field of strength g, the weight W is given by

weight = mass × gravitational field strength

or

$$W = mg$$

Although mass is invariant, weight depends on gravitational field strength. For example, a person of mass 60 kg has a weight of approximately 600 N on Earth, but only 100 N on the Moon, although the mass is still 60 kg.

13.4 Gravitational potential

In Topic 18.5, we will meet ideas about electric potential energy and electric potential. There is a very strong analogy between gravitational and electric fields, and this will help when we talk about electrical potential energy and electrical potential.

Gravitational potential at a point in a gravitational field is defined as the work done per unit mass in bringing a small test mass from infinity to the point.

The symbol for gravitational potential is Φ and its unit is the joule per kilogram (J kg^{-1}). For a field produced by a point mass, the equation for the potential at a point distance r from the mass M in the field is

$$\Phi = -GM/r$$

Note the minus sign.

The gravitational potential at infinity is defined as being zero. The gravitational force is always attractive and so, as the test mass moves from infinity, work can be done by the test mass and as a result its potential decreases. The gravitational potential is negative.

This is entirely consistent with the electrical case introduced in Topic 18.5. We have a negative electric potential when the field is produced by a negative charge, so that the force between the negative field-producing charge and the positive test charge is attractive. Here the attractive gravitational force between the field-producing mass and the test mass also gives a negative potential.

Gravitational potential is work done per unit mass. For an object of mass m, then the gravitational potential energy of the object will be m times as large as for an object of unit mass.

gravitational potential energy = mass × gravitational potential

$$= m\Phi = -GMm/r$$

For two isolated point masses m_1 and m_2 situated a distance r apart in a vacuum, then the gravitational potential energy E_p of the two masses is given by

$$E_p = -\frac{Gm_1m_2}{r}$$

WORKED EXAMPLE 13D

How much work is done by the gravitational field in moving a mass m of 2.0 kg from infinity to a point which is a distance 0.40 m from a mass M of 30 kg? (Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.)

Answer

The work which would have to be done by an external force is simply the change in gravitational potential energy. The potential energy at infinity is zero, so

$$W = m\Phi = -mGM/r = -2.0 \times 6.67 \times 10^{-11} \times 30/0.40 = -1.0 \times 10^{-8}\,\mathrm{J}$$

This is the work which would be done by an external force and is negative, so the work done by the field is positive and is equal to $\mathbf{1.0 \times 10^{-8}\,J}$.

Note the similarity in the method of calculation with the electric potential energy calculation in Topic 18.5.

Question

4 The Earth has mass 6.0×10^{24} kg and radius 6.4×10^{6} m. A meteorite of mass 220 kg moves from an infinite distance to the Earth's surface. The meteorite starts from rest. Calculate:

a the change in gravitational potential energy of the meteorite

b the speed at which it strikes the Earth.

(Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.)

SUMMARY

- A gravitational field is a region around a mass where another mass feels a force.
- The direction of gravitational field lines shows the direction of the force on a mass placed in the field and the separation shows the field strength.
- From a point outside a spherical mass, the mass of the sphere can be treated as a point mass at its centre.
- Gravitational field strength g is the force per unit mass: $g = F/m$ is also the acceleration of free fall.
- The attractive force between two point masses is proportional to the product of their masses and inversely proportional to the square of the distance between them. This is Newton's law of gravitation: $F = Gm_1m_2/r^2$.
- The gravitational field strength g at a point in the gravitational field of a point mass M is $g = GM/r^2$.
- Over a small area close to the Earth's surface g is approximately constant.
- For a circular orbit in a gravitational field: gravitational field strength = centripetal acceleration.
- The gravitational potential at a point in a gravitational field is the work done per unit mass in bringing a small test mass from infinity to the point.
- The potential at a point in a field produced by a point mass is: $\Phi = -GM/r$.
- Gravitational potential energy at a point in a field produced by a point mass is: $E_P = \Phi m = GMm/r$.

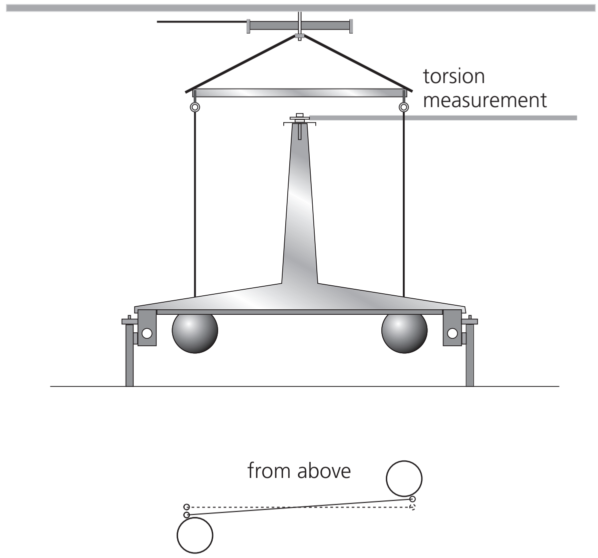
END OF TOPIC QUESTIONS

1 The radius of Mars is approximately 3.4×10^6 m. The acceleration of free fall at the surface of Mars is $3.7\,\mathrm{m\,s^{-2}}$. The gravitational constant G is $6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. Use this information to estimate the mean density of Mars.

2 The times for Mars and Jupiter to orbit the Sun are 687 days and 4330 days respectively. The radius of the orbit of Mars is 228×10^6 km. Calculate the radius of the orbit of Jupiter.

3 The weight of a passenger in an aircraft on the runway is W. His weight when the aircraft is flying at an altitude of 16 km above the Earth's surface is W_a. The percentage change F in his weight is given by $F = 100(W_a - W)/W\%$. Taking the radius of the Earth as 6.378×10^3 km, calculate F. Calculate also the percentage change P in his gravitational potential energy.

4 Fig. 13.6 illustrates the apparatus used by Cavendish in 1798 to find a value for the gravitational constant G. In a school experiment using similar apparatus, two lead spheres are attached to a light horizontal beam which is suspended by a wire. When a flask of mercury is brought close to each sphere, the gravitational attraction causes the beam to twist through a small angle. From measurements of the twisting (torsional) oscillations of the beam, a value can be found for the force producing a measured deflection. G can then be calculated if the large and small masses are known.



▲ **Figure 13.6 Cavendish's experiment for** G

a In such an experiment, one lead sphere has mass 6.50×10^{-3} kg and the mass of the mercury flask is 0.740 kg. Calculate the force between them when they are 72.0 mm apart. (Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.)

b Comment on the size of the force.

5 The gravitational field strength at the surface of the Moon is $1.62\,\mathrm{N\,kg^{-1}}$. The radius of the Moon is 1740 km.

a Show that the mass of the Moon is 7.35×10^{22} kg.

b The Moon rotates about its axis (as well as moving in orbit about the Earth). In the future, scientists may wish to put a satellite into an orbit about the Moon, such that the satellite remains stationary above one point on the Moon's surface.

i Explain why this orbit must be an equatorial orbit.

ii The period of rotation of the Moon about its axis is 27.4 days. Calculate the radius of the required orbit. ($G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.)

6 **a** State Newton's law of gravitation. **[2]**

b Some planets in the Solar System have several moons (satellites) that have circular orbits about the planet.

The planet and each of its moons may be considered to be point masses.

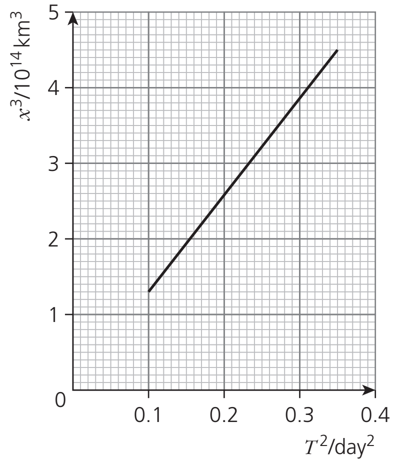
Show that the radius x of a moon's orbit is related to the period T of the orbit by the expression

$$GM = \frac{4\pi^2 x^3}{T^2}$$

where G is the gravitational constant and M is the mass of the planet.

Explain your working. **[3]**

c The planet Neptune has eight moons, each in a circular orbit of radius x and period T. The variation with T^2 of x^3 for some of the moons is shown in Fig. 13.7.



▲ **Figure 13.7**

Use Fig. 13.7 and the expression in **b** to determine the mass of Neptune. **[4]**

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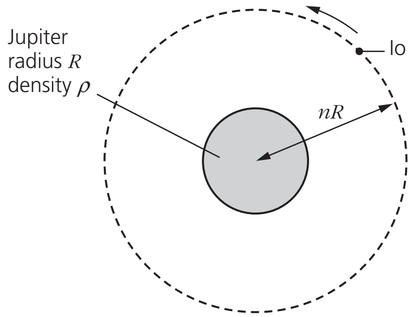
7 **a** State Newton's law of gravitation. **[2]**

b The planet Jupiter and one of its moons, Io, may be considered to be uniform spheres that are isolated in space. Jupiter has radius R and mean density ρ. Io has mass m and is in a circular orbit about Jupiter with radius nR, as illustrated in Fig. 13.8.



▲ **Figure 13.8**

The time for Io to complete one orbit of Jupiter is T. Show that the time T is related to the mean density ρ of Jupiter by the expression

$$\rho T^2 = \frac{3\pi n^3}{G}$$

where G is the gravitational constant. **[4]**

c **i** The radius R of Jupiter is 7.15×10^4 km and the distance between the centres of Jupiter and Io is 4.32×10^5 km. The period T of the orbit of Io is 42.5 hours. Calculate the mean density ρ of Jupiter. **[3]**

ii The Earth has a mean density of 5.5×10^3 kg m^{-3}. It is said to be a planet made of rock. By reference to your answer in **i**, comment on the possible composition of Jupiter. **[1]**

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14 Temperature

Learning outcomes

By the end of this topic, you will be able to:

14.1 Thermal equilibrium

1. understand that (thermal) energy is transferred from a region of higher temperature to a region of lower temperature
2. understand that regions of equal temperature are in thermal equilibrium

14.2 Temperature scales

1. understand that a physical property that varies with temperature may be used for the measurement of temperature and state examples of such properties, including the density of a liquid, volume of a gas at constant pressure, resistance of a metal, e.m.f. of a thermocouple
2. understand that the scale of thermodynamic temperature does not depend on the property of any particular substance
3. convert temperatures between kelvin and degrees Celsius and recall that $T/\mathrm{K} = \theta/°\mathrm{C} + 273.15$
4. understand that the lowest possible temperature is zero kelvin on the thermodynamic temperature scale and that this is known as absolute zero

14.3 Specify heat capacity and specific latent heat

1. define and use specific heat capacity
2. define and use specific latent heat and distinguish between specific latent heat of fusion and specific latent heat of vaporisation

Starting points

- ★ Temperature measures the degree of hotness of an object, not the amount of thermal energy in it.
- ★ Temperature is measured using a thermometer.
- ★ All matter consists of atoms or groups of atoms called molecules.
- ★ Solid, liquid and gas are three different states of matter.
- ★ The state of matter depends on the forces between the atoms or molecules and their spacing.
- ★ In a solid the atoms or molecules are held in fixed positions by strong forces and they vibrate about these positions with energy that depends on temperature.
- ★ In a liquid the forces between atoms or molecules are still strong but are no longer rigid and thus the atoms or molecules can move freely within the body of the liquid with energy that depends on temperature.
- ★ In a gas the forces between atoms or molecules are negligible. The particles are far apart and in rapid, random motion with energy that depends on temperature.
- ★ A substance can change state when energy is involved.

14.1 Thermal equilibrium

Our everyday idea of temperature is based on our sense of touch. Putting your hand into a bowl containing ice immediately gives a sense of cold; putting the other hand into a bowl of warm water gives the sense of something that is hot (Figure 14.1, overleaf). Intuitively, we would say that the water is at a higher temperature than the ice.

▲ **Figure 14.1**

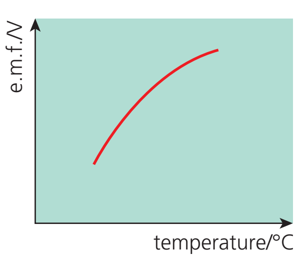
Think about a thick metal bar with one end at a higher temperature than the other. For example, one end can be heated by pouring hot water over it, and the other end cooled by holding it under a cold-water tap. The effect of the temperature difference between the ends is that thermal energy is transferred along the bar from the high temperature end to the low temperature end. We can think of this in terms of the vibrations of the atoms of the metal. One atom passes on some of its vibrational energy to its neighbour, which originally had less. If the bar is removed from the arrangement for keeping the ends at different temperatures, eventually the whole of the bar will end up in equilibrium at the same temperature. When different regions in thermal contact are at the same temperature, they are said to be in **thermal equilibrium**.

Think about a swimming pool and a cup of water that are at the same temperature. The swimming pool will contain much more thermal energy than the cup of water. However, because they are at the same temperature, there is no movement of thermal energy from the pool to the cup of water. The pool and the cup of water are in thermal equilibrium because no thermal energy is being transferred and they are at the same temperature.

14.2 Temperature scales

In Physics, we look for ways of defining and measuring quantities. In the case of temperature, we will first look at ways of measuring this quantity, and then think about the definition.

Many physical properties change with temperature. Most materials (solids, liquids and gases) expand as their temperature is increased. This means that the density of a liquid or the volume of a gas change with temperature change. The electrical resistance of a metal wire increases as the temperature of the wire is increased (see Topic 9.3). In a **thermocouple**, one end of each of two wires of different metals are twisted together and the other ends are connected to the terminals of a sensitive voltmeter. An e.m.f. is produced and the reading on the voltmeter depends on the temperature of the junction of the wires (Figure 14.2). All these properties may be used in different types of thermometer. The relationship between the physical property measured and temperature is not always proportional, so a **calibration curve** must be obtained before the thermometer can be used to measure temperature.



▲ **Figure 14.2** Calibration curve for a thermocouple

EXTENSION

A thermometer is an instrument for measuring temperature. The physical property on which a particular thermometer is based is called the **thermometric property**, and the working material of the thermometer, the property of which varies with temperature, is called the thermometric substance. Thus, in the familiar mercury-in-glass thermometer, the **thermometric substance** is mercury and the thermometric property is the length of the mercury thread in the capillary tube of the thermometer.

Remember that temperature measures the degree of 'hotness' of a body. It does not measure the *amount* of thermal energy (heat energy).

Each type of thermometer can be used to establish its own temperature scale. To do this, the fact that substances change state (from solid to liquid, or from liquid to gas) at fixed temperatures is used to define reference temperatures, which are called **fixed points**. By taking the value of the thermometric property at two fixed points, and dividing the range of values into a number of equal steps (or degrees), we can set up what is called an **empirical scale of temperature** for that thermometer. ('Empirical' means 'derived by experiment'.) If the fixed points are the melting point of ice (the ice point) and the temperature of steam above water boiling at normal atmospheric pressure (the steam point), and if we choose to have one hundred equal degrees between the temperatures corresponding to these fixed points, taken as 0 degrees and 100 degrees respectively, we arrive at the empirical centigrade scale of temperature for that thermometer. If the values of the thermometric property P are P_i and P_s at the ice- and steam-points respectively, and if the property has the value P_θ at an unknown temperature θ, the unknown temperature is given by

$$\theta = \frac{100(P_\theta - P_i)}{(P_s - P_i)}$$

on the empirical centigrade scale of this particular thermometer. This equation is illustrated in graphical form in Figure 14.3.

It is important to realise that the choice of a different thermometric substance and thermometric property would lead to a different centigrade scale. Agreement between scales occurs only at the two fixed points. This happens because the property may not vary linearly with temperature.

This situation, with temperature values depending on the type of thermometer on which they are measured, is clearly unsatisfactory for scientific purposes. It is found that the differences between empirical scales are small in the case of thermometers based on gases as thermometric substances. In the constant-volume gas thermometer (Figure 14.4), the pressure of a fixed volume of gas (measured by the height difference h) is used as the thermometric property.

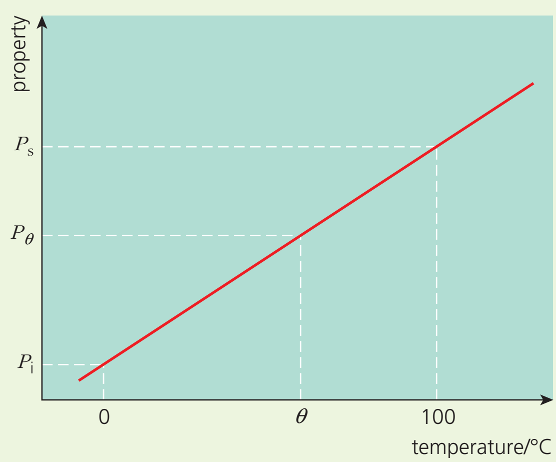


Figure 14.3 Empirical centigrade scale

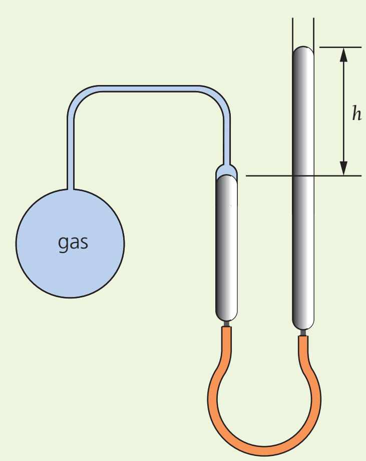
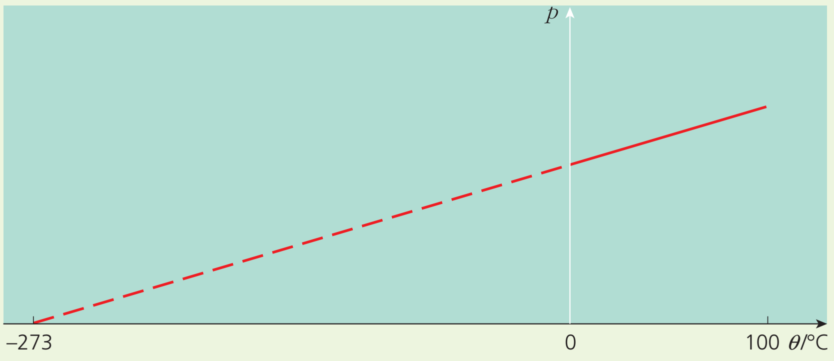


Figure 14.4 Constant-volume gas thermometer

The differences between the scales of different gas thermometers become even less as the pressures used are reduced. This is because the lower the pressure of a real gas, the more linear is the variation of pressure or volume. If we set up an empirical centigrade scale for a real gas in a constant-volume thermometer by obtaining the pressures of the gas at the ice-point (0 °C) and the steam-point (100 °C) we can extrapolate the graph of pressure p against the centigrade temperature θ to find the temperature at which the pressure of the gas would become zero. This is shown in Figure 14.5.



▲ **Figure 14.5** Graph of p against θ for constant-volume gas thermometer

The extrapolated temperature will be found to be close to –273 degrees on the empirical centigrade constant-volume gas thermometer scale. If the experiment is repeated with lower and lower pressures of gas in the thermometer, the extrapolated temperature tends to a value of –273.15 degrees. This temperature is the lowest theoretically possible temperature and is known as **absolute zero**. It does not depend on the properties of any particular substance. At absolute zero, molecules would have zero kinetic energy.

Thermodynamic temperature

We can see from the idea of extrapolating the *pressure–temperature* graph for a constant-volume gas thermometer (Figure 14.5) and Figure 15.2 (page 255) that there seems to be a natural zero of temperature, absolute zero. This is used as one of the fixed points of the **thermodynamic temperature scale**. The thermodynamic temperature scale starts with zero at absolute zero (–273.15 °C). We shall see in Topic 15.2 that the thermodynamic temperature scale is based on the theoretical behaviour of a so-called ideal gas.

EXTENSION

The upper fixed point is taken as the 'triple point' of water – the temperature at which ice, water and water vapour are in equilibrium. This is found to be less dependent on environmental conditions, such as pressure, than the ice-point. The thermodynamic temperature of the triple point of water is taken as 273.16 units, by international agreement. This defines the kelvin (symbol K), the unit of thermodynamic temperature.

One **kelvin** is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Since the variation with temperature of the property of a substance is not used when defining thermodynamic temperature, then thermodynamic temperature (kelvin temperature) does not depend on the property of a particular substance.

Thus, if a constant-volume gas thermometer gives a pressure reading of p_{tr} at the triple point, and a pressure reading of p at an unknown temperature T, the unknown temperature (in K) is given by

$$T = 273.16(p/p_{tr})$$

The Celsius scale

Why choose 273.16 as the number of units between the two fixed points of this scale? The reason is that this number will give 100 K between the ice- and steam-points, allowing agreement between the thermodynamic temperature scale and a centigrade scale based on the pressure of an ideal gas. The ideal-gas centigrade scale is based on experiments with real gases at decreasing pressures. This agreement is based on a slightly awkward linking up of the theoretical thermodynamic scale and the empirical constant-volume gas thermometer scale. To avoid this complication, a new scale, the Celsius scale, was defined by international agreement.

The unit of temperature on the Celsius scale is the degree Celsius (°C), which is exactly equal to the kelvin.

The equation linking temperature θ on the Celsius scale and thermodynamic temperature T is

$$\theta/°\text{C} = T/\text{K} - 273.15 \quad \text{or} \quad T/\text{K} = \theta/°\text{C} + 273.15$$

In this equation, θ is measured in °C and T in K. Note that the degree sign ° always appears with the Celsius symbol, but it is never used with the kelvin symbol K.

WORKED EXAMPLE 14A

The temperature of the liquid is measured as 35 °C. What is the temperature, to 3 significant figures, on the thermodynamic (kelvin) scale of temperature?

Answer

We use the equation $T/\text{K} = \theta/°\text{C} + 273.15$ but to 3 significant figures, this becomes

$$T/\text{K} = \theta/°\text{C} + 273$$

$$= 35 + 273 = \mathbf{308\ K}$$

Question

1 A block of metal is heated so that its temperature rises from 27°C to 150°C. Determine, to 3 significant figures on the thermodynamic (kelvin) scale of temperature:

a the temperature of 150°C

b the temperature rise of the block.

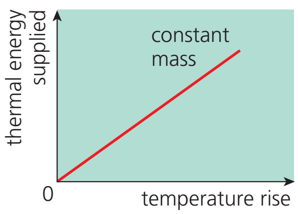
14.3 Specific heat capacity and specific latent heat

In this section, we will be looking at the effect of heating on temperature.

Specific heat capacity

When a solid, a liquid or a gas is heated, its temperature rises. Plotting a graph of thermal energy supplied against temperature rise (Figure 14.6), it is seen that the temperature rise $\Delta\theta$ is proportional to the thermal energy ΔQ supplied, for a particular mass of a particular substance.

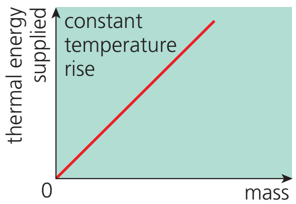
$$\Delta Q \propto \Delta\theta$$



▲ Figure 14.6

Similarly, the thermal energy required to produce a particular temperature rise is proportional to the mass m of the substance being heated (Figure 14.7, overleaf).

$$\Delta Q \propto m$$



▲ **Figure 14.7**

Combining these two relations gives

$$\Delta Q \propto m\Delta\theta$$

or

$$\Delta Q = mc\Delta\theta$$

where c is the constant of proportionality known as the **specific heat capacity** of the substance. In this case, *specific* means *per unit mass*.

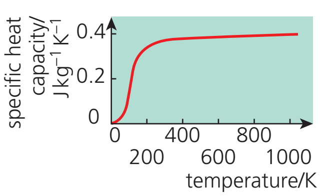
The numerical value of the specific heat capacity of a substance is the quantity of thermal energy required to raise the temperature of unit mass of the substance by one degree.

The SI unit of specific heat capacity is $\mathrm{J\,kg^{-1}\,K^{-1}}$. The unit of specific heat capacity is *not* the joule and this is why, in the definition of specific heat capacity, it is important to make reference to the *numerical value*. Specific heat capacity is different for different substances. Some values are given in Table 14.1.

The specific heat capacity of a substance is the **thermal energy per unit mass** required to raise the temperature of the substance by one degree.

material	specific heat capacity / $\mathrm{J\,kg^{-1}\,K^{-1}}$
ethanol	2500
glycerol	2420
ice	2100
mercury	140
water	4200
aluminium	913
copper	390
glass	640

▲ **Table 14.1 Values of specific heat capacity for different materials**



▲ **Figure 14.8**

It should be noted that, for relatively small changes in temperature, specific heat capacity is approximately constant. However, over a wide range of temperature, the value for a substance may vary considerably (Figure 14.8). Unless stated otherwise, specific heat capacity is assumed to be constant.

WORKED EXAMPLE 14B

Calculate the quantity of thermal energy required to raise the temperature of a mass of 590 g of copper from 25 °C to 90 °C. The specific heat capacity of copper is $390\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

Answer

thermal energy required $= m \times c \times \Delta\theta$

$= 0.59 \times 390 \times (90 - 25)$

$= \mathbf{1.5 \times 10^4\,J}$

Questions

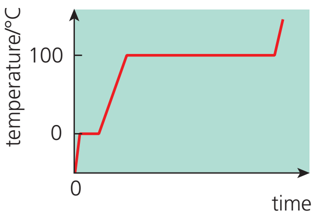
2 Calculate the thermal energy gained or lost for the following temperature changes. Use Table 14.1 to obtain values for specific heat capacity.

- a 1.4 kg of aluminium heated from 20 °C to 85 °C
- b 2.3 g of ice at 0 °C cooled to −18° C.

3 Calculate the specific heat capacity of water given that 0.25 MJ of energy are required to raise the temperature of a mass of 690 g of water by 86 K.

Specific latent heat

Figure 14.9 illustrates how the temperature of a mass of ice varies with time when it is heated at a constant rate to become steam.



▲ **Figure 14.9**

At times when the substance is changing phase (ice to water or water to steam), thermal energy (heat) is being supplied without any change of temperature. Because the heat transferred does not change the temperature of the substance as it changes state, it is said to be *latent* (i.e. hidden). The latent heat required to melt (fuse) a solid is known as **latent heat of fusion**.

The numerical value of the **specific latent heat of fusion** is the quantity of thermal energy required to convert unit mass of solid to liquid without any change in temperature.

Note: The meaning of 'fusion' is melting.

The SI unit of specific latent heat of fusion is $J\,kg^{-1}$. For a substance with latent heat of fusion L_f, the quantity of thermal energy ΔQ required to fuse (melt) a mass m of solid is given by

$$\Delta Q = mL_f$$

The latent heat required to vaporise a liquid without any change of temperature is referred to as **latent heat of vaporisation**.

The numerical value of the **specific latent heat of vaporisation** is the quantity of thermal energy required to convert unit mass of liquid to vapour without any change in temperature.

The SI unit of specific latent heat of vaporisation is the same as that for fusion, i.e. $J\,kg^{-1}$. For a substance with latent heat of vaporisation L_v, the quantity of thermal energy ΔQ required to vaporise a mass m of liquid is given by

$$\Delta Q = mL_v$$

When a vapour condenses (vapour becomes liquid), the latent heat of vaporisation is released. Similarly, when a liquid solidifies (liquid becomes solid), the latent heat of fusion is released. Some values of specific latent heat of fusion and of vaporisation are given in Table 14.2.

The specific latent heat of a substance is the **quantity of heat per unit mass** required to change the state of a substance at constant temperature.

material	specific latent heat of fusion $kJ\,kg^{-1}$	specific latent heat of vaporisation $kJ\,kg^{-1}$
ice/water	330	2260
ethanol	108	840
copper	205	4840
sulfur	38.1	

▲ **Table 14.2 Values of specific latent heat**

Note that the unit $kJ\,kg^{-1}$ is numerically equal to the unit $J\,g^{-1}$.

It can be seen that, for the same mass, specific latent heat of vaporisation is significantly greater than specific latent heat of fusion. For fusion (melting), the thermal energy enables the strong forces that make the solid rigid between atoms/molecules to be overcome. Forces between atoms/molecules still exist in the liquid and volume does not change significantly. In the case of vaporisation, the atoms/molecules are completely separated so that forces between atoms/molecules are negligible, thus a greater amount of energy is required to overcome the forces that hold the liquid together. Furthermore, volume increases significantly (1 cm^3 of liquid water becomes approximately 1800 cm^3 of steam at atmospheric pressure), resulting in additional thermal energy transfer to do work against the atmosphere.

WORKED EXAMPLE 14C

Use the information given in Table 14.2 to determine the thermal energy required to melt 78 g of sulfur at its normal melting point.

Answer

$$\text{heat energy required} = m \times L_f$$
$$= 0.078 \times 38.1$$
$$= \mathbf{3.0\,kJ = 3000\,J}$$

Question

4 Where appropriate, use the information given in Table 14.2.

a Calculate the thermal energy required to:

i melt 76 g of ice at 0 °C

ii evaporate 76 g of water at 100 °C.

b Using your answers to **a**, determine how many times more energy is required to evaporate a mass of water than to melt the same mass of ice.

Exchanges of thermal energy

When a hot object and a cold object come into contact, thermal energy passes from the hot object to the cold one so that the two objects reach the same temperature. The law of conservation of energy applies in that the thermal energy gained by the cold object is equal to the thermal energy lost by the hot object. This does, of course, assume that no energy is lost to the surroundings. This simplification enables temperatures to be calculated.

WORKED EXAMPLE 14D

1 A mass of 0.30 kg of water at 95 °C is mixed with 0.50 kg of water at 20 °C. Calculate the final temperature of the water, given that the specific heat capacity of water is 4200 J kg^{-1} K^{-1}.

2 A mass of 15 g of ice at 0 °C is placed in a drink of mass 240 g at 25 °C. Calculate the final temperature of the drink, given that the specific latent heat of fusion of ice is 334 kJ kg^{-1} and that the specific heat capacity of water and the drink is 4.2 kJ kg^{-1} K^{-1}.

Answers

1 *Hint*: Always start by writing out a word equation containing all the gains and losses of heat energy.

heat energy lost by hot water = heat energy gained by cold water

$(m \times c \times \theta_1) = (M \times c \times \theta_2)$

$0.30 \times 4200 \times (95 - \theta) = 0.50 \times 4200 \times (\theta - 20)$

where θ is the final temperature of the water.

$1260 \times (95 - \theta) = 2100 \times (\theta - 20)$
$119\,700 - 1260\theta = 2100\theta - 42\,000$
$161\,700 = 3360\theta$
$\theta = 48\,°C$

2 energy lost by drink = energy gained by melting ice + energy gained by ice water
$(m \times c \times \Delta\theta_1) = (M \times L_f) + (M \times c \times \Delta\theta_2)$
$0.240 \times 4.2 \times (25 - \theta) = (0.015 \times 334) + (0.015 \times 4.2 \times \{\theta - 0\})$
where θ is the final temperature of the drink. Simplifying,
$\theta = 19\,°C$

Questions

Use the data in Table 14.1 and in Table 14.2 where appropriate.

5 A lump of copper of mass 120 g is heated in a gas flame. It is then transferred to a mass of 450 g of water, initially at 20 °C. The final temperature of the copper and the water is 31 °C. Calculate the temperature of the gas flame.

6 Steam at 100 °C is passed into a mass of 450 g of water, initially at 25 °C. The steam condenses. Calculate the mass of steam required to raise the temperature of the water to 80 °C.

SUMMARY

- Thermal energy is transferred from a region of high temperature to one of lower temperature.
- Two regions that are at the same temperature are said to be in thermal equilibrium.
- A physical property that varies with temperature may be used to measure temperature. These properties include the density of a liquid, volume of a gas, resistance of a metal, e.m.f. of a thermocouple.
- Thermodynamic (kelvin or absolute) temperature does not depend on the property of any particular substance.
- Celsius scale: $\theta = T - 273.15$, where θ is the Celsius temperature (in °C) and T is the thermodynamic temperature (in K).
- The absolute zero of temperature is the lowest possible temperature and is zero kelvin.
- Melting, boiling and evaporation are all examples of changes of phase (solid to liquid, and liquid to vapour).
- All these changes of phase require an input of energy (latent heat) to overcome the interatomic forces.
- Specific heat capacity is the thermal energy per unit mass required to raise the temperature of the substance by one degree.
- The SI unit of specific heat capacity is $J\,kg^{-1}\,K^{-1}$.
- The thermal energy ΔQ required to raise the temperature of a mass m of substance of specific heat capacity c by an amount $\Delta\theta$ is given by the expression: $\Delta Q = mc\Delta\theta$.
- Specific latent heat of fusion is the quantity of thermal energy per unit mass required to convert mass of solid to liquid without any change in temperature.
- Specific latent heat of vaporisation is the quantity of thermal energy per unit mass required to convert mass of liquid to vapour without any change in temperature.
- When a substance of mass m changes its state the quantity of thermal energy required is given by $\Delta Q = mL$ where L is the appropriate specific latent heat.
- Specific latent heat has the SI unit of $J\,kg^{-1}$.

END OF TOPIC QUESTIONS

In the following questions, use the data in Table 14.1 and in Table 14.2 where appropriate.

1 A piece of aluminium of mass 260 g is cooled in a freezer. It is then dropped into 140 g of water at 5 °C, causing 4.0 g of water to freeze. Determine the temperature inside the freezer.

2 **a** A jet of steam at 100 °C is directed into a hole in a large block of ice. After the steam has been switched off, the condensed steam and the melted ice are both at 0 °C. The mass of water collected in the hole is 206 g. Calculate the mass of steam condensed.

b Suggest why a scald with steam is much more serious than one involving boiling water.

3 A kettle, rated at 2.8 kW, contains a mass of 465 g of water at a temperature of 24 °C.

The kettle is switched on for a time of 2.5 minutes. During that time, 92% of the input energy is transferred to the water. Calculate the mass of water that is evaporated.

4 **a** Define *specific latent heat.* **[2]**

b The heater in an electric kettle has a power of 2.40 kW.

When the water in the kettle is boiling at a steady rate, the mass of water evaporated in 2.0 minutes is 106 g.

The specific latent heat of vaporisation of water is 2260 J g^{-1}.

Calculate the rate of loss of thermal energy to the surroundings of the kettle during the boiling process. **[3]**

Cambridge International AS and A Level Physics (9702) Paper 41 Q3 May/June 2012

15 Ideal gases

Learning outcomes

By the end of this topic, you will be able to:

15.1 The mole

1 understand that amount of substance is an SI base quantity with the base unit mol

2 use molar quantities where one mole of any substance is the amount containing a number of particles of that substance equal to the Avogadro constant N_A

15.2 Equation of state

1 understand that a gas obeying $pV \propto T$, where T is the thermodynamic temperature, is known as an ideal gas

2 recall and use the equation of state for an ideal gas expressed as $pV = nRT$, where n = amount of substance (number of moles) and as $pV = NkT$, where N = number of molecules

3 recall that the Boltzmann constant k is given by $k = R/N_A$

15.3 Kinetic theory of gases

1 state the basic assumptions of the kinetic theory of gases

2 explain how molecular movement causes the pressure exerted by a gas and derive and use the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$, where $\langle c^2\rangle$ is the mean-square speed (a simple model considering one-dimensional collisions and then extending to three dimensions using $\frac{1}{3}\langle c^2\rangle = \langle c_x^2\rangle$ is sufficient)

3 understand that the root-mean-square speed $c_{r.m.s.}$ is given by $\sqrt{\langle c^2\rangle}$

4 compare $pV = \frac{1}{3}Nm\langle c^2\rangle$, with $pV = NkT$ to deduce that the average translational kinetic energy of a molecule is $\frac{3}{2}kT$

Starting points

- ★ All matter consists of atoms or groups of atoms called molecules.
- ★ The spacing of these atoms/molecules and the forces between them determine whether the matter is solid, liquid or gas.
- ★ In a gas, the forces between atoms or molecules are negligible. On average the particles are far apart and in rapid, random motion with kinetic energy that depends on temperature.
- ★ Gases have no fixed volume and no fixed shape.

15.1 The mole

In Topic 1, we saw that SI is founded on fundamental (base) quantities and their **base units**. These fundamental quantities used so far in the Cambridge International AS & A Level Physics course include mass, length, time, electric current and thermodynamic temperature.

One additional base quantity is **amount of substance** with the base unit **mol**.

The **mole** (abbreviated mol) is the amount of substance which contains $6.02214076 \times 10^{23}$ elementary entities, usually atoms or molecules but could also be ions or electrons.

The **Avogadro constant** N_A is the number of elementary entities in 1 mole of any substance.

The numerical value of the Avogadro constant is usually given to three significant figures as 6.02×10^{23}. Thus, $N_A = 6.02 \times 10^{23}\,mol^{-1}$.

The mass of 1 mole of substance is known as the **molar mass**.

WORKED EXAMPLE 15A

1 The molar mass of a sample of neon is 20.0 g. Calculate the number of atoms in a sample of 2.5 g of this neon.

2 A gas cylinder contains 1.51×10^{24} molecules of oxygen. The mass of 1 mol of oxygen atoms is 16.0 g.

   a Determine the amount, in mol, of oxygen molecules.

   b Determine the mass of oxygen present.

Answers

1 A mass of 20.0 g of the sample of neon contains the Avogadro number of atoms. So, 2.5 g contains $2.5/20.0 \times 6.02 \times 10^{23}$ = **7.53×10^{22} atoms**.

2 a amount (mol) = number of particles/Avogadro constant
   = $1.51 \times 10^{24}/6.02 \times 10^{23}$ = 2.51 mol

   b mass = number of particles × mass of one particle
   = 2.51×32.0 (since 1 oxygen molecule contains 2 atoms)
   = 80.3 g

Question

1 The mass of 1 mol of nitrogen atoms is 14.0 g. Determine:

   a the mass, in kg, of 1 atom of nitrogen

   b the number of molecules of nitrogen present in a cylinder containing 290 g nitrogen.

15.2 Equation of state

Experiments in the seventeenth and eighteenth centuries showed that the volume, pressure and temperature of a given sample of gas are all related.

For a given mass of gas, Robert Boyle (1627–91) found that the volume V of a gas is inversely proportional to its pressure p, provided that the temperature is held constant.

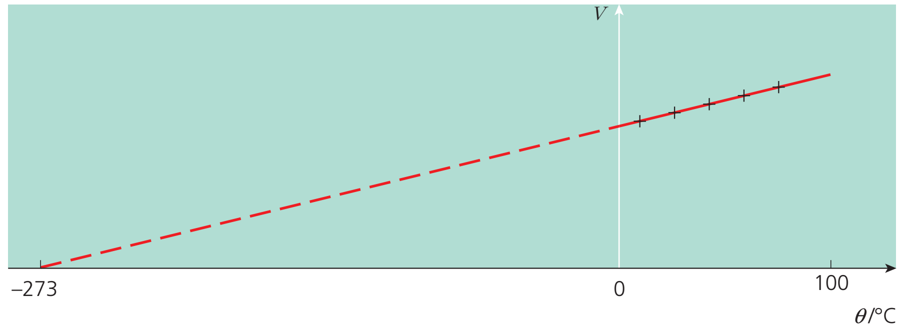
This relation is known as **Boyle's law**.

Expressed mathematically

$$p_1V_1 = p_2V_2$$

where p_1 and V_1 are the initial pressure and volume of the gas, and p_2 and V_2 are the final values after a change of pressure and volume is carried out for a fixed mass of gas at constant temperature.

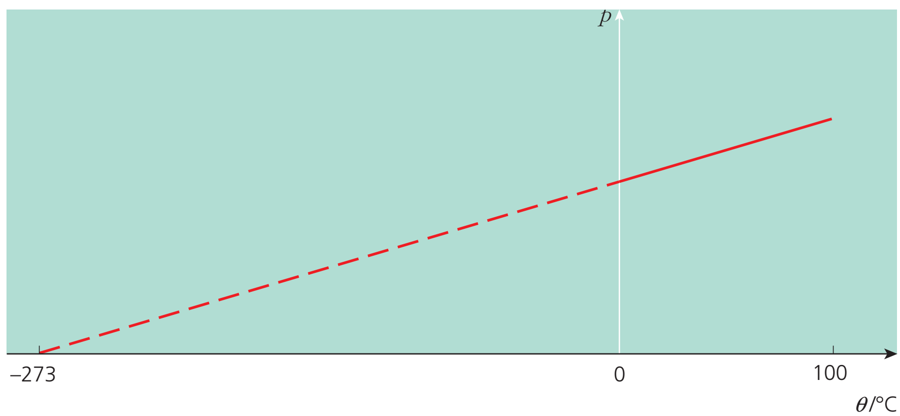
The effect of temperature on the volume of a gas was investigated by the French scientist Jacques Charles (1746–1823). Charles found that the graph of volume V against temperature θ is a straight line (see Figure 15.1). Because gases liquefy when the temperature is reduced, experimental points could not be obtained below the liquefaction temperature. But if the graph was projected backwards, it was found that it cut the temperature axis at about −273°C.



▲ **Figure 15.1 Graph of *V* against *θ***

The effect of temperature on the pressure of a gas was investigated by another Frenchman, Joseph Gay-Lussac (1778–1850).

The graph of pressure p against temperature θ is a straight line, which, if projected like the volume–temperature graph, also meets the temperature axis at about –273°C (Figure 15.2). This fact was used in Topic 14.2 to introduce the thermodynamic scale of temperature and the idea of the absolute zero of temperature.



▲ **Figure 15.2 Graph of *p* against *θ***

If Celsius temperatures are converted to thermodynamic temperatures T, Charles' results can be expressed as **Charles' law**.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 and T_1 are the initial volume and temperature, and V_2 and T_2 are the final values. The pressure is constant.

The corresponding relation between pressure and thermodynamic temperature of a fixed mass of gas at constant volume is given by **Gay-Lussac's law**, or the **law of pressures**. That is,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

The volume is constant.

We can combine the three gas laws into a single relation between pressure p, volume V and thermodynamic temperature T for a fixed mass of gas. This relation is

$$pV \propto T$$

or

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

Strictly speaking, the laws of Boyle, Charles and Gay-Lussac are not really laws, as their validity is restricted. They are accurate for real gases only if the pressure of the sample is not too great, and if the gas is well above its liquefaction temperature. However, they can be used to define an 'ideal' gas.

An **ideal gas** is one which obeys the equation of state $pV \propto T$ at all pressures p, volumes V and thermodynamic temperatures T.

Note that, when defining what is meant by an ideal gas, the temperature must be stated to be the thermodynamic temperature.

For approximate calculations, the ideal gas equation can be used with real gases if the gas is well above the temperature at which it would liquefy and the pressure is not high.

The laws relate to a fixed mass of gas. Another series of experiments could be carried out to find out how the volume of a gas, held at constant pressure and temperature, depends on the mass of gas present. It would be found that the volume is proportional to the mass. This would give the combined relation

$$pV \propto mT$$

where m is the mass of gas, or

$$pV = AmT$$

where A is a constant of proportionality. However, this is not a very useful way of expressing the relation, as the constant A has different values for different gases. We need to find a way of including the quantity of gas. The way to do this is to express the fixed mass of gas in Boyle's, Charles' and Gay-Lussac's laws in terms of the number of moles of gas present.

Combining the three gas laws and using the number n of moles of the gas, we have

$$pV \propto nT$$

or, putting in a new constant of proportionality R,

$$pV = nRT$$

R is called the **molar gas constant** (sometimes called the **universal gas constant**, because it has the same value for all gases). It has the value $8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

The quantity n is the number of moles of gas and is a constant for a fixed mass of gas.

The equation $pV = nRT$ is known as the **universal gas equation** or the **equation of state for an ideal gas**.

Sometimes the equation $pV = nRT$ is expressed in the form

$$pV = NkT$$

where N is the number of molecules in the gas and k is a constant called the **Boltzmann constant**. The Boltzmann constant has the value $1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$. Note that the molar gas constant R and the Boltzmann constant k are connected through the Avogadro constant N_A.

$$k = R/N_A$$

WORKED EXAMPLE 15B

1 Calculate the volume occupied by 1 mole of air at standard temperature and pressure (273 K and 1.01×10^5 Pa), taking R as $8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$ for air.

2 Calculate the number of molecules per cubic metre of air at standard temperature and pressure.

3 A syringe contains $25 \times 10^{-6}\,\mathrm{m}^3$ of helium gas at a temperature of 20°C and a pressure of 5.0×10^4 Pa. The temperature is increased to 400°C and the pressure on the syringe is increased to 2.4×10^5 Pa. Find the new volume of gas in the syringe.

Answers

1 Since $pV = nRT$, $V = nRT/p$. Substituting the values $n = 1$ mol, $R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$, $T = 273$ K and $p = 1.01 \times 10^5$ Pa, $V = (1 \times 8.31 \times 273)/1.01 \times 10^5$
$= \mathbf{2.25 \times 10^{-2}\,m^3}$.

2 We have just shown that the volume occupied by one mole of air at standard temperature and pressure is $2.25 \times 10^{-2}\,\mathrm{m}^3$. One mole of air contains N_A molecules, where N_A is the Avogadro constant ($6.02 \times 10^{23}\,\mathrm{mol^{-1}}$). Thus the number of molecules per cubic metre of air is $6.02 \times 10^{23}/2.25 \times 10^{-2}$
$= \mathbf{2.68 \times 10^{25}\,m^{-3}}$.
Note: It is useful to remember these two quantities, the *molar volume* of a gas and the *number density* of molecules in it. They give an idea of the relatively small volume occupied by a mole of gas at standard temperature and pressure (a cube of side about 28 cm), and the enormous number of molecules in every cubic metre of a gas under these conditions. See also question 2 below.

3 Assuming the gas to be ideal, then the gas equation is given as pV/T = constant. This is written in the form $p_1V_1/T_1 = p_2V_2/T_2$ and re-arranged as $V_2 = p_1V_1T_2/p_2T_1$. Substituting the values $p_1 = 5.0 \times 10^4$ Pa, $V_1 = 25 \times 10^{-6}\,\mathrm{m}^3$, $T_2 = 673$ K, $p_2 = 2.4 \times 10^5$ Pa, $T_1 = 293$ K (again note that temperatures are converted from °C to K),
$V_2 = (5.0 \times 10^4 \times 25 \times 10^{-6} \times 673)/(2.4 \times 10^5 \times 293)$
$= \mathbf{12 \times 10^{-6}\,m^3}$.

Questions

2 The number of molecules per cubic metre of air at standard temperature and pressure is about $2.7 \times 10^{25}\,\mathrm{m}^{-3}$. What is the average separation of these molecules?

3 The mean mass of one mole of air (which is made up mainly of nitrogen, oxygen and argon) is 0.029 kg. What is the density of air at standard temperature and pressure, that is, 273 K and 1.01×10^5 Pa? ($R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$.)

4 The volume of a sample of gas is $3.2 \times 10^{-2}\,\mathrm{m}^3$ when the pressure is 8.6×10^4 Pa and the temperature is 27°C. How many moles of gas are there in the sample? How many molecules? What is the number of molecules per cubic metre? ($R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$.)

5 A sample of air has volume $30 \times 10^{-6}\,\mathrm{m}^3$ when the pressure is 4.0×10^5 Pa. The pressure is reduced to 1.5×10^5 Pa, without changing the temperature. What is the new volume?

6 A sample of gas, originally at standard temperature and pressure (273 K and 1.01×10^5 Pa), has volume $4.5 \times 10^{-5}\,\mathrm{m}^3$ under these conditions. The pressure is increased to 5.87×10^5 Pa and the temperature rises to 34°C. Calculate the new volume.

15.3 Kinetic theory of gases

One of the aims of Physics is to describe and explain the behaviour of various systems. For mechanical systems, this involves calculating the motion of the parts of the system in detail. For example, we have already seen how to predict the motion of a stone thrown in a uniform gravitational field. Using the equations of uniformly accelerated motion, it is not too difficult to calculate the position and velocity of the stone at any time (Topic 2.1).

However, there are some cases in which it is quite impossible to describe what happens to each component of the system.

This sort of problem arises if we try to describe the properties of a gas in terms of the motion of each of its molecules. The difficulty is that the numbers are so large. One cubic metre of atmospheric air contains about 3×10^{25} molecules. There is no practical method of determining the position and velocity of every single molecule at a given time. Even the most advanced computer would be unable to handle the calculation of the motions of such a very large number of molecules.

In some ways, the fact that the gas is made up of such an enormous number of molecules is an advantage. It means that we can give a large-scale description of the gas in terms of only a few variables. These variables are quantities such as pressure p, volume V and thermodynamic temperature T. They tell us about average conditions in the gas, instead of describing the behaviour of each molecule. We have already met the experimental laws relating to these quantities. Our aim now is to relate the ideal gas equation, which deals with the large-scale (macroscopic) quantities p, V and T, to the small-scale (microscopic) behaviour of the particles of the gas. We shall do this by taking averages over the very large numbers of molecules involved. We shall find that we can derive the equation for Boyle's law when we make very simple assumptions about the atoms or molecules which make up the gas. This is the **kinetic theory of an ideal gas**. We shall also see that temperature can be related to the kinetic energy of the molecules of the gas.

The kinetic theory

An explanation of how a gas exerts a pressure was developed by Robert Boyle in the seventeenth century and, in greater detail, by Daniel Bernoulli in the eighteenth century. The basic idea was that the gas consists of atoms or molecules randomly moving about at great speed (later visualised by Robert Brown).

When a gas molecule hits the wall of its containing vessel, it rebounds. There is a change in momentum of the molecule and it experiences an impulse. By Newton's third law, the wall of the vessel also experiences an impulse. There are many collisions per unit time of molecules with the wall and all these collisions and the associated impulses average out to give a constant force on the wall. Force per unit area is pressure.

We shall make some simplifying assumptions about the molecules of the gas. The assumptions of the kinetic theory of an ideal gas are:

- All molecules behave as identical, hard, perfectly elastic spheres.
- The volume of the molecules is negligible compared with the volume of the containing vessel.
- There are no forces of attraction or repulsion between molecules.
- There are many molecules, all moving randomly.

The number of molecules must be very large, so that average behaviour can be considered.

Suppose that the container is a cube of side L (Figure 15.3). The motion of each molecule can be resolved into x-, y- and z-components. For convenience we shall take the x-, y- and z-directions to be parallel to the edges of the cube.

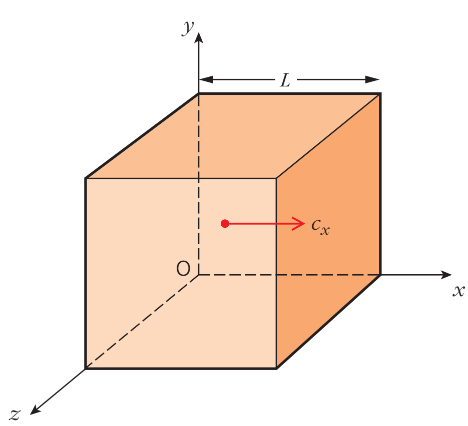


Figure 15.3 A gas molecule in a cubic container

Consider one molecule. Let the x-component of its velocity be c_x. When this molecule collides with the wall of the container perpendicular to the x-axis, the x-component of velocity will be exactly reversed, because (from our assumptions) the collision of the molecule with the wall is perfectly elastic. The time taken for the molecule to move between the two walls perpendicular to the x-axis is L/c_x. To make the round trip from one wall to the opposite one and back again takes $2L/c_x$. This is the time between one collision of the molecule with a wall and its next collision with the same wall.

When the molecule strikes a wall, the component of velocity is reversed in direction, from c_x to $-c_x$. Thus, during each collision with a wall, the x-component of momentum changes by

$$\Delta p_x = 2mc_x$$

where m is the mass of the molecule. The rate at which this molecule changes momentum at the wall is

$$\text{change of momentum/time between collisions} = 2mc_x/(2L/c_x) = mc_x^2/L$$

From Newton's second law, this rate of change of momentum is the average force exerted by this particular molecule on the wall through its collisions with the wall. If there are N molecules in the container, the total force is Nmc_x^2/L. Pressure is force divided by area, and the area of the wall is L^2, so the pressure exerted by all N molecules on this wall is Nmc_x^2/L^3.

The volume V of the container is L^3, giving us

$$p = Nmc_x^2/V$$

as an expression for the pressure. This expression relates only to the x-component of velocity of the molecules. For a molecule moving with velocity c, an extension of Pythagoras' theorem to three dimensions gives the relation between c and the three components of velocity c_x, c_y and c_z as $c^2 = c_x^2 + c_y^2 + c_z^2$. Because we are dealing with a large number of molecules in random motion, the average value of the component in the x-direction will be the same as for those in the y-direction or in the z-direction. Therefore, taking the averages, $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$ and $\langle c_x^2 \rangle = 1/3 \langle c^2 \rangle$. The notation $\langle c_x^2 \rangle$. means the average value of c_x^2. Our expression for the pressure can now be written as

$$pV = \frac{1}{3} Nm\langle c^2 \rangle$$

Since N is the total number of molecules in the container, then Nm is the total mass of gas and Nm/V is the density ρ of the gas. So,

$$p = \frac{1}{3}\rho\langle c^2 \rangle$$

Since the average kinetic energy of a molecule is

$$\langle E_k \rangle = \frac{1}{2}m\langle c^2 \rangle$$

there seems to be a link between our kinetic theory equation for pV and energy. We can find this relation by re-writing the equation $pV = \frac{1}{3} Nm\langle c^2 \rangle$ as

$$pV = \frac{2}{3}N\left(\frac{1}{2}m\langle c^2 \rangle\right) = \frac{2}{3}N\left(E_k\right)$$

We have already stated the ideal gas equation in the forms $pV = nRT = NkT$. Indeed, even real gases obey this law reasonably well under normal conditions. If our kinetic theory model and the subsequent theoretical derivation of the equation are correct, we can bring together the two equations for pV. This will allow us to relate the temperature of a gas to the average kinetic energy of its molecules.

$$pV = \tfrac{2}{3}N\left(\tfrac{1}{2}m <c^2>\right) = \tfrac{2}{3}N<E_k> = NkT$$

and

$$<E_k> = \tfrac{1}{2}m <c^2> = \tfrac{3}{2}kT$$

This is an important result. We have derived a relation between the average kinetic energy of a molecule in a gas and the thermodynamic temperature of the gas. This will allow us to obtain an idea of the average speed of the molecules. Since

$$<E_k> = \tfrac{1}{2}m <c^2> = \tfrac{3}{2}kT$$

we have

$$<c^2> = \frac{3kT}{m}$$

and

$$\sqrt{<c^2>} = \sqrt{\left(\frac{3kT}{m}\right)}$$

The quantity $\sqrt{<c^2>}$ is called the **root-mean-square speed or r.m.s. speed** of the molecules. It is not exactly equal to the average speed of the molecules, but is often taken as being so. The average speed is about 0.92 of the root-mean-square speed. The difference between the r.m.s. speed and the average speed is highlighted in Worked Example 15C.

Note that the r.m.s. speed is proportional to the square root of the thermodynamic temperature of the gas, and inversely proportional to the square root of the mass of the molecule. Thus, at a given temperature, less massive molecules move faster, on average, than more massive molecules. For a given gas, the higher the temperature, the faster the molecules move.

WORKED EXAMPLE 15C

The speeds of seven molecules in a gas are numerically equal to 2, 4, 6, 8, 10, 12 and 14 units. Determine the numerical values of:

a the mean speed $<c>$,

b the mean speed squared $<c>^2$,

c the mean-square speed $<c^2>$,

d the r.m.s. speed.

Answers

a $<c> = (2 + 4 + 6 + 8 + 10 + 12 + 14)/7 =$ **8.0 units**

b $<c>^2 = 8^2 =$ **64 units²**

c $<c^2> = (4 + 16 + 36 + 64 + 100 + 144 + 196)/7 =$ **80 units²**

d r.m.s. speed = **8.9 units**

WORKED EXAMPLE 15D

Calculate the total kinetic energy of the molecules in one mole of an ideal gas at standard temperature (273 K).

Answer

We know that the average kinetic energy of one molecule is $\frac{3}{2}kT$.

For one mole of molecules, that is N_A molecules, the energy is

$$\frac{3}{2}N_A kT = \frac{3}{2}RT$$

Substituting the values $R = 8.3\,\text{J K}^{-1}\,\text{mol}^{-1}$ and $T = 273\,\text{K}$, we have

$$E_k = 1.5 \times 8.3 \times 273 = \mathbf{3400\,J}$$

WORKED EXAMPLE 15E

The mass of a nitrogen molecule is 4.6×10^{-26} kg. Determine the root-mean-square speed of the molecules in nitrogen gas at 27°C.

Answer

Since $c_{rms} = \sqrt{(3kT/m)}$ we have

$$c_{rms} = \sqrt{(3 \times 1.38 \times 10^{-23} \times 300/4.6 \times 10^{-26})} = \mathbf{520\,m\,s^{-1}}$$

Questions

7 Determine the average kinetic energy of a molecule in an ideal gas at a temperature of 260°C.

8 Calculate the root-mean-square speed of the molecules in neon-20 gas at 50°C. Neon-20 gas may be assumed to be an ideal gas.

SUMMARY

- Amount of substance is an SI base quantity with the base unit of moles (mol).
- One mole of any substance contains a number of particles equal to the Avogadro constant.
- The pressure exerted by a gas on the walls of its container is caused by collisions of gas molecules with the walls.
- An ideal gas obeys the expression $pV \propto T$ where T is the thermodynamic temperature.
- The equation of state for an ideal gas relates the pressure p, volume V and thermodynamic temperature T of n moles of gas: $pV = nRT$ where $R = 8.3\,\text{J K}^{-1}\,\text{mol}^{-1}$, the molar gas constant.
- For N molecules of gas: $pV = NkT$ where $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$, the Boltzmann constant.
- The relation between R and k is $k = R/N_A$ where $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, the Avogadro constant.
- The assumptions of the kinetic theory of gases are
  - molecules behave as identical, hard, perfectly elastic spheres
  - volume of the molecules is negligible compared with the volume of the containing vessel
  - there are no forces of attraction or repulsion between molecules
  - there are many molecules, all moving randomly.
- The mean-square speed of a large number of molecules is represented by $<c^2>$.
- The kinetic theory equation: $pV = \frac{1}{3}Nm<c^2>$ can be derived using Newton's laws of motion. A molecule's change of momentum when it collides with the walls of its container gives rise to a force exerted by the wall and hence to pressure.
- Average kinetic energy of a molecule is directly proportional to thermodynamic temperature: $<E_k> = \frac{1}{2}m<c^2> = \frac{3}{2}kT$.
- The root-mean-square speed or r.m.s. speed (c_{rms}) of a large number of molecules is given by $\sqrt{<c^2>}$.
- Root-mean-square speed of molecules is found from $c_{rms} = \sqrt{(3kT/m)}$.

END OF TOPIC QUESTIONS

1 On a day when the atmospheric pressure is 102 kPa and the temperature is 8°C, the pressure in a car tyre is 190 kPa above atmospheric pressure. After a long journey the temperature of the air in the tyre rises to 29°C. Calculate the pressure above atmospheric of the air in the tyre at 29°C. Assume that the volume of the tyre remains constant.

2 A helium-filled balloon is released at ground level, where the temperature is 17°C and the pressure is 1.0 atmosphere. The balloon rises to a height of 2.5 km, where the pressure is 0.75 atmospheres and the temperature is 5°C. Calculate the ratio of the volume of the balloon at 2.5 km to that at ground level.

3 Estimate the root-mean-square speed of helium atoms near the surface of the Sun, where the temperature is about 6000 K. (Mass of helium atom = 6.6×10^{-27} kg.)

4 **a** An ideal gas is said to consist of molecules that are hard elastic identical spheres.
State two further assumptions of the kinetic theory of gases. **[2]**

b The number of molecules per unit volume in an ideal gas is n.
If it is assumed that all the molecules are moving with speed v, the pressure p exerted by the gas on the walls of the vessel is given by

$p = \frac{1}{3}nmv^2$

where m is the mass of one molecule.
Explain the reasoning by which this expression is modified to give the formula

$p = \frac{1}{3}nm\langle c^2\rangle$ **[1]**

c The density of an ideal gas is 1.2 kg m^{-3} at a pressure of 1.0×10^5 Pa and a temperature of 207°C.

i Calculate the root-mean-square (r.m.s.) speed of the molecules of the gas at 207°C. **[3]**

ii Calculate the mean-square speed of the molecules at 207°C **[2]**

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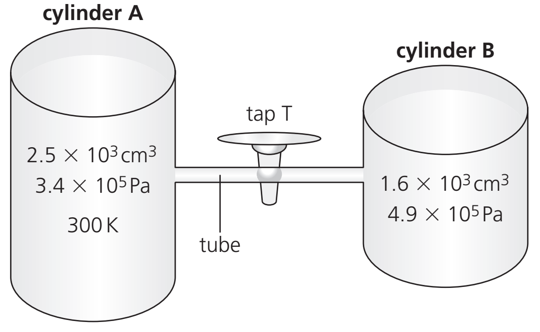
5 **a** State what is meant by an *ideal gas*. **[3]**

b Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 15.4.



▲ **Figure 15.4**

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

i Cylinder A has a constant volume of 2.5×10^3 cm^3 and contains a gas at pressure 3.4×10^5 Pa and temperature of 300 K.
Show that cylinder A contains 0.34 mol of gas. **[1]**

ii Cylinder B has a constant volume of $1.6 \times 10^3\,cm^3$ and contains 0.20 mol of gas.
When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5\,Pa$.
No thermal energy enters or leaves the gas.
Determine the final temperature of the gas. **[2]**

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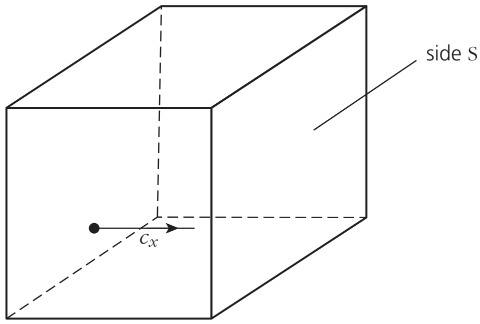
6 a The kinetic theory of gases is based on some simplifying assumptions.
The molecules of the gas are assumed to behave as hard elastic identical spheres.
State the assumption about idea gas molecules based on:

i the nature of their movement, **[1]**

ii their volume. **[2]**

b A cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_x of velocity normal to one side S of the cube, as shown in Fig. 15.5.



▲ **Figure 15.5**

The pressure p of the gas due to the component c_x of velocity is given by the expression

$$pV = Nmc_x^{\,2}$$

where m is the mass of a molecule.
Explain how the expression leads to the relation

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

where $\langle c^2\rangle$ is the mean square speed of the molecules. **[3]**

c The molecules of an ideal gas have a root-mean-square (r.m.s.) speed of $520\,m\,s^{-1}$ at a temperature of 27°C.
Calculate the r.m.s. speed of the molecules at a temperature of 100°C. **[3]**

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16 Thermodynamics

Learning outcomes

By the end of this topic, you will be able to:

16.1 Internal energy

1. understand that internal energy is determined by the state of the system and that it can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of a system
2. relate a rise in temperature of an object to an increase in its internal energy

16.2 The first law of thermodynamics

1. recall and use $w = p\Delta V$ for the work done when the volume of a gas changes at constant pressure and understand the difference between the work done by the gas and the work done on the gas
2. recall and use the first law of thermodynamics $\Delta U = q + w$ expressed in terms of the increase in internal energy, the heating of the system (energy transferred to the system by heating) and the work done on the system

Starting points

- ★ The differences between the three states of matter are due to the arrangement and spacing of the particles and their motion.
- ★ When work is done on a system energy is transferred.
- ★ The collisions of gas molecules with the walls of a container cause pressure.
- ★ Pressure = force/area.
- ★ Work done = force × distance moved by the force in the direction of the force.
- ★ For an ideal gas there are no intermolecular forces.
- ★ Potential energy is the energy stored in an object due to its position or shape.

16.1 Internal energy

We have seen that the molecules of a gas possess kinetic energy, and that for an ideal gas the mean kinetic energy of molecules is proportional to the thermodynamic temperature of the gas (Topic 15.3).

Not all molecules have the same kinetic energy, because they are moving with different speeds, but the sum of all the kinetic energies will be a constant if the gas is kept at a constant temperature.

For a real gas, the molecules also possess potential energy. Because the molecules exert forces on each other, at any instant there will be a certain potential energy associated with the positions that the molecules occupy in space. Because the molecules are moving, the potential energy of a given molecule will also vary. But at a given temperature the total potential energy of all of the molecules will remain constant. If the temperature changes, the total potential energy will also change. Furthermore, in a gas, the molecules collide with each other and will interchange kinetic energy during the collisions.

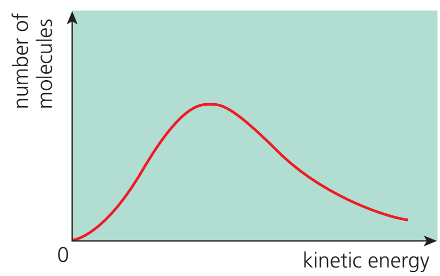
For a gas, the internal energy is defined as the sum of the potential energies and the kinetic energies of all the molecules.

The sum of the potential energies and kinetic energies of all the molecules, owing to their random motion, is called the **internal energy** of the gas.

For an ideal gas there are no intermolecular forces so no potential energies.

The internal energy of an ideal gas is due only to the kinetic energies of its molecules.

It is important to realise that looking at a single molecule will give us very little information. In a gas, the kinetic energy will be changing all the time as the molecule collides with other gas molecules, and its potential energy is also changing as its position relative to the other molecules in the gas changes. This single molecule has a kinetic energy which is part of the very wide range of kinetic energies of the molecules of the gas.



▲ **Figure 16.1 Distribution of molecular kinetic energies**

We say that there is a *distribution* of molecular kinetic energies. The distribution is illustrated in Figure 16.1. Similarly, there is a distribution of molecular potential energies. But by adding up the kinetic and potential energies of all the molecules in the gas, the random nature of the kinetic and potential energies of the single molecule is removed.

Internal energy for solids and liquids

The idea of internal energy can be extended to all states of matter. In a liquid, intermolecular forces are stronger as the molecules are closer together, so the potential energy contribution to internal energy becomes more significant. The kinetic energy contribution is still due to the random motion of the molecules in the liquid. In a solid, we can think of the solid as being made up of atoms or molecules which oscillate (vibrate) about equilibrium positions. Here, the potential energy contribution is caused by the strong binding (attractive) forces between atoms, and the kinetic energy contribution is due to the motion of the vibrating atoms.

Thus, the amount of internal energy within a system is determined by the state of the system.

The amount of internal energy is determined by the state of the system and can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of the system.

Thermal energy and temperature

The concept of internal energy is particularly useful as it helps us to distinguish between temperature and heat (thermal energy). Using an ideal gas as an example, temperature is a measure of the average (translational) kinetic energy of the molecules. It, therefore, does not depend on how many molecules are present in the gas. Internal energy (again for an ideal gas), however, is the *total* kinetic energy of the molecules, and clearly does depend on how many molecules there are. In general, a rise in temperature causes an increase in the kinetic energy of the molecules and, if the substance is not an ideal gas, a rise in potential energy of the molecules and hence an increase in the internal energy of the system.

A rise in temperature of an object is related to an increase in the internal energy of the object.

16.2 The first law of thermodynamics

We have already met the law of conservation of energy (Topic 5.1). There, it was stated in the following form: energy can be neither created nor destroyed, it can only be transformed from one form to another. In this section we shall see how this conservation law may be re-stated in relation to terms such as work, thermal energy and internal energy. This will lead to an understanding of the first law of thermodynamics.

Thermodynamics is the study of processes involving the transfer of thermal energy and the doing of work. In thermodynamics, it is necessary to define the system under consideration. For example, the system may be an ideal gas in a cylinder fitted with a piston, or an electric heating coil in a container of liquid.

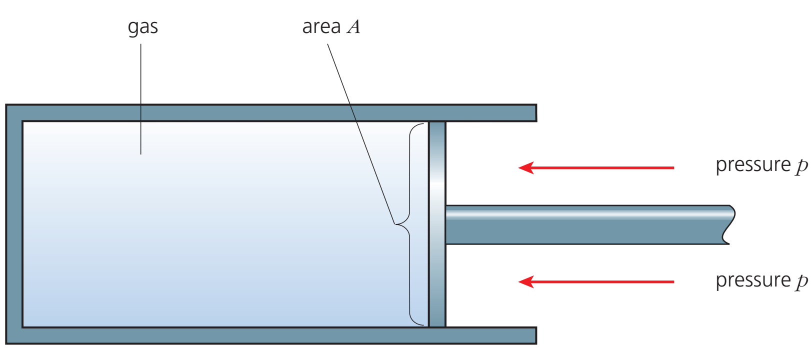
In Topic 5.1 we established the scientific meaning of **work**. Work is done when energy is transferred by mechanical means. We have seen (Topic 14) that heating is a transfer of energy due to a difference in temperature. Work and heating both involve a transfer of energy, but by different means.

▲ **Figure 16.2** Explosives produce large quantities of high-pressure gas. When the gas expands, it does work in demolishing the building.

Work done by an expanding gas

A building can be demolished with explosives (Figure 16.2). When the explosives are detonated, large quantities of gas at high pressure are produced. As the gas expands, it does work by breaking down the masonry. In this section we will derive an equation for the work done when a gas changes its volume.

Consider a gas contained in a cylinder by means of a frictionless piston of area A, as shown in Figure 16.3. The pressure p of the gas in the cylinder is equal to the atmospheric pressure outside the cylinder. This pressure may be thought to be constant.



▲ **Figure 16.3**

Since pressure = force/area, the gas produces a force F on the piston given by

$$F = pA$$

When the gas expands at constant pressure, the piston moves outwards through a distance x. So,

work done by the gas = force × distance moved in the direction of the force

$$w = pAx$$

However, Ax is the increase in volume ΔV of the gas. Hence

$$w = p\Delta V$$

When the volume of a gas changes at constant pressure,

work done = pressure × change in volume

Note that when the volume increases, the gas pushes against the external pressure and work is done by the gas. When the volume of the gas decreases, the external pressure forces a reduction in the volume of the gas and work is done on the gas.

Remember that the unit of work done is the joule (J). The pressure must be in pascals (Pa) or newtons per metre squared ($N\,m^{-2}$) and the change in volume in metres cubed (m^3).

WORKED EXAMPLE 16A

A sample of gas has a volume of $750\,cm^3$. The gas expands at a constant pressure of $1.4 \times 10^5\,Pa$ so that its volume becomes $900\,cm^3$. Calculate the work done by the gas during the expansion.

Answer

change in volume $\Delta V = (900 - 750) = 150\,cm^3 = 150 \times 10^{-6}\,m^3$

work done by gas $= p\Delta V$

$= (1.4 \times 10^5) \times (150 \times 10^{-6}) = \mathbf{21\,J}$

Questions

1 The volume of air in a tyre is $9.0 \times 10^{-3}\,m^3$. Atmospheric pressure is $1.0 \times 10^5\,Pa$. Calculate the work done against the atmosphere by the air when the tyre bursts and the air expands to a volume of $2.7 \times 10^{-2}\,m^3$.

2 High-pressure gas in a spray-can has a volume of $250\,cm^3$. The gas escapes into the atmosphere through a nozzle, so that its final volume is four times the volume of the can. Calculate the work done by the gas, given that atmospheric pressure is $1.0 \times 10^5\,Pa$.

Changing internal energy

We know that the internal energy of a system is the total energy, kinetic and potential, of all the atoms and molecules in the system. For a system consisting of an ideal gas, the internal energy is simply the total kinetic energy of all the atoms or molecules of the gas. For such a system, we would expect the internal energy to increase if the gas was heated, or if work was done on it. In both cases we are adding energy to the system. By the law of conservation of energy, this energy cannot just disappear; it must be transformed to another type of energy. It appears as an increase in the internal energy of the gas; that is, the total kinetic energy of the molecules is increased. But if the total kinetic energy of the molecules increases, their average kinetic energy is also increased. Because the average kinetic energy is a measure of temperature, the addition of energy to the ideal gas shows up as an increase in its temperature. We can express this transformation of energy as an equation.

The increase in internal energy of a system is equal to the sum of the thermal energy added to the system and the work done on it.

This is a statement of the **first law of thermodynamics**.

The increase in internal energy is given the symbol ΔU, thermal energy added is represented by q and work done on the system by w. The equation is then

$$\Delta U = q + w$$

Note the sign convention which has been adopted. A positive value of q means that thermal energy has been added to the system. A positive value of w means that work is done on the system. A positive value for ΔU means an increase in internal energy.

If the system does work, then we show this by writing $-w$. If thermal energy leaves the system, we show this by writing $-q$. Take care! There is an alternative sign convention that takes the work done *by* the system as a positive quantity. To avoid confusion, write down the sign convention you are using every time you quote the first law.

Applying the first law of thermodynamics

Let's see how the first law of thermodynamics applies to some simple processes. First, think about a change in the pressure and volume of a gas in a cylinder fitted with a piston. The cylinder and piston are insulated, so that no thermal energy can enter or leave the gas. (The thermodynamic name for such a change is an **adiabatic** change. That is, no thermal energy is allowed to enter or leave the system.) If no thermal energy enters or leaves the gas, q in the first law equation is zero. Thus

$$\Delta U = 0 + w$$

If work is done on the gas by pushing the piston in, w is positive (remember the sign convention!) and ΔU will also be positive. That is, the internal energy increases and, because temperature is proportional to internal energy, the temperature of the gas rises. An adiabatic change can be achieved even if the cylinder and piston are not well insulated. Moving the piston rapidly, so that there is no time for thermal energy to enter or leave, is just as effective. You will have noticed that a cycle pump gets hot as a result of brisk pumping. This is because the gas in the pump is being compressed adiabatically. Work is being done on the gas, the pump strokes are too rapid for the thermal energy to escape, and the internal energy, and hence the temperature, increases. Another example is the diesel engine, where air in the cylinder is compressed so rapidly that the temperature rises to a point that, when fuel is injected into the cylinder, it is above its ignition temperature.

Now think about an electric kettle containing water. Here the element provides thermal energy to the system. The quantity q in the first law equation is positive (the sign convention is 'thermal energy added, q positive'). No mechanical work is done on or by the water, so w in the first law equation is zero. Thus

$$\Delta U = q + 0$$

The fact that q is positive means that ΔU is also positive. Internal energy, and hence temperature, increases.

WORKED EXAMPLE 16B

250 J of thermal energy is added to a system, which does 100 J of work. Find the change in internal energy of the system.

Answer

We use the first law of thermodynamics in the form $\Delta U = q + w$ with the sign convention that q is positive if energy is supplied to the system by heating and w is positive if work is done on the system. Here $q = 250$ J and $w = -100$ J (the system is doing the work, hence the minus sign). Therefore, $\Delta U = 250 - 100 =$ **150 J**. This change in internal energy is an increase.

Question

3 An **isothermal** change is one which takes place at constant temperature. Explain why, in any isothermal change, the change in internal energy is zero. In such a change, 350 J of thermal energy is added to a system. How much work is done on or by the system?

Internal energy during changes of state

When a substance changes from solid to liquid, intermolecular bonds are broken, thus increasing the potential energy component of the internal energy. During the melting process, the temperature does not change, and therefore the kinetic energy of the molecules does not change. Most substances expand on melting, and thus external work is done. By the first law, thermal energy must be supplied to the system, and this thermal energy is the latent heat of fusion.

Similarly, when a substance changes from liquid to gas, by boiling or evaporation, temperature does not change so there is no change in the kinetic energies of the molecules. However the separation of the molecules is increased, thus increasing the potential energy component of the internal energy.

Volume changes associated with evaporation are much greater than those associated with melting. The external work done is much greater during vaporisation, and thus latent heat of vaporisation is much greater than latent heat of fusion.

WORKED EXAMPLE 16C

The specific latent heat of vaporisation of a liquid is $2300\,J\,g^{-1}$. When 1.0 g of the liquid evaporates, its volume increases by $1600\,cm^3$ against the atmospheric pressure of 1.0×10^5 Pa.

Determine, for the evaporation of 1.0 g of the liquid:

a the work done against the atmosphere

b the increase in internal energy.

Answers

a work done $= p\Delta V$

$= 1.0 \times 10^5 \times 1600 \times 10^{-6} =$ **160 J**

b increase in internal energy, $\Delta U = q + w$

$\Delta U = 2300 - 160$

$=$ **2140 J**

SUMMARY

» The internal energy of a system is the sum of the random kinetic and potential energies of the molecules in the system.

» For an ideal gas, the internal energy is the total kinetic energy of random motion of the molecules.

» Internal energy is a measure of the temperature of the system.

» The work done w by a gas when its volume changes ΔV at constant pressure p is $w = p\Delta V$.

» The first law of thermodynamics expresses the law of conservation of energy. The increase in the internal energy ΔU *of a* system is equal to the sum of the energy transferred to the system by heating q and the work w done on it:

$\Delta U = q + w$

(Sign convention: positive q, energy is added to the system by heating; positive w, work is done on the system; positive ΔU, increase in internal energy.)

END OF TOPIC QUESTIONS

1 An ideal gas expands isothermally, doing 250 J of work. What is the change in internal energy? How much thermal energy is absorbed in the process?

2 50 J of thermal energy is supplied to a fixed mass of gas in a cylinder. The gas expands, doing 20 J of work. Calculate the change in internal energy of the gas.

3 When water boils at an atmospheric pressure of 101 kPa, $1.00\,cm^3$ of liquid becomes $1560\,cm^3$ of steam. Calculate the work done against the atmosphere when a saucepan containing $45\,cm^3$ of water is allowed to boil dry.

4 a i State the basic assumption of the kinetic theory of gases that leads to the conclusion that the potential energy between the atoms of an ideal gas is zero. **[1]**

ii State what is meant by the *internal energy* of a substance. **[2]**

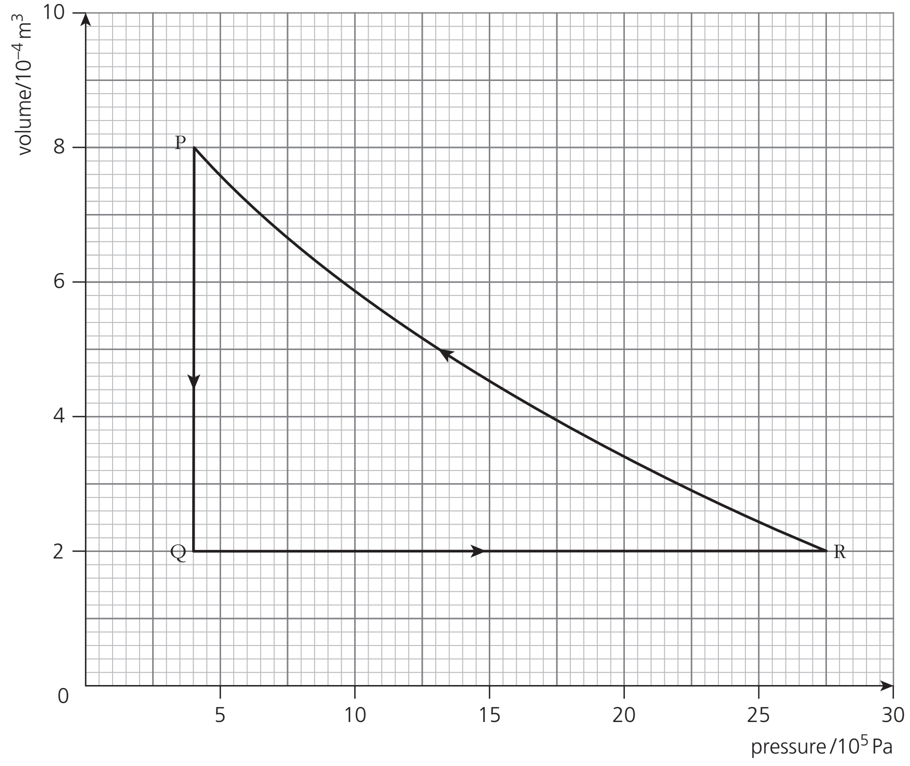
iii Explain why an increase in internal energy of an ideal gas is directly related to a rise in temperature of the gas. **[2]**

b A fixed mass of an ideal gas undergoes a cycle PQRP of changes as shown in Fig. 16.4.

i State the change in internal energy of the gas during one complete cycle PQRP. **[1]**

ii Calculate the work done on the gas during the change from P to Q. **[2]**

iii Some energy changes during the cycle PQRP are shown in Fig. 16.5.



▲ **Figure 16.4**

change	work done on gas/J	heating supplied to gas/J	increase in internal energy/J
P→Q	....	−600	....
Q→R	0	+720	....
R→P	....	+480	....

▲ **Figure 16.5**

Complete a copy of Fig. 16.5 to show all of the energy changes. **[3]**

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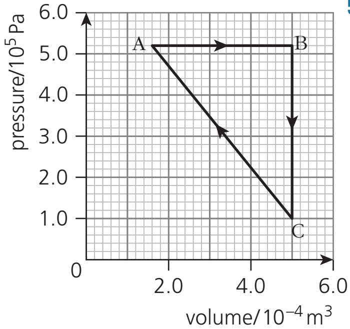

▲ **Figure 16.6**

5 **a** The first law of thermodynamics can be represented by the expression

$\Delta U = q + w$

State what is meant by the symbols in the expression. [2]

b A fixed mass of an ideal gas undergoes a cycle ABCA of changes, as shown in Fig. 16.6.

i During the change from A to B, the energy supplied to the gas by heating is 442 J. Use the first law of thermodynamics to show that the internal energy of the gas increases by 265 J. [2]

ii During the change from B to C, the internal energy of the gas decreases by 313 J. By considering molecular energy, state and explain qualitatively the change, if any, in the temperature of the gas. [3]

iii For the change from C to A, use the data in **b i** and **b ii** to calculate the change in internal energy. [1]

iv The temperature of the gas at point A is 227°C. Calculate the number of molecules in the fixed mass of the gas. [2]

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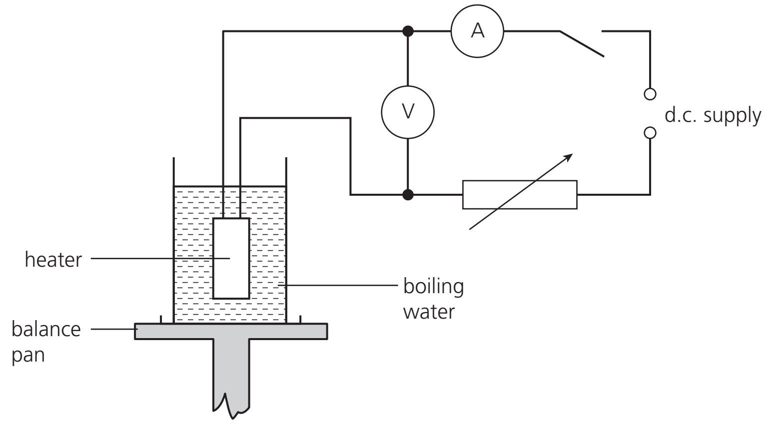
6 **a** State what is meant by *specific latent heat*. [2]

b A beaker of boiling water is placed on the pan of a balance, as illustrated in Fig. 16.7.



▲ **Figure 16.7**

The water is maintained at its boiling point by means of a heater. The change M in the balance reading in 300 s is determined for two different input powers to the heater. The results are shown in Fig. 16.8.

voltmeter reading/V	ammeter reading/A	M/g
11.5	5.2	5.0
14.2	6.4	9.1

▲ **Figure 16.8**

i Energy is supplied continuously by the heater. State where, in this experiment:

1 external work is done,

2 internal energy increases. Explain your answer. [3]

ii Use the data in Fig. 16.8 to determine the specific latent heat of vaporisation of water. [3]

Cambridge International AS and A Level Physics (9702) Paper 42 Q2 Oct/Nov 2017

17 Oscillations

Learning outcomes

By the end of this topic, you will be able to:

17.1 Simple harmonic oscillations

1. understand and use the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of oscillations, and express the period in terms of both frequency and angular frequency
2. understand that simple harmonic motion occurs when acceleration is proportional to displacement from a fixed point and in the opposite direction
3. use $a = -\omega^2 x$ and recall and use, as a solution to this equation, $x = x_0 \sin \omega t$
4. use the equations $v = v_0 \cos \omega t$ and $v = \pm\omega\sqrt{(x_0^2 - x^2)}$
5. analyse and interpret graphical illustrations of the variations of displacement, velocity and acceleration for simple harmonic motion

17.2 Energy in simple harmonic motion

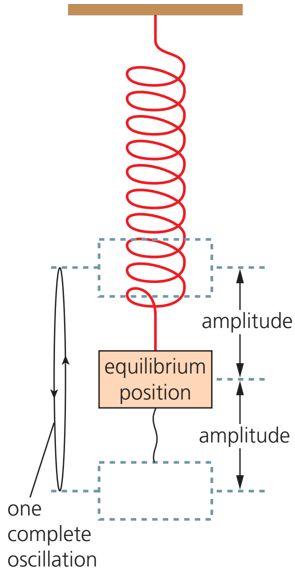
1. describe the interchange between kinetic and potential energy during simple harmonic motion
2. recall and use $E = \frac{1}{2}m\omega^2 x_0^2$ for the total energy of a system undergoing simple harmonic motion

17.3 Damped and forced oscillations, resonance

1. understand that a resistive force acting on an oscillating system causes damping
2. understand and use the terms light, critical and heavy damping and sketch displacement–time graphs illustrating these types of damping
3. understand that resonance involves a maximum amplitude of oscillations and that this occurs when an oscillating system is forced to oscillate at its natural frequency

Starting points

- ★ An object that moves to-and-fro continuously is said to be oscillating or vibrating.
- ★ Oscillations occur in many different systems from the very small (e.g. atoms) to the very large (e.g. buildings).
- ★ Waves can be described by the quantities period, frequency, displacement and amplitude.
- ★ Angles may be measured in radians (rad): 2π rad = 360°.



▲ **Figure 17.1** Oscillation of a mass on a spring

17.1 Simple harmonic oscillations

Some movements involve repetitive to-and-fro motion, such as a pendulum, the beating of a heart, the motion of a child on a swing and the vibrations of a guitar string. Another example would be a mass bouncing up and down on a spring, as illustrated in Figure 17.1. One complete movement from the starting or rest position, move up, then down and finally back up to the rest position, is known as an **oscillation**.

The time taken for one complete oscillation or vibration is referred to as the **period** T of the oscillation.

The oscillations repeat themselves.

The number of oscillations or vibrations per unit time is the **frequency** f.

Frequency may be measured in hertz (Hz), where one hertz is one oscillation per second ($1 Hz = 1 s^{-1}$). However, frequency may also be measured in min^{-1}, $hour^{-1}$, etc. For example, it would be appropriate to measure the frequency of the tides in h^{-1}.

Since period T is the time for one oscillation then

frequency $f = 1/T$

As the mass oscillates, it moves from its rest or equilibrium position.

The distance from the equilibrium position is known as the **displacement.**

This is a vector quantity and therefore has magnitude and direction relative to the equilibrium position. Displacement may be on either side of the equilibrium position.

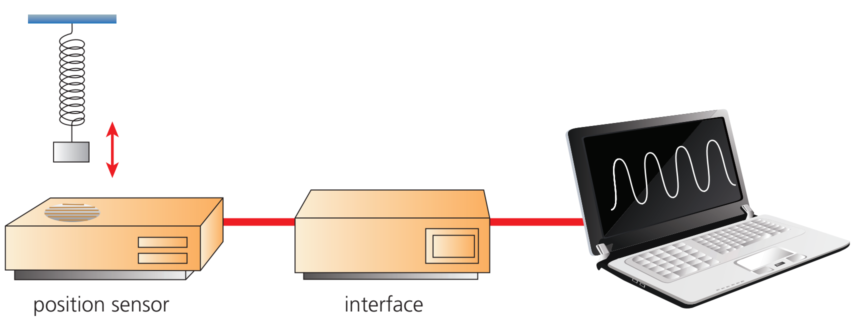
The **amplitude** (a scalar quantity) is the maximum displacement.

Some oscillations maintain a constant period even when the amplitude of the oscillation changes. Galileo discovered this fact for a pendulum. He timed the swings of an oil lamp in Pisa Cathedral, using his pulse as a measure of time. Oscillators that have a constant time period are called isochronous, and may be made use of in timing devices. For example, in quartz watches the oscillations of a small quartz crystal provide constant time intervals. Galileo's experiment was not precise, and we now know that a pendulum swinging with a large amplitude is not isochronous.

The quantities period, frequency, displacement and amplitude should be familiar from our study of waves in Topic 7. It should not be a surprise to meet them again, as the idea of oscillations is vital to the understanding of waves.

Displacement–time graphs

It is possible to plot displacement–time graphs (as we did for waves in Topic 7.1) for oscillators. One experimental method is illustrated in Figure 17.2. A mass on a spring oscillates above a position sensor that is connected to a computer through a datalogging interface, causing a trace to appear on the monitor.



▲ **Figure 17.2** Apparatus for plotting displacement–time graphs for a mass on a spring

The graph describing the variation of displacement with time may have different shapes, depending on the oscillating system. For many oscillators the graph is approximately a sine (or cosine) curve. A sinusoidal displacement–time graph is a characteristic of an important type of oscillation called **simple harmonic motion** (s.h.m.). Oscillators which move in s.h.m. are called **harmonic oscillators**. We shall analyse simple harmonic motion in some detail, because it successfully describes many oscillating systems, both in real life and in theory. Fortunately, the mathematics of s.h.m. can be approached through a simple defining equation. The properties of the motion can be deduced from the relations between graphs of displacement against time, and velocity against time, which we met in Topic 2.

Simple harmonic motion (s.h.m.)

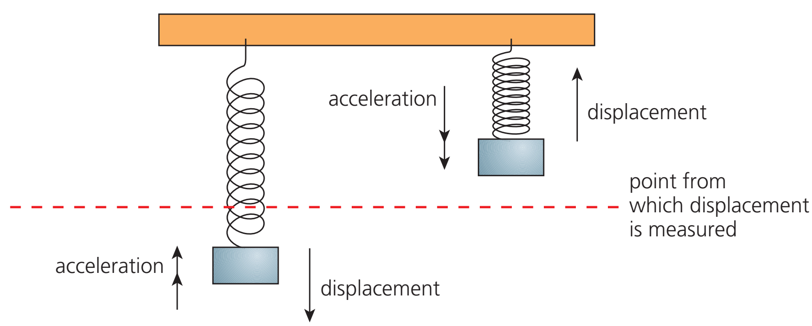
Simple harmonic motion is defined as the motion of a particle about a fixed point such that its acceleration a is proportional to its displacement x from the fixed point, and is in the opposite direction.

Note that, since the acceleration and the displacement are in opposite directions then *acceleration is always directed towards the fixed point from which displacement is measured*.

Mathematically, we write this definition as

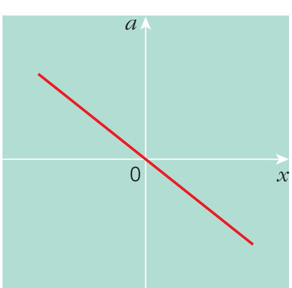
$$a = -\omega^2 x$$

where ω^2 is a constant. We take the constant as a squared quantity, because this will ensure that the constant is always positive (the square of a positive number, or of a negative number, will always be positive). Why worry about keeping the constant positive? This is because the minus sign in the equation must be preserved. It has a special significance, because it tells us that the acceleration a is always in the opposite direction to the displacement x. Remember that both acceleration and displacement are vector quantities, so the minus sign is shorthand for the idea that the acceleration is always directed towards the fixed point from which the displacement is measured. This is illustrated in Figure 17.3.



▲ **Figure 17.3** Directions of displacement and acceleration are always opposite

The defining equation is represented in a graph of a against x as a straight line, of negative gradient, through the origin, as shown in Figure 17.4. *The gradient is negative because of the minus sign in the equation*. Note that both positive and negative values for the displacement should be considered.



▲ **Figure 17.4** Graph of the defining equation for simple harmonic motion

The square root of the constant ω^2 (that is, ω) is known as the **angular frequency** of the oscillation. This angular frequency ω is related to the frequency f of the oscillation by the expression

$$\omega = 2\pi f$$

where one complete oscillation is described as 2π radians.

Since period T is related to frequency f by the expression

$$\text{frequency } f = 1/T$$

then

$$\text{angular frequency } \omega = \frac{2\pi}{T}$$

By Newton's second law, the force acting on an object is proportional to the acceleration of the object. The defining equation for simple harmonic motion can thus be related to the force acting on the particle. If the acceleration of the particle is proportional to its displacement from a fixed point, the resultant force acting on the particle is also proportional to the displacement. We can bring in the idea of the direction of the acceleration by specifying that the force is always acting towards the fixed point, or by calling it a **restoring force**.

Solution of equation for simple harmonic motion

In order to find the displacement–time relation for a particle moving in a simple harmonic motion, we need to solve the equation $a = -\omega^2 x$. To derive the solution requires mathematics which is beyond the requirements of Cambridge International AS & A Level Physics. However, you need to know the form of the solution. This is

$$x = x_0 \sin \omega t$$

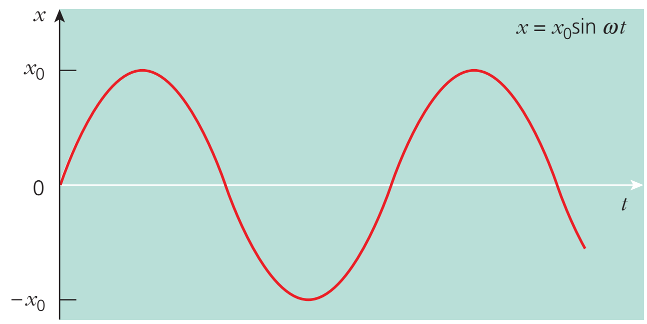
where x_0 is the amplitude of the oscillation and at time $t = 0$, the particle is at its equilibrium position defined as displacement $x = 0$. The variation with time t of the displacement x for this solution is shown in Figure 17.5.

MATHS NOTE

There are actually *two* solutions to the defining equation of simple harmonic motion, $a = -\omega^2 x$, depending on whether the timing of the oscillation starts when the particle has zero displacement or is at its maximum displacement. If at time $t = 0$ the particle is at its maximum displacement, $x = x_0$, the solution is $x = x_0 \cos \omega t$ (not shown in Figure 17.5). The two solutions are identical apart from the fact that they are out of phase with each other by one quarter of a cycle or $\pi/2$ radians.

The variation of velocity with time is sinusoidal if the cosinusoidal displacement solution is taken:

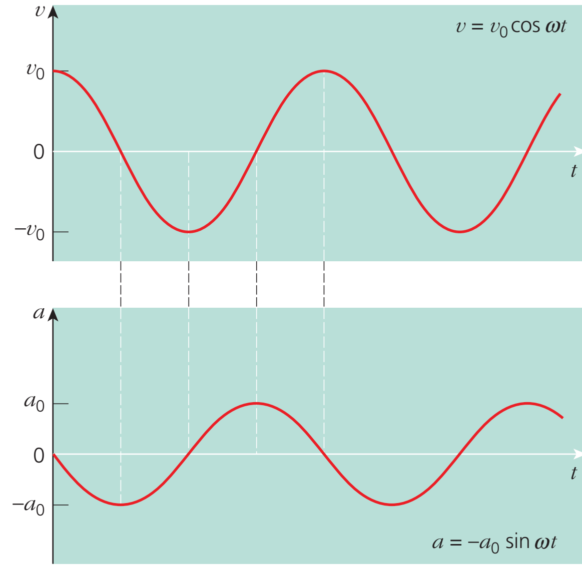
$$v = -v_0 \sin \omega t \text{ when } x = x_0 \cos \omega t$$



▲ **Figure 17.5** Displacement–time curve for simple harmonic motion

In Topic 2.1 it was shown that the gradient of a displacement–time graph may be used to determine velocity at any point (the instantaneous velocity) by taking a tangent to the curve. Referring to Figure 17.5, it can be seen that, at each time at which $x = x_0$, the gradient of the graph is zero (a tangent to the curve would be horizontal). Thus, the velocity is zero whenever the particle has its maximum displacement. If we think about a mass vibrating up and down on a spring, this means that when the spring is fully stretched and the mass has its maximum displacement, the mass stops moving downwards and has zero velocity. Also from Figure 17.5, we can see that the gradient of the graph is at a maximum whenever $x = 0$. This means that when the spring is neither under- or over-stretched the speed of the mass is at a maximum. After passing this point, the spring forces the mass to slow down until it changes direction.

If a full analysis is carried out, it is found that the variation of velocity with time is cosinusoidal when the displacement is sinusoidal. This is illustrated in Figure 17.6.



▲ **Figure 17.6** **Velocity–time and acceleration–time graph for simple harmonic motion**

The velocity v of the particle is given by the expression

$$v = v_0 \cos \omega t \text{ when } x = x_0 \sin \omega t$$

There is a phase difference between velocity and displacement. The velocity curve is $\pi/2$ rad ahead of the displacement curve. The maximum speed v_0 is given by

$$v_0 = x_0\omega$$

There is an alternative expression for the velocity:

$$v_0 = \pm\omega\sqrt{(x_0{}^2 - x^2)}$$

which is derived and used in Topic 17.2.

For completeness, Figure 17.6 also shows the variation with time of the acceleration a of the particle. This could be derived from the velocity–time graph by taking the gradient. The equation for the acceleration is

$$a = -a_0 \sin \omega t \text{ when } x = x_0 \sin \omega t$$

WORKED EXAMPLE 17A

The displacement x at time t of a particle moving in simple harmonic motion is given by $x = 0.36 \sin 10.7t$, where x is in metres and t is in seconds.

a Use the equation to find the amplitude, frequency and period for the motion.

b Find the displacement when $t = 0.35$ s.

Answers

a Compare the equation with $x = x_0 \sin \omega t$. The amplitude $x_0 =$ **0.36 m**. The angular frequency $\omega = 10.7\,\text{rad}\,\text{s}^{-1}$. Remember that $\omega = 2\pi f$, so the frequency $f = \omega/2\pi = 10.7/2\pi =$ **1.7 Hz**. The period $T = 1/f = 1/1.7 =$ **0.59 s**.

b Substitute $t = 0.35$ s in the equation, remembering that the angle ωt is in radians and not degrees. $\omega t = 10.7 \times 0.35 = 3.75\,\text{rad} = 215°$. So $x = 0.36 \sin 215° =$ **−0.21 m**.

Question

1 A mass oscillating on a spring has an amplitude of 0.20 m and a period of 1.5 s.

a Deduce the equation for the displacement x if timing starts at the instant when the mass has its zero displacement.

b Calculate the time interval from $t = 0$ before the displacement is 0.17 m.

17.2 Energy in simple harmonic motion

Kinetic energy

In Topic 17.1, we saw that the velocity of a particle vibrating with simple harmonic motion varies with time and, consequently, with the displacement of the particle. For the case where displacement x is zero at time $t = 0$, displacement and velocity are given by

$$x = x_0 \sin \omega t$$

and

$$v = x_0 \omega \cos \omega t \text{ or } v = v_0 \cos \omega t$$

There is a trigonometrical relation between the sine and the cosine of an angle θ, which is $\sin^2 \theta + \cos^2 \theta = 1$. Applying this relation, we have

$$x^2/x_0^2 + v^2/x_0^2\omega^2 = 1$$

which leads to

$$v^2 = x_0^2\omega^2 - x^2\omega^2$$

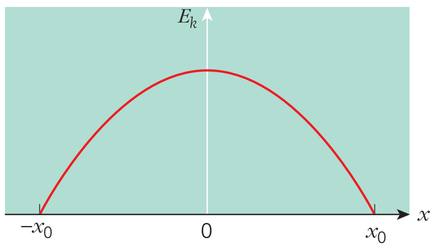
and so

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

The kinetic energy of the particle (of mass m) oscillating with simple harmonic motion is $\frac{1}{2}mv^2$. Thus, the kinetic energy E_k at displacement x is given by

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$

The variation with displacement of the kinetic energy is shown in Figure 17.7.



▲ **Figure 17.7** Variation of kinetic energy in s.h.m.

Potential energy

The defining equation for simple harmonic motion can be expressed in terms of the restoring force F_{res} acting on the particle. Since $F = ma$ and $a = -\omega^2 x$ then at displacement x, this force is

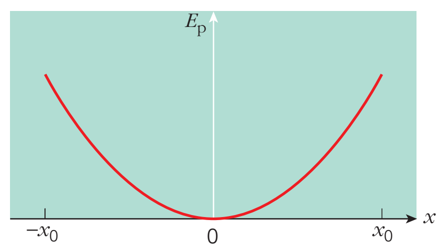
$$F_{res} = -m\omega^2 x$$

where m is the mass of the particle. To find the change in potential energy of the particle when the displacement increases by Δx, we need to find the work done against the restoring force.

The work done in moving the point of application of a force F by a distance Δx is $F\Delta x$. In the case of the particle undergoing simple harmonic motion, we know that the restoring force is directly proportional to displacement. To calculate the work done against the restoring force in giving the particle a displacement x, we take account of the fact that F_{res} depends on x by taking the average value of F_{res} during this displacement. The average value of force is just $\frac{1}{2}m\omega^2x$, since the value of F_{res} is zero at $x = 0$ and increases linearly to $m\omega^2x$ at displacement x. Thus, the potential energy E_p at displacement x is given by *average restoring force* × *displacement*, or

$$E_p = \frac{1}{2}m\omega^2x^2$$

The variation with displacement of the potential energy is shown in Figure 17.8.



▲ **Figure 17.8** Variation of potential energy in s.h.m.

Total energy

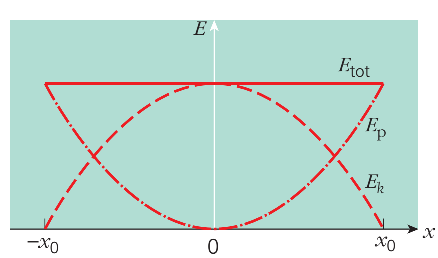
The total energy E_{tot} of the oscillating particle is given by

$$E_{tot} = E_k + E_p$$
$$= \frac{1}{2}m\omega^2(x_0^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

$$E_{tot} = \frac{1}{2}m\omega^2x_0^2$$

This total energy is constant since m, ω and x_0 are all constant. We might have expected this result, as it merely expresses the law of conservation of energy.

The variations with displacement x of the total energy E_{tot}, the kinetic energy E_k and the potential energy E_p are shown in Figure 17.9.



▲ **Figure 17.9** Energy variations in s.h.m.

WORKED EXAMPLE 17B

A particle of mass 95 g oscillates in simple harmonic motion with angular frequency 12.5 rad s^{-1} and amplitude 16 mm. Calculate:

a the total energy

b the kinetic and potential energies at half-amplitude (at displacement x = 8.0 mm).

Answers

a Using $E_{tot} = \frac{1}{2}m\omega^2 x_0^{\,2}$,

$$E_{tot} = \frac{1}{2} \times 0.095 \times 12.5^2 \times (16 \times 10^{-3})^2$$
$$= \mathbf{1.90 \times 10^{-3}\,J}$$

(Don't forget to convert g to kg and mm to m.)

b Using $E_k = \frac{1}{2}m\omega^2(x_0^{\,2} - x^2)$

$$E_k = \frac{1}{2} \times 0.095 \times 12.5^2 \times [(16 \times 10^{-3})^2 - (8 \times 10^{-3})^2]$$
$$= \mathbf{1.43 \times 10^{-3}\,J}$$

Using $E_{tot} = E_k + E_p$

$$1.90 \times 10^{-3} = 1.43 \times 10^{-3} + E_p$$
$$E_p = \mathbf{0.47 \times 10^{-3}\,J}$$

Question

2 A particle of mass 0.35 kg oscillates in simple harmonic motion with frequency 4.0 Hz and amplitude 8.0 cm. Calculate, for the particle at displacement 7.0 cm:

a the kinetic energy

b the potential energy

c the total energy.

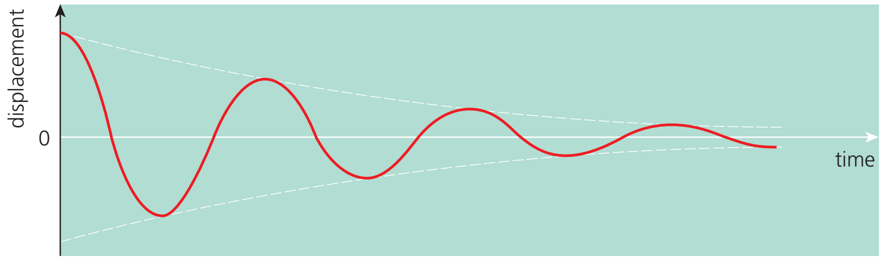
17.3 Damped and forced oscillations, resonance

A particle is said to be undergoing **free oscillations** when the only external force acting on it is the restoring force.

There are no forces to dissipate energy and so the oscillations have constant amplitude. Total energy remains constant. This is the situation we have been considering so far. Simple harmonic oscillations are free oscillations.

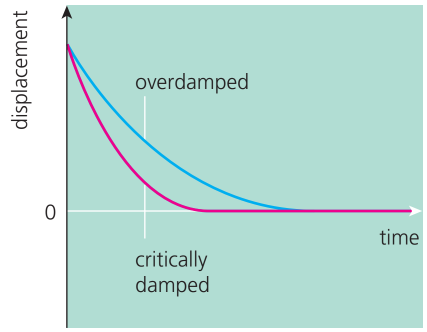
In real situations, however, frictional and other resistive forces cause the oscillator's energy to be dissipated, and this energy is converted eventually into thermal energy. The oscillations are said to be **damped**.

The total energy of the oscillator decreases with time. The damping is said to be light when the amplitude of the oscillations decreases gradually with time. This is illustrated in Figure 17.10. The decrease in amplitude is, in fact, exponential with time. The period of the oscillation is slightly greater than that of the corresponding free oscillation.

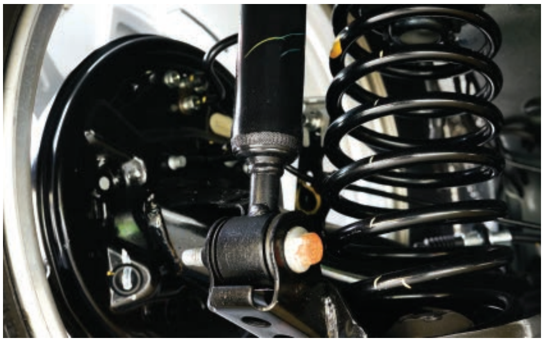


▲ **Figure 17.10** Lightly damped oscillations

Heavier damping causes the oscillations to die away more quickly. If the damping is increased further, then the system reaches **critical damping** point. Here the displacement decreases to zero in the shortest time, without any oscillation (Figure 17.11, overleaf).



▲ **Figure 17.11** Critical damping and overdamping



▲ **Figure 17.12** Vehicle suspension system showing springs and dampers

Any further increase in damping produces **overdamping** or **heavy damping**. The displacement decreases to zero in a longer time than for critical damping (Figure 17.11).

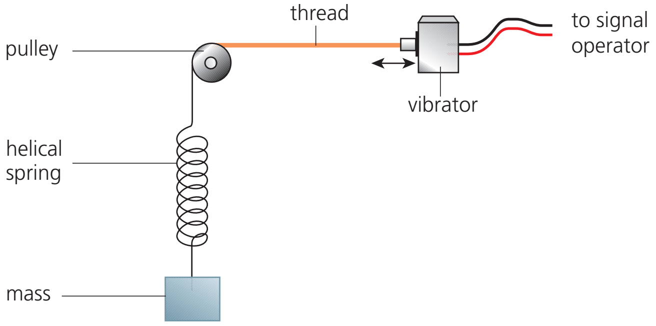
Damping is often useful in an oscillating system. For example, vehicles have springs between the wheels and the frame to give a smoother and more comfortable ride (Figure 17.12). If there was no damping, a vehicle would move up and down for some time after hitting a bump in the road. Dampers (shock absorbers) are connected in parallel with the springs so that the suspension has critical damping and comes to rest in the shortest time possible. Dampers often work through hydraulic action. When the spring is compressed, a piston connected to the vehicle frame forces oil through a small hole in the piston, so that the energy of the oscillation is dissipated as thermal energy in the oil.

Many swing doors have a damping mechanism fitted to them. The purpose of the damper is so that the open door, when released, does not overshoot the closed position with the possibility of injuring someone approaching the door. Most door dampers operate in the overdamped or heavily damped mode.

Forced oscillations and resonance

When a vibrating object undergoes free (undamped) oscillations, it vibrates at its **natural frequency**. We met the idea of a natural frequency in Topic 8, when talking about stationary waves on strings. The natural frequency of such a system is the frequency of the first mode of vibration; that is, the fundamental frequency. A practical example is a guitar string, plucked at its centre, which oscillates at a particular frequency that depends on the speed of progressive waves on the string and the length of the string. The speed of progressive waves on the string depends on the mass per unit length of the string and the tension in the string.

Vibrating objects may have periodic forces acting on them. These periodic forces will make the object vibrate at the frequency of the applied force, rather than at the natural frequency of the system. The object is then said to be undergoing **forced vibrations**. Figure 17.13 illustrates apparatus which may be used to demonstrate the forced vibrations of a mass on a helical spring. The vibrator provides the forcing (driving) frequency and has a constant amplitude of vibration.

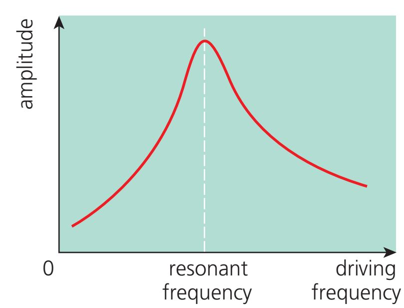


▲ **Figure 17.13** Demonstration of forced oscillations

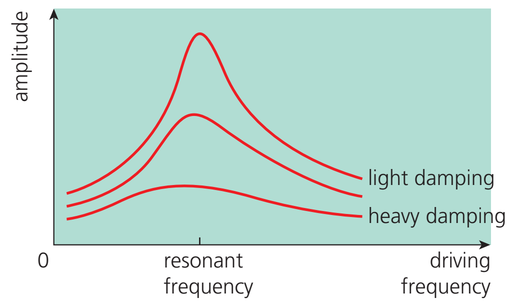
As the frequency of the vibrator is gradually increased from zero, the mass begins to oscillate. At first the amplitude of the oscillations is small, but it increases with increasing frequency. When the driving frequency equals the natural frequency of oscillation of the mass–spring system, the amplitude of the oscillations reaches a maximum. The frequency at which this occurs is called the **resonant frequency**, and **resonance** is said to occur.

> Resonance occurs when the natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration.

If the driving frequency is increased further, the amplitude of oscillation of the mass decreases. The variation with driving frequency of the amplitude of vibration of the mass is illustrated in Figure 17.14. This graph is often called a **resonance curve**.



▲ **Figure 17.14** Resonance curve

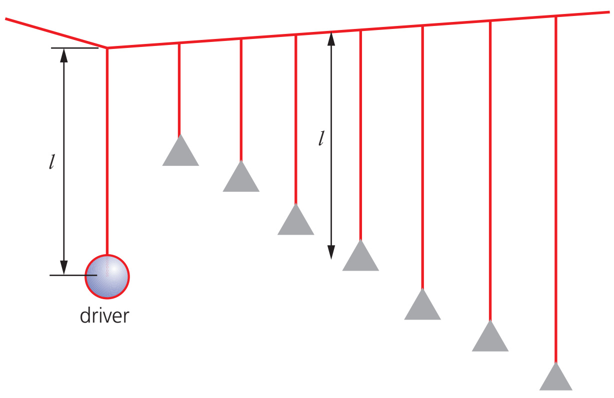


▲ **Figure 17.15** Effect of damping on the resonance curve

The effect of damping on the amplitude of forced oscillations can be investigated by attaching a light but stiff card to the mass in Figure 17.13. Movement of the card gives rise to air resistance and thus damping of the oscillations. The degree of damping may be varied by changing the area of the card. The effects of damping are illustrated in Figure 17.15. It can be seen that, as the degree of damping increases:

- the amplitude of oscillation at all frequencies is reduced
- the frequency at maximum amplitude shifts gradually towards lower frequencies
- the peak becomes flatter.

Barton's pendulums may be used to demonstrate resonance and the effects of damping. The apparatus consists of a set of light pendulums, made (for example) from paper cones, and a more massive pendulum (the driver), all supported on a taut string. The arrangement is illustrated in Figure 17.16. The lighter pendulums have different lengths, but one has the same length as the driver. This has the same natural frequency as the driver and will, therefore, vibrate with the largest amplitude of all the pendulums (Figure 17.17).



▲ **Figure 17.16** Barton's pendulums

▲ **Figure 17.17** Time-exposure photographs of Barton's pendulums with light damping, taken end-on. The longest arc, in the middle, is the driver.

Adding weights to the paper cones reduces the effect of damping. With less damping, the amplitude of the resonant pendulum is much larger.

There are many examples in everyday life of forced oscillations causing resonance. One of the simplest is that of pushing a child on a swing. We push at the same frequency as the natural frequency of oscillation of the swing and child, so that the amplitude of the motion increases.

The operation of the engine of a vehicle causes a periodic force on the parts of the vehicle, which can cause them to resonate. For example, at particular frequencies of rotation of the engine, the mirrors may resonate. To prevent excessive vibration, the mountings of the mirrors provide damping.

A spectacular example of resonance that is often quoted is the failure in 1940 of the first suspension bridge over the Tacoma Narrows in Washington State, USA. Wind caused the bridge to oscillate. It was used for months even though the roadway was oscillating with transverse vibrations. Approaching vehicles would appear, and then disappear, as the bridge deck vibrated up and down. One day, strong winds set up twisting vibrations (Figure 17.18) and the amplitude of vibration increased due to resonance, until eventually the bridge collapsed. The driver of a car that was on the bridge managed to walk to safety before the collapse, although his dog could not be persuaded to leave the car.

▲ **Figure 17.18** The Tacoma Narrows bridge disaster

EXTENSION

Musical instruments rely on resonance to amplify the sound produced. The sound from a tuning fork is louder when it is held over a tube of just the right length, so that the column of air resonates. We met this phenomenon in Topic 8.1, in connection with the resonance tube method of measuring the speed of sound in air. Stringed instruments have a hollow wooden box with a hole under the strings which acts in a similar way. To amplify all notes from all of the strings, the sounding-box has to be a complex shape so that it resonates at many different frequencies.

SUMMARY

- The period of an oscillation is the time taken to complete one oscillation.
- Frequency is the number of oscillations per unit time.
- Frequency f is related to period T by the expression $f = 1/T$.
- The displacement of a particle is its distance (in a stated direction) from the equilibrium position.
- Amplitude is the maximum displacement.
- Simple harmonic motion (s.h.m.) is defined as the motion of a particle about a fixed point such that its acceleration a is proportional to its displacement x from the fixed point, and is directed towards the fixed point, $a \propto -x$ or $a = -\omega^2 x$.
- The constant ω in the defining equation for simple harmonic motion is known as the angular frequency.

- For a particle oscillating in s.h.m. with frequency f, then $\omega = 2\pi f$ and $T = 2\pi/\omega$.
- Simple harmonic motion is described in terms of displacement x, amplitude x_0, frequency f, and angular frequency ω by the following relations.
  displacement: $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$
  velocity: $v = x_0 \omega \cos \omega t$ or $v = -x_0 \omega \sin \omega t$ or $v = \pm\omega\sqrt{(x_0^2 - x^2)}$
  acceleration: $a = -x_0\omega^2 \sin \omega t$ or $a = -x_0\omega^2 \cos \omega t$.
- Remember that $\omega = 2\pi f$, and the equations above may appear in either form.
- For a particle oscillating in s.h.m., graphs of the displacement, velocity and acceleration are all sinusoidal but have a phase difference.
- Velocity is out of phase with displacement by $\pi/2$ radians, meaning velocity is zero at maximum displacement and maximum when displacement is zero.
- Acceleration is out of phase with displacement by π radians, meaning acceleration is maximum at maximum displacement but in the opposite direction.
- The kinetic energy E_k of a particle of mass m oscillating in simple harmonic motion with angular frequency ω and amplitude x_0 is $E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$ where x is the displacement.
- The potential energy E_p of a particle of mass m oscillating in simple harmonic motion with angular frequency ω is $E_p = \frac{1}{2}m\omega^2 x^2$ where x is the displacement.
- The total energy E_{tot} of a particle of mass m oscillating in simple harmonic motion with angular frequency ω and amplitude x_0 is $E_{tot} = \frac{1}{2}m\omega^2 x_0^2$.
- For a particle oscillating in simple harmonic motion $E_{tot} = E_k + E_p$ and this expresses the law of conservation of energy.
- Free oscillations are oscillations where there are no resistive forces acting on the oscillating system.
- Damping is produced by resistive forces which dissipate the energy of the vibrating system.
- Light damping causes the amplitude of vibration of the oscillation to decrease gradually. Critical damping causes the displacement to be reduced to zero in the shortest time possible, without any oscillation of the object. Overdamping or heavy damping also causes an exponential reduction in displacement, but over a greater time than for critical damping.
- The natural frequency of vibration of an object is the frequency at which the object will vibrate when allowed to do so freely.
- Forced oscillations occur when a periodic driving force is applied to a system which is capable of vibration.
- Resonance occurs when the driving frequency on the system is equal to its natural frequency of vibration. The amplitude of vibration is a maximum at the resonant frequency.

END OF TOPIC QUESTIONS

1 A particle is oscillating in simple harmonic motion with period 2.5 ms and amplitude 4.0 mm.

At time $t = 0$, the particle is at the equilibrium position. Calculate, for this particle:

a the frequency,
b the angular frequency,
c the maximum speed,
d the magnitude of the maximum acceleration,
e the displacement at time $t = 0.8$ ms,
f the speed at time $t = 1.0$ ms.

2 A spring stretches by 69 mm when a mass of 45 g is hung from it. The spring is then stretched a further distance of 15 mm from the equilibrium position, and the mass is released at time $t = 0$.

When the spring is released, the mass oscillates with simple harmonic motion of period T.

The period T is given by the expression

$T = 2\pi\sqrt{(m/k)}$

where k is the spring constant and m is the mass on the spring.

Calculate:

a the spring constant,

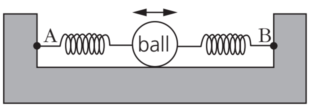
b the amplitude of the oscillations,

c the period,

d the displacement at time $t = 0.20\,\text{s}$.

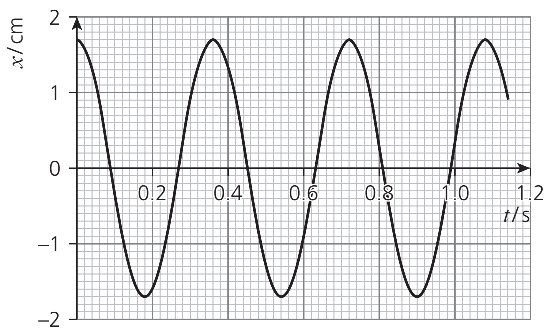
3 One particle oscillating in simple harmonic motion has ten times the total energy of another particle, but the frequencies and masses are the same. Calculate the ratio of the amplitudes of the two motions.

4 A ball is held between two fixed points A and B by means of two stretched springs, as shown in Fig. 17.19.



▲ **Figure 17.19**

The ball is free to oscillate along the straight line AB. The springs remain stretched and the motion of the ball is simple harmonic. The variation with time t of the displacement x of the ball from its equilibrium position is shown in Fig. 17.20.



▲ **Figure 17.20**

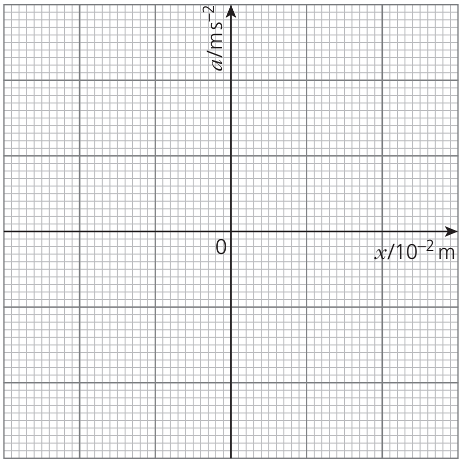
a **i** Use Fig. 17.20 to determine, for the oscillations of the ball:

1 the amplitude, **[1]**

2 the frequency. **[2]**

ii Show that the maximum acceleration of the ball is $5.2\,\text{m}\,\text{s}^{-2}$. **[2]**

b Use your answers in **a** to plot, on a copy of Fig. 17.21, the variation with displacement x of the acceleration a of the ball. **[2]**



▲ **Figure 17.21**

c Calculate the displacement of the ball at which its kinetic energy is equal to one half of the maximum kinetic energy. **[3]**

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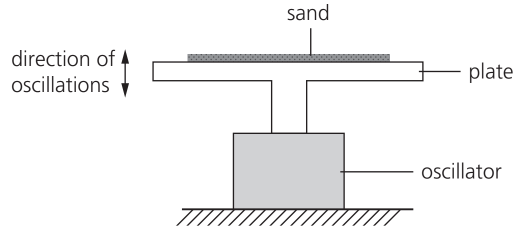
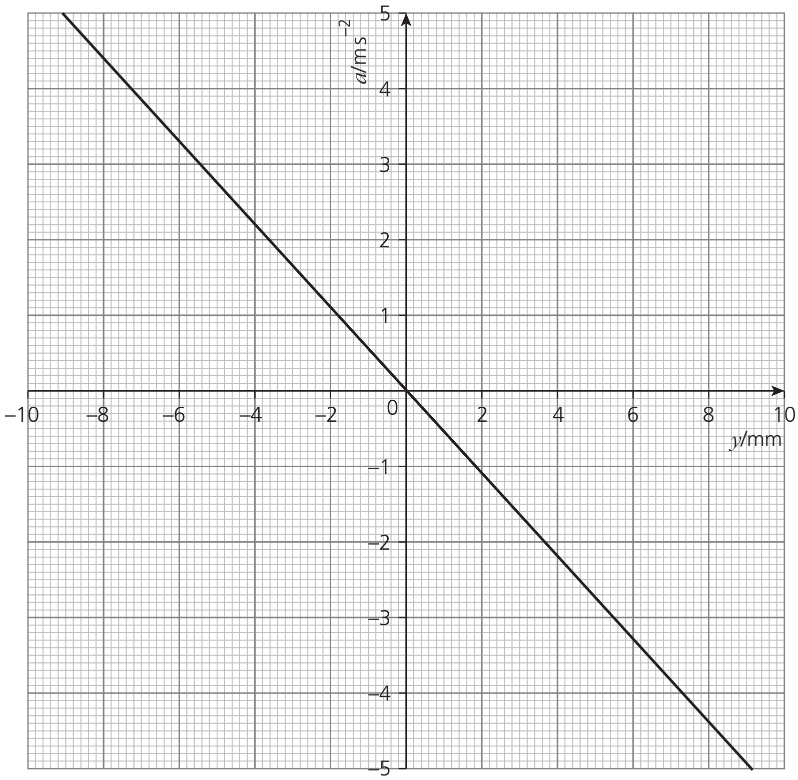
5 A metal plate is made to vibrate vertically by means of an oscillator, as shown in Fig. 17.22.



▲ **Figure 17.22**

Some sand is sprinkled on to the plate.

The variation with displacement y of the acceleration a of the sand on the plate is shown in Fig. 17.23.



▲ **Figure 17.23**

a **i** Use Fig. 17.23 to show how it can be deduced that the sand is undergoing simple harmonic motion. [2]

ii Calculate the frequency of oscillation of the sand. [2]

b The amplitude of oscillation of the plate is gradually increased beyond 8 mm. The frequency is constant.

At one amplitude, the sand is seen to lose contact with the plate.

For the plate when the sand first loses contact with the plate:

i state the position of the plate, [1]

ii calculate the amplitude of oscillation. [3]

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18 Electric fields

Learning outcomes

By the end of this topic, you will be able to:

18.1 Electric fields and field lines

1. understand that an electric field is an example of a field of force and define electric field as force per unit positive charge
2. recall and use $F = qE$ for the force on a charge in an electric field
3. represent an electric field by means of field lines

18.2 Uniform electric fields

1. recall and use $E = \Delta V/\Delta d$ to calculate the field strength of the uniform field between charged parallel plates
2. describe the effect of a uniform electric field on the motion of charged particles

18.3 Electric force between point charges

1. understand that, for a point outside a spherical conductor, the charge on the sphere may be considered to be a point charge at its centre
2. recall and use Coulomb's law $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$ for the force between two point charges in free space

18.4 Electric field of a point charge

1. recall and use $E = Q/(4\pi\varepsilon_0 r^2)$ for the electric field strength due to a point charge in free space

18.5 Electric potential

1. define the electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to the point
2. recall and use the fact that the electric field strength at a point is equal to the negative of the potential gradient at that point
3. use $V = Q/(4\pi\varepsilon_0 r)$ for the electric potential in the field due to a point charge
4. understand how the concept of electric potential leads to the electric potential energy of two point charges and use $E_p = Qq/(4\pi\varepsilon_0 r)$

Starting points

- ★ There are two types of charge, positive and negative, and the unit of charge is the coulomb.
- ★ Objects can be charged by friction or induction.
- ★ Electric forces hold electrons in atoms, and bind atoms together in molecules and in solids.
- ★ Work is done when a force moves its point of application in the direction of the force.
- ★ Potential difference is the work done (energy transferred) per unit charge as it moves from one point to the other.

18.1 Electric fields and field lines

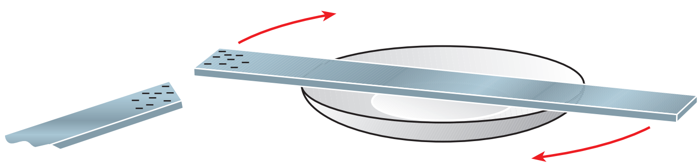
Some effects of static electricity are familiar in everyday life. For example, a balloon rubbed on a woollen jumper will stick to a wall, dry hair crackles (and may actually spark!) when brushed and you may feel a shock when you touch the metal door handle of a car when getting out after a journey in dry weather. All these are examples of insulated objects that have gained an electric charge by friction – that is, by being rubbed against other objects.

Insulators that are charged by friction will attract other objects. Some of the effects have been known for centuries. Greek scientists experimented with amber that was charged by rubbing it with fur. Today, electrostatics experiments are often carried out with plastic materials that are moisture repellent and stay charged for longer times.

Charging by friction can be hazardous. For example, a tanker which carries a bulk powder must be earthed before unloading, otherwise, electric charge may build up on the tanker. This could then lead to a spark between the tanker and earth, causing an explosion. Similarly, pipes used for movement of highly flammable liquids (for example, petrol) are metal-clad. An aircraft moving through air will also become charged. To prevent the first person touching the aircraft on landing from becoming injured, the tyres are made to conduct, so that, on landing, the aircraft loses its charge.

There are two kinds of electric charge. Polythene becomes negatively charged when rubbed with wool and cellulose acetate becomes positively charged, also when rubbed with wool. To understand this, we need to consider the model of the atom. An atom consists of a positively charged nucleus with negatively charged electrons orbiting it. When the polythene is rubbed with wool, friction causes some electrons to be transferred from the wool to the polythene. The polythene has a negative charge and the wool is left with a positive charge. Cellulose acetate becomes positive because it loses some electrons to the wool when it is rubbed. Polythene and cellulose acetate are poorly conducting materials and so the charges remain static on their surfaces.

Putting two charged polythene rods close to one another, or two charged acetate rods close to one another, shows that similar charges repel one another (Figure 18.1). Conversely, unlike charges attract. A charged polythene rod attracts a charged acetate rod.

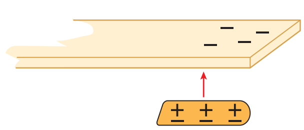


▲ **Figure 18.1** Like charges repel.

This is the basic law of the force between charges:

Like charges repel, unlike charges attract.

Charged rods will also attract uncharged objects. For example, a charged polythene rod will pick up small pieces of paper. The presence of charge on the rod causes a redistribution of charge on the paper. Electrons are repelled to the side away from the rod so that the side nearest the rod is positive and is, therefore, attracted to the rod (Figure 18.2). The paper is said to be **charged by induction**. When the rod is removed, the electrons will move back and cancel the positive charge.



▲ **Figure 18.2** A charged rod can induce charges in an uncharged object.

Electric fields

Electric charges exert forces on one another even when they are separated by a distance. The concept of an **electric field** is used to explain this force at a distance.

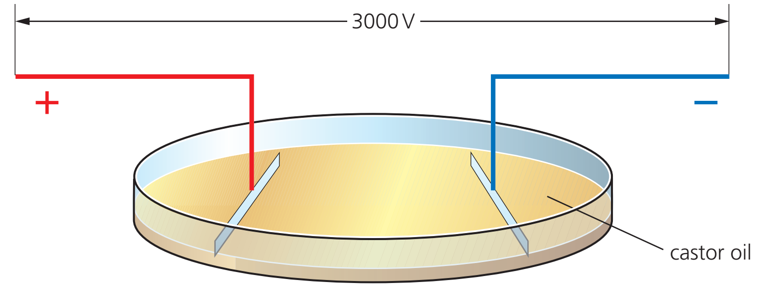
An electric field is a region of space where a stationary electric charge experiences a force.

Electric fields are invisible but they can be represented by electric lines of force just as gravitational fields can be represented by gravitational lines of force (Topic 13.1) and magnetic fields can be represented by magnetic lines of force (see Topic 20.1). The direction of the electric field is defined as the direction in which a small positive charge would move if it were free to do so. Hence, the lines of force are drawn with arrows that point from positive charge to negative charge.

For an electric field:

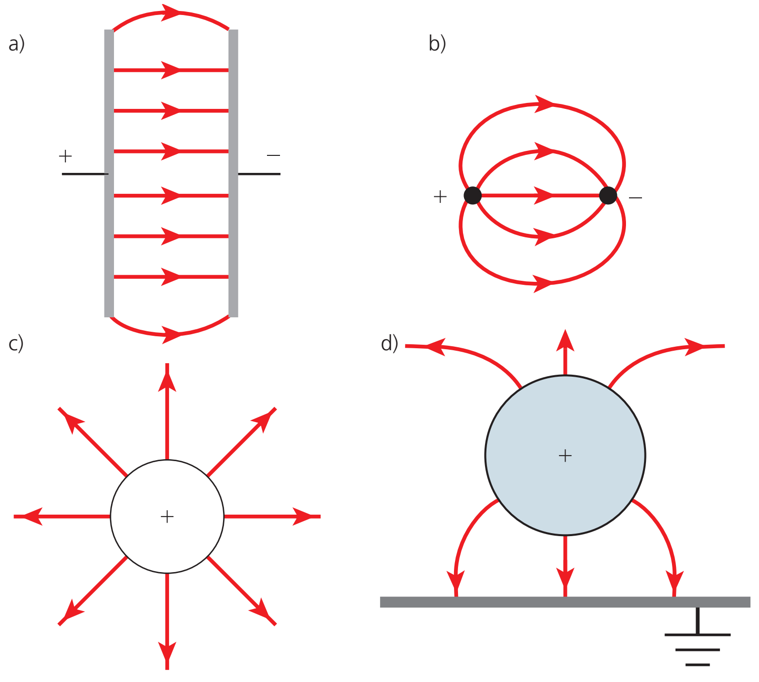
- a line of force starts on a positive charge and ends on a negative charge
- the lines of force are smooth curves that never touch or cross
- the strength of the electric field is indicated by the closeness of the lines: the closer the lines, the stronger the field.

Note that electric field and electric field strength are vector quantities.



▲ **Figure 18.3** Apparatus for investigating electric field patterns

The apparatus of Figure 18.3 can be used to show electric field patterns. Semolina is sprinkled on to the surface of a non-conducting oil and a high voltage is applied between the plates. The semolina becomes charged by induction and lines up along the lines of force. Some electric field patterns are illustrated in Figure 18.4. Note that the lines are always normal to a conducting surface. The pattern for a charged conducting sphere (Figure 18.4c) is of particular importance and will be considered in more detail in Topic 18.3.

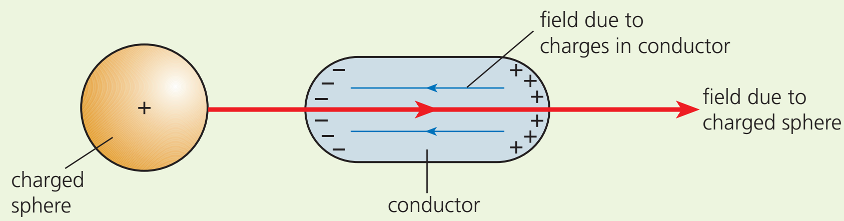


▲ **Figure 18.4** Some electric field patterns

EXTENSION

Induced fields and charges

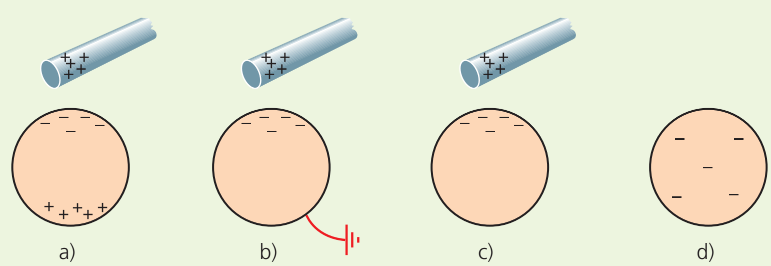
There is no resultant electric field inside a conductor unless this is being maintained by a source of electromotive force (e.m.f.). The reason for this is that the electrons are free to move in the conductor. As soon as a charged body is placed close to the conductor, the electric field of the charged body causes electrons in the conductor to move in the opposite direction to the electric field (the electrons are negatively charged). This is illustrated in Figure 18.5. The electrons create an electric field in the opposite direction to the field due to the charged body. The induced charges (the electrons) will stop moving when the two fields are equal and opposite. The consequence is that there is no electric field in the conductor.



▲ **Figure 18.5** Induced charges

Charging by induction

The effect illustrated in Figure 18.5 may be used to charge a conductor. The process is shown in Figure 18.6. A positively charged rod is placed near to an uncharged conductor that is insulated from earth. Induced charges appear on the insulated conductor, as shown in Figure 18.6a. The conductor is now earthed, as in Figure 18.6b. Electrons move from earth to neutralise the positive charge on the conductor. The earth connection is removed. The negative charge is still held on the conductor by the positively charged rod, as in Figure 18.6c. Finally, the charged rod is removed. The electrons on the conductor distribute themselves over its surface, as in Figure 18.6d. Note that if a negatively charged rod is used, the final charge on the conductor is positive.



▲ **Figure 18.6** Charging by induction

Electric field strength

The **electric field strength** at a point is defined as the force per unit charge acting on a small stationary positive charge placed at that point.

If the force experienced by a positive test charge $+q$ placed in an electric field is F, then the field strength E is given by

$$E = F/q$$

Note: Do not be confused by the use of the symbol E for field strength. This symbol is also used for energy!

The unit of field strength can be deduced from this expression. Since force is measured in newtons (N) and charge in coulombs (C), then the SI unit of field strength is newton per coulomb ($N\,C^{-1}$). We shall see later that the volt per metre ($V\,m^{-1}$) is another common SI unit for field strength. These two units are equivalent.

From the definition of electric field strength, the force F on a charge q in an electric field of strength is given by

$$F = qE$$

WORKED EXAMPLE 18A

Two parallel flat metal plates are separated by a distance of 5.0 cm. The uniform electric field between the plates is $2.0 \times 10^4\ Vm^{-1}$. Calculate the force on a charge of +5.0 nC situated mid-way between the plates.

Answer

force on electron = $qE = 5.0 \times 10^{-9} \times 2.0 \times 10^4$

$= \mathbf{1.0 \times 10^{-4}\,N}$

Questions

1 a Explain what is meant by an *electric field*.

b Sketch the electric field patterns:

i between two negatively charged particles

ii between a point positive charge and a negatively charged flat metal plate.

2 A positive and a negative charge of the same magnitude are on the same straight line as shown in Figure 18.7. State the direction of the electric field strength:

a at point A

b at point B

c at point C

d at point D.

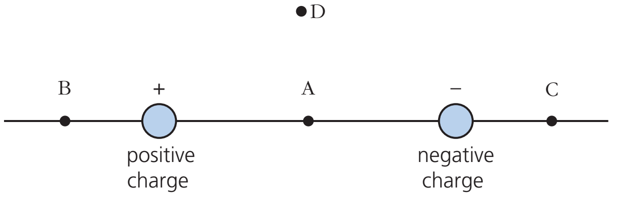


Figure 18.7

3 Calculate the acceleration on an electron that is in a uniform electric field of field strength $5.0 \times 10^2\,V\,m^{-1}$.

18.2 Uniform electric fields

A uniform field is where the electric field strength is the same at all points in the field.

In Figure 18.4a, the electric field pattern between the charged parallel plates consists of parallel, equally spaced lines, except near the edges of the plates. This shows that the electric field between charged parallel plates (as, for example, in a parallel plate capacitor [see Topic 19.1]) is uniform. This means that the force experienced by a charge is the same, no matter where the charge is placed within the field.

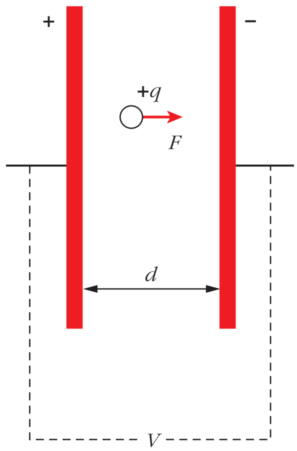


Figure 18.8

Figure 18.8 illustrates charged parallel plates that are a distance d apart with a potential difference V between them. A charge $+q$ in the uniform field between the plates has a force F acting on it. To move the charge towards the positive plate requires work to be done on the charge. Work is defined as the product of force and distance moved in the direction of the force. To move the charge from one plate to the other requires work W given by

$$W = Fd$$

From the definition of potential difference as the energy transferred per unit charge (Topic 9.2),

$$W = Vq$$

Thus, $W = Fd = Vq$ and, re-arranging,

$$F/q = V/d$$

But, F/q is the force per unit charge which is the field strength. Thus, for the uniform field, the field strength is given by

$$E = V/d$$

The equation gives an alternative unit for field strength, $\mathrm{V\,m^{-1}}$. The two units, $\mathrm{V\,m^{-1}}$ and $\mathrm{N\,C^{-1}}$ are equivalent.

It is assumed that the potential difference V changes at a constant rate over the distance d.

Where the change in potential difference varies with distance, then the small change ΔV of potential difference over a small distance Δd should be considered and then the electric field strength E is given by

$$E = \Delta V/\Delta d$$

WORKED EXAMPLE 18B

Two parallel metal plates are separated by a distance of 5.0 cm. The potential difference between the plates is 1000 V.

Calculate the electric field strength between the plates.

Answer

From $E = V/d$, $E = 1000/5.0 \times 10^{-2} = \mathbf{2.0 \times 10^4\,V\,m^{-1}}$.

Question

4 Two metal plates 15 mm apart have a potential difference of 750 V between them. The force on a small charged sphere placed between the plates is 1.2×10^{-7} N. Calculate:

a the strength of the electric field between the plates

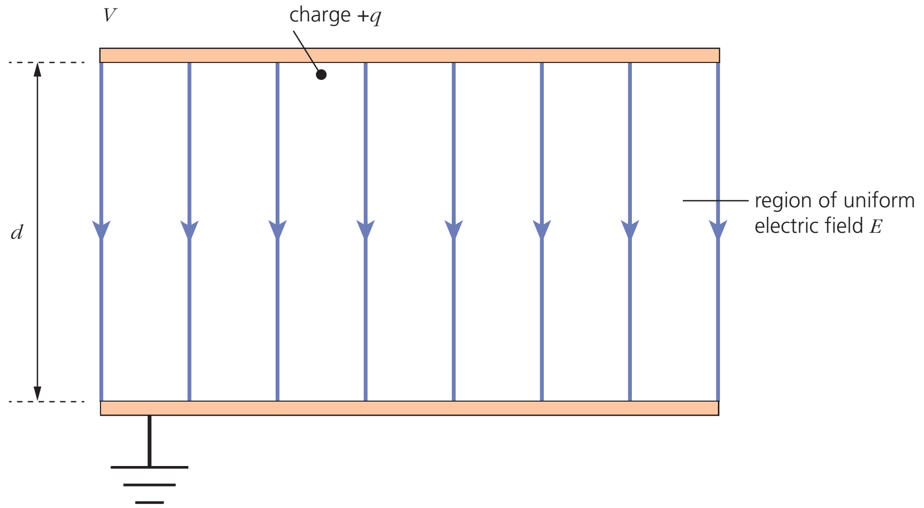
b the charge on the sphere.

Motion of charged particle in an electric field

Charged particle moving parallel to the field

A charged particle either at rest or moving parallel to an electric field experiences an electric force. The direction of the force depends on the sign of the charge. A particle having charge $+q$ is accelerated in the direction of the field by the electric force. For a uniform field, the force and hence the acceleration of the particle are constant. The equations of uniformly accelerated motion, studied in Topic 2, can be used to determine the motion of the particle.

A uniform electric field E is produced by a potential difference V across two horizontal parallel metal plates separated by distance d, as shown in Figure 18.9. A particle, charge $+q$ and mass m, initially at rest on the top plate, moves to the lower plate. The field does work on the particle and the charge gains kinetic energy.



▲ **Figure 18.9**

For a final speed v,

work done = gain in kinetic energy

$$Vq = \frac{1}{2}mv^2$$

and

$$v = \sqrt{(2Vq/m)}$$

Alternatively, the motion of the particle can be determined by considering the acceleration a produced by the force F due to the electric field. Acceleration a is given by

$$a = F/m = qE/m$$

For the particle moving from rest through distance d from the top plate to the lower plate, using $v^2 = u^2 + 2as$

$$v^2 = 2 \times qE/m \times d = 2 \times qV/dm \times d$$

$$v = \sqrt{(2Vq/m)}.$$

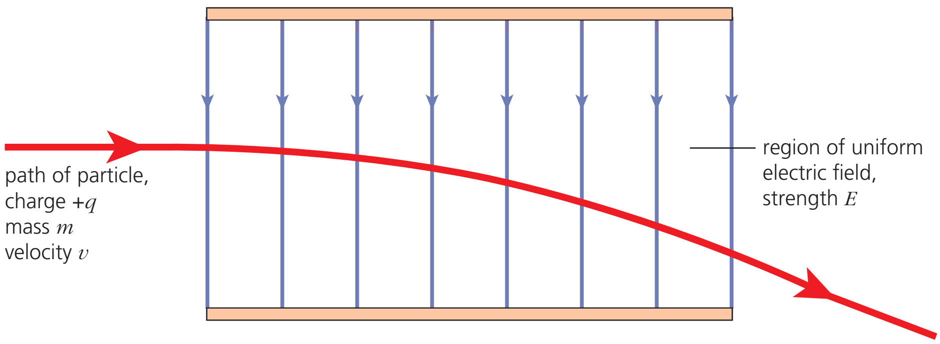
As expected, the same result!

A negatively charged particle would be accelerated in the opposite direction to the positively charged particle.

Note that gravitational effects on the particle have not been considered. The reason is that, for charged particles, the weight of the particle is negligible when compared to any force due to the electric field.

Charged particle moving with initial velocity perpendicular to the field direction

A charged particle, having charge $+q$ and mass m, enters a uniform electric field with velocity v normal to the direction of the electric field, as shown in Figure 18.10. The particle experiences a force at right angles to its initial direction. The particle will follow a parabolic path as it passes through the field.



▲ **Figure 18.10**

The analysis of the motion is similar to that of a particle of mass *m* moving in a uniform gravitational field with constant velocity in one direction and a constant acceleration in a perpendicular direction as described for projectile motion in Topic 2.

WORKED EXAMPLE 18C

Two parallel metal plates are separated by a distance of 15 mm. The plates are in a vacuum and the potential difference between the plates is 600 V.

An α-particle, mass 6.7×10^{-27} kg and charge $+3.2 \times 10^{-19}$ C, is initially at rest at the positively charged plate. Determine:

a the force on the particle

b the acceleration of the particle

c the speed of the particle as it reaches the negatively charged plate.

Answers

a force $= Eq = V/d \times q = (600 \times 3.2 \times 10^{-19})/(15 \times 10^{-3}) = \mathbf{1.28 \times 10^{-14}\,N}$

b acceleration $= F/m = (1.28 \times 10^{-14})/(6.7 \times 10^{-27}) = \mathbf{1.91 \times 10^{12}\,m\,s^{-2}}$

c $v^2 = 2as$, $v^2 = 2 \times 1.91 \times 10^{12} \times 15 \times 10\,\text{m}\,\text{s}^{-3}$

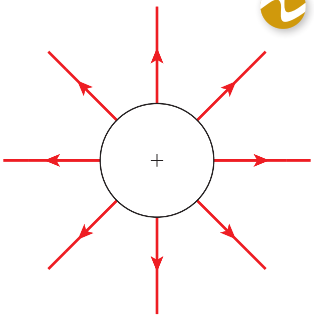
$v = \mathbf{2.4 \times 10^5\,m\,s^{-1}}$

Question

5 An electron, mass 9.1×10^{-31} kg and charge -1.6×10^{-19} C, enters an evacuated region between two horizontal plates, similar to that in Figure 18.10, with a horizontal velocity of $6.5 \times 10^7\,\text{m}\,\text{s}^{-1}$. The uniform vertical electric field has field strength $4.2 \times 10^5\,\text{V}\,\text{m}^{-1}$ between the plates. The length of the plates is 2.5 cm. Calculate the vertical displacement of the electron for its travel between the plates.

18.3 Electric force between point charges

Point charges



▲ **Figure 18.11** Electric field near an isolated charged sphere

Figure 18.11 shows an isolated, positively charged sphere. The electric field surrounding the sphere is shown by lines of force and since the sphere is positively charged the direction of the electric field is outwards from the sphere.

The sphere is a conductor and so the charge on the sphere distributes itself evenly around the surface of the sphere. However, from any position outside the sphere, the electric field lines appear to radiate from the centre of the sphere. Consequently:

For any point outside a spherical conductor, the charge on the sphere may be considered to act as a point charge at the centre of the sphere.

This is very similar to the idea that the mass of a uniform sphere can be considered to be point mass at the centre of the sphere (see Topic 13.2).

Force between point charges

We have already met the 'law of charges', namely

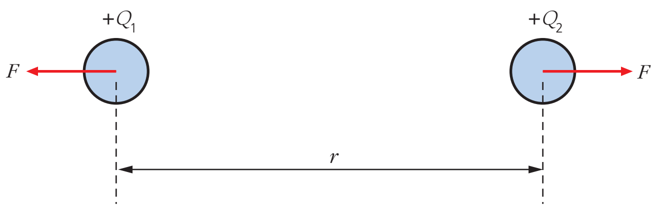
- Like charges repel.
- Unlike charges attract.

However, this law is purely qualitative and does not give any indication as to the magnitude of the forces.

In the late eighteenth century, the French scientist Charles Coulomb investigated the magnitude of the force between charges, and how this force varies with the charges involved and the distance between them. He discovered the following rule.

The force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them. This is known as **Coulomb's law**.

Coulomb's experiments made use of small, charged insulated spheres. Strictly speaking, the law applies to point charges, but it can be used for charged spheres provided that their radii are small compared with their separation.



▲ **Figure 18.12** Force between charged spheres

For point charges Q_1 and Q_2 situated a distance r apart (Figure 18.12), Coulomb's law gives the force F as

$$F \propto Q_1Q_2/r^2$$

or

$$F = kQ_1Q_2/(r^2)$$

where k is a constant of proportionality, the value of which depends on the insulating medium around the charges and the system of units employed. In SI units, F is measured in newtons, Q in coulombs and r in metres. Then the constant k is given as

$$k = \frac{1}{(4\pi\varepsilon_0)}$$

and so

$$F = \frac{Q_1Q_2}{(4\pi\varepsilon_0 r^2)}$$

when the charges are in a vacuum (free space). The quantity ε_0 is called the **permittivity of free space** (or the permittivity of a vacuum).

Notice that this equation has a similar form to that for Newton's law of gravitation between two point masses (see Topic 13.2). Both Newton's law of gravitation and Coulomb's law are inverse square laws of force. We say that the equations are analogous, or that there is an analogy between this aspect of electric fields and gravitational fields.

However, there are important differences:

- The electric force acts on charges, whereas the gravitational force acts on masses.
- The electric force can be attractive or repulsive, depending on the signs of the interacting charges, whereas two masses always attract each other.

The value of the permittivity of air is very close to that of a vacuum ($1.0005\varepsilon_0$), so the equation can be used for the force between charges in a vacuum or in air.

The value of the permittivity of free space is given as

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$$

where the unit represented by 'F' is called the farad and 1 F is $1\,\mathrm{C\,V^{-1}}$ (we shall find out about this unit in Topic 19).

Note that, when writing down the expression for Coulomb's law, in the first instance, it should be given in terms of ε_0 and not k. The value of k is $1/(4\pi\varepsilon_0) = 8.99 \times 10^9\,\mathrm{m\,F^{-1}}$.

Coulomb's law is often referred to as an **inverse square law** of force, because the variation of force with distance r between the charges is proportional to $1/r^2$. We have met another important inverse square law of force when we considered the gravitational force between two point masses (Topic 13.2).

WORKED EXAMPLE 18D

Calculate the force between two point charges, each of 1.4 μC, which are 6.0 cm apart in a vacuum.

(Permittivity of free space = $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.)

Answer

Using $F = Q_1Q_2/4\pi\varepsilon_0 r^2$,

$$F = (1.4 \times 10^{-6})^2/4\pi \times 8.85 \times 10^{-12} \times (6.0 \times 10^{-2})^2)$$

$$= \mathbf{4.9\,N}$$

Question

6 Calculate the force on an α-particle, charge $+3.2 \times 10^{-19}$ C, that is situated in a vacuum a distance of 3.2×10^{-7} m from a gold nucleus of charge $+1.3 \times 10^{-17}$ C in a vacuum.

(Permittivity of free space = $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.)

18.4 Electric field of a point charge

The **electric field strength** at a point is defined as the force per unit charge acting on a small stationary positive charge placed at that point (see Topic 18.1).

We have seen that the electric field due to an isolated point charge is **radial** (see Figure 18.11). We have to mention that the point charge is isolated. If any other object, charged or otherwise, is near it, the field would be distorted.

From Coulomb's law, the force on a test charge q a distance r from the isolated point charge Q is given by

$$F = Qq/4\pi\varepsilon_0 r^2$$

The electric field E at the location of the test charge q is given by $E = F/q$. Thus, the electric field strength due to the isolated point charge in a vacuum (free space) is

$$E = Q/4\pi\varepsilon_0 r^2$$

WORKED EXAMPLE 18E

In a simplified model of the hydrogen atom, the electron is at a distance of 5.3×10^{-11} m from the proton. The proton charge is $+1.6 \times 10^{-19}$ C. Calculate the electric field strength of the proton at this distance.

Answer

Assuming that the field is radial,

$$E = Q/4\pi\varepsilon_0 r^2 = 1.6 \times 10^{-19}/(4\pi \times 8.85 \times 10^{-12} \times (5.3 \times 10^{-11})^2)$$

$$= \mathbf{5.1 \times 10^{11}\ N\,C^{-1}}$$

Question

7 A Van de Graaff generator has a spherical metal dome of diameter 36 cm. The maximum permissible electric field strength at the surface of the dome is $2.0 \times 10^4\ \text{V m}^{-1}$.

Assuming that the charge on the dome acts as if it were all concentrated at the centre of the spherical dome, calculate the magnitude of this charge.

(Permittivity of free space = $8.85 \times 10^{-12}\ \text{F m}^{-1}$.)

18.5 Electric potential

We define electric potential in a similar way as we defined gravitational potential in Topic 13.4. That is, in terms of the change in energy, or work done, when a small charge q is moved between two points A and B in an electric field. It is the work done by the electric force in moving a small positive charge towards a point charge Q.

We already know that only *differences* in potential energy are measurable. We need to specify a reference point to act as a zero of electric potential energy and potential. In dealing with gravitational energy we often take the floor of the laboratory or the Earth's surface as zero, and measure $mg\Delta h$ from one of these. Similarly, in electrical problems, it is often convenient to take earth potential as zero, especially if part of the circuit is earthed. But the 'official' definition of the zero of electric potential is the potential of a point an infinite distance away. This means that

The **electrical potential** at a point in an electric field is defined as the work done per unit positive charge in bringing a small test charge from infinity to the point.

The symbol for potential is V, and its unit is the volt which is equivalent to the joule per coulomb (J C^{-1}).

Two points in an electric field are at the same potential if the work done moving a charge between them along any path is zero.

Two points may be at different potentials. The difference in the potentials is known as the *potential difference*. We have already met the idea in Topic 9.2 that the potential difference across a component in a circuit is the energy transferred per unit charge in moving charge between these points in the circuit. We can now use this idea to describe the potential difference between any points in an electric field.

WORKED EXAMPLE 18F

The electric potential at point A is 450 V and at point B it is –150 V. Calculate:

a the potential difference between points A and B

b the work done in moving a proton from point B to point A.

Answers

a potential difference = 450 – (–150) = **600 V**

b work is done on the proton to move it towards a positive charge (potential increases)

work done = $q\Delta V = 1.60 \times 10^{-19} \times 600$

$= \mathbf{9.6 \times 10^{-17}\,J}$

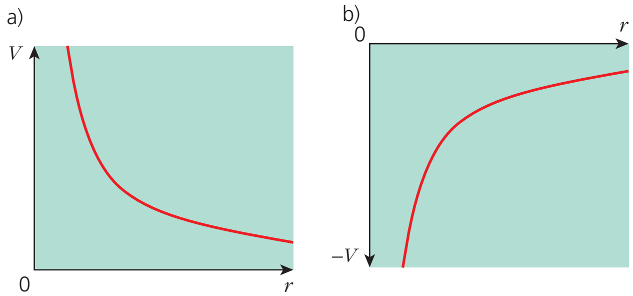
Electric potential in the field due to a point charge

The electric potential V at points in the field of a point charge Q is given by the expression

$$V = \frac{Q}{(4\pi\varepsilon_0 r)}$$

where r is the distance from the charge Q.

The potential very near a positive point charge is large, and decreases towards zero as we move away from the charge which is the source of the field. If the charge producing the field is negative, the potential is also negative and increases towards zero as the distance from the charge increases. Note that the variation of potential with distance is an inverse proportionality, and not the inverse square relationship that applies for the variation of field strength with distance.



▲ **Figure 18.13** Variation of the electric potential a) for a positive point charge and b) for a negative point charge

Electric field strength and electric potential

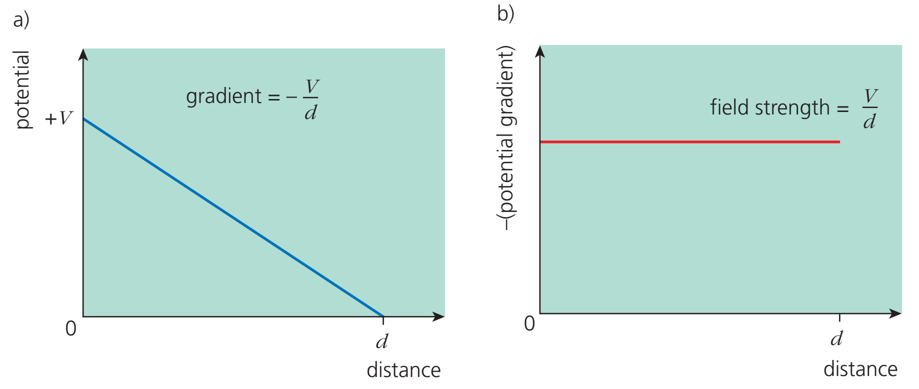
The rate of change of electric potential with distance is called the *potential gradient*. It can be shown that there is an important link between the electric field strength at any point and the electric potential gradient at that point.

The electric field strength is equal to the negative of the potential gradient at that point.

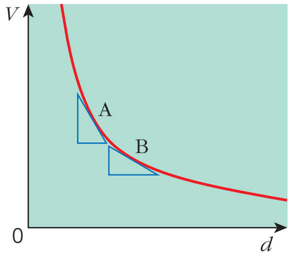
Thus, if we have a graph showing how the potential changes with distance in a field, the gradient of this graph at any point gives us the numerical value of the field strength at that point.

Note that the field strength and potential gradient have opposite signs because they are in opposite directions. If the difference in potential between two points in a field is negative (potential is decreasing) a small positive charge would tend to move in the direction of the electric field. If the difference in potential between two points in a field is positive, a small positive charge would tend to move in the opposite direction to that of the electric field.

For the special case of a uniform electric field, the field strength is constant and so the potential gradient is constant. This can be seen in Figure 18.14.



▲ **Figure 18.14 Graphs of the potential and the (negative of the) potential gradient for a uniform electric field**



▲ **Figure 18.15 Finding the gradient of the potential–distance curve**

For the electric field of a point charge, the field strength is *not* constant. Figure 18.15 shows the variation of the electric potential for a positive point charge. The electrical potential decreases more rapidly close to the point charge, as shown by the steep (negative) slope of the tangent to the curve at point A. At point B the potential gradient is still negative, but the slope of the tangent to the curve is not as steep. The potential gradient decreases with increasing distance from the point charge, showing (as expected) that the electric field strength decreases with distance. The gradient of the potential–distance curve at any point, in volts per metre ($V\,m^{-1}$), is equal to the negative of the electric field strength, measured in $V\,m^{-1}$ or newtons per coulomb ($N\,C^{-1}$).

Electric potential energy

We have seen that the **electrical potential** at a point in an electric field is defined as the work done per unit positive charge in bringing a small test charge from infinity to the point. For an object with charge q, then the electric potential energy of the object will be q times as large as for a charge of unit charge.

electric potential energy = charge × electric potential

We have also seen that the electric potential at a distance r from a point charge Q_1 is $Q_1/(4\pi\varepsilon_0 r)$.

Thus, if a point charge q is placed a distance r from the point charge Q, then the electric potential energy E of the two point charges is given by

$$E_P = \frac{Qq}{(4\pi\varepsilon_0 r)}$$

This means that the work done to bring together two isolated point charges q and Q so that their separation is r is $Qq/(4\pi\varepsilon_0 r)$.

Question

8 Two $+30\,\mu C$ charges are placed on a straight line 0.40 m apart. A $+0.5\,\mu C$ charge is to be moved a distance of 0.10 m along the line from a point midway between the charges. How much work must be done?

SUMMARY

- Like charges repel; unlike charges attract each other.
- When charged objects are placed near insulated conductors, there is a redistribution of charges giving rise to charging by induction.
- An electric field is a region of space where a stationary electric charge experiences a force.
- The direction of electric field lines shows the direction of the force on a positive charge placed in the field and the separation indicates the field strength – greater separation, smaller field strength.
- From a point outside a spherical conductor, the charge on the sphere can be treated as a point charge at its centre.
- Electric field strength is the force per unit positive charge $E = F/Q$.
- The electric field between parallel charge plates is uniform and given by the expression $E = \Delta V/\Delta d$
- An electric charge moving initially at right angles to a uniform electric field experiences a constant force and so is deflected into a parabolic path.
- The force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

  This is Coulomb's law: $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$ when the charges are in free space (a vacuum) or air.

  ε_0 is the permittivity of free space; its value is $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.
- The electric field strength E at a point in the field of an isolated point charge is given by $E = Q/(4\pi\varepsilon_0 r^2)$.
- The electric potential at a point in an electric field is the work done per unit charge in bringing a small positive test charge from infinity to the point
- The electric field strength at any point in a field is equal to the negative of potential gradient at that point.
- The electric potential V at a point in the field of an isolated point charge is given by $V = Q/(4\pi\varepsilon_0 r)$.
- Electric potential energy E_P of two point charges is given by $E_P = Qq/(4\pi\varepsilon_0 r)$.

END OF TOPIC QUESTIONS

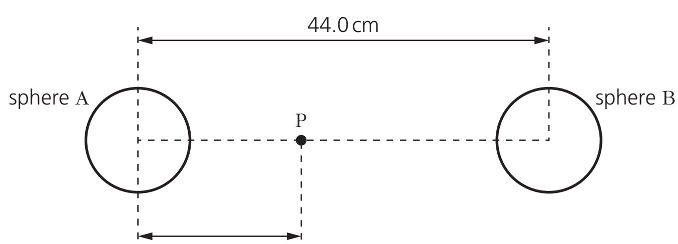
1 An α-particle has an initial energy of 6.2 MeV. It approaches head-on a nucleus of gold-197 ($^{197}_{79}$Au). The electronic charge is 1.6×10^{-19} C.

a Calculate the distance of closest approach of the α-particle to the gold nucleus.

b Suggest why your answer in **a** indicates an upper limit for the radius of a gold nucleus.

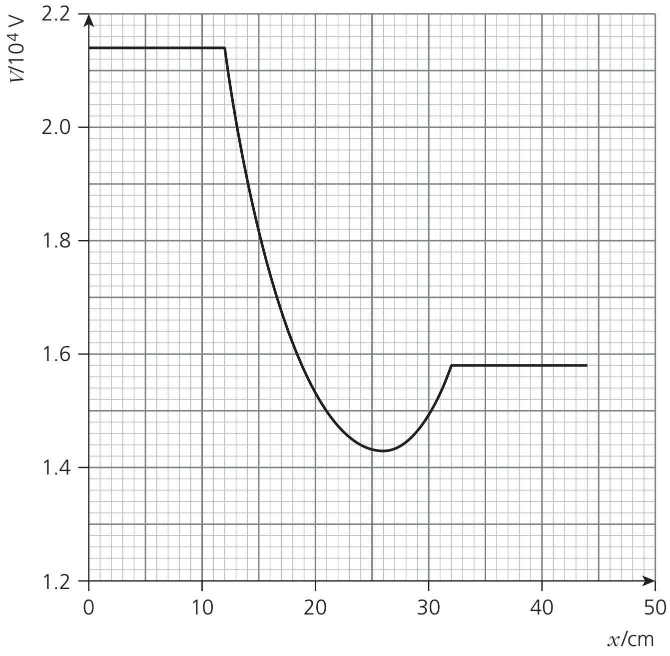
2 **a** State what is meant by *electric potential* at a point. [2]

b The centres of two charged metal spheres A and B are separated by a distance of 44.0 cm, as shown in Fig. 18.16.



▲ **Figure 18.16** (not to scale)

A moveable point P lies on the line joining the centres of the two spheres. Point P is a distance x from the centre of sphere A. The variation with distance x of the electric potential V at point P is shown in Fig. 18.17.



▲ **Figure 18.17**

i Use Fig. 18.17 to state and explain whether the two spheres have charges of the same, or opposite, sign. **[1]**

ii A positively-charged particle is at rest on the surface of sphere A. The particle moves freely from the surface of sphere A to the surface of sphere B.

1 Describe qualitatively the variation, if any, with distance x of the speed of the particle as it:

moves from x = 12 cm to x = 25 cm

passes through x = 26 cm

moves from x = 27 cm to x = 31 cm

reaches x = 32 cm **[4]**

2 The particle has charge 3.2×10^{-19} C and mass 6.6×10^{-27} kg. Calculate the maximum speed of the particle. **[2]**

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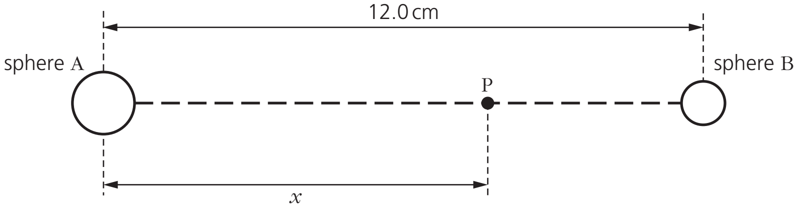
3 Two small charged metal spheres A and B are situated in a vacuum. The distance between the centres of the spheres is 12.0 cm, as shown in Fig. 18.18.



▲ **Figure 18.18** (not to scale)

The charge on each sphere may be assumed to be a point charge at the centre of the sphere.

Point P is a movable point that lies on the line joining the centres of the spheres and is distance x from the centre of sphere A.

The variation with distance x of the electric field strength E at point P is shown in Fig. 18.19.

a State the evidence provided by Fig. 18.19 for the statements that:

i the spheres are conductors, **[1]**

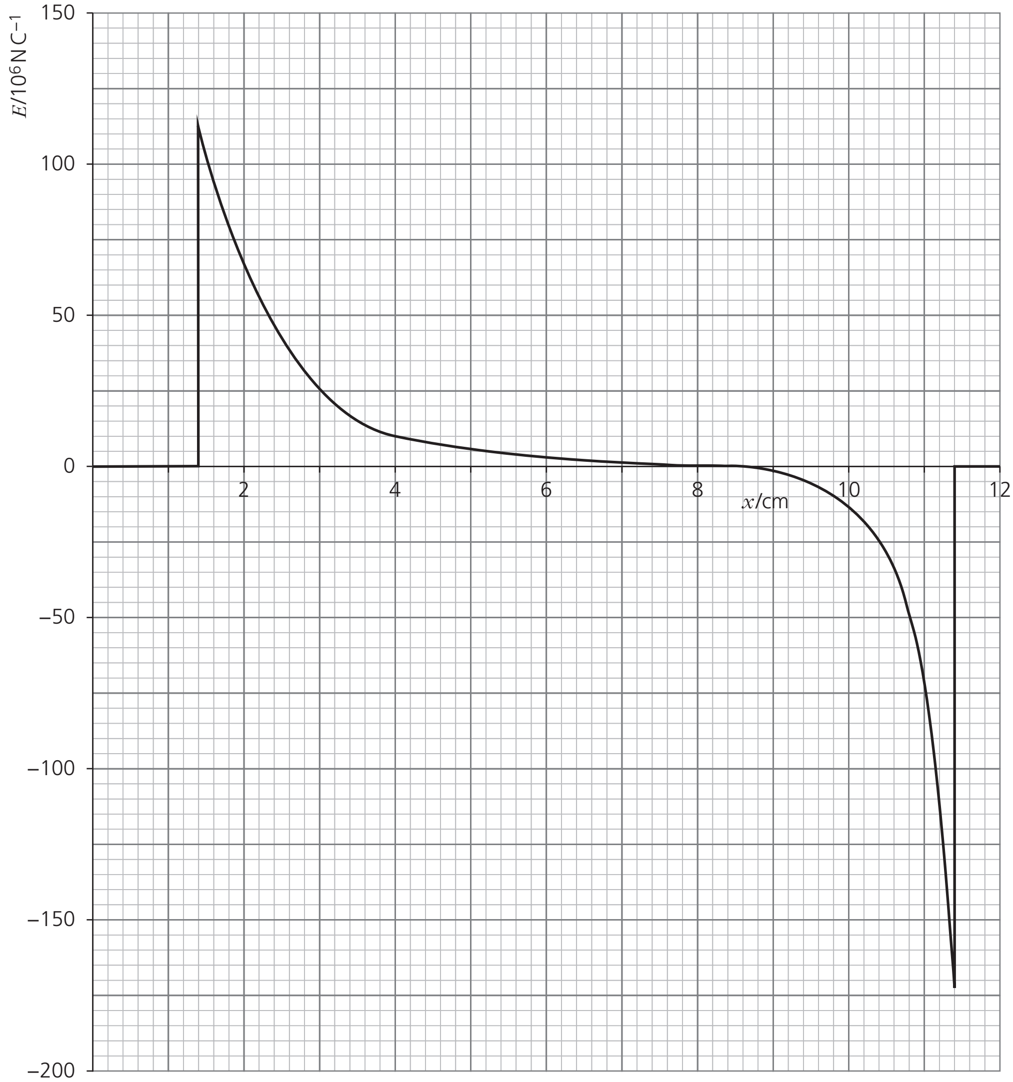
ii the charges on the spheres are either both positive or both negative. **[2]**

b **i** State the relation between electric field strength E and potential gradient at a point. **[1]**

ii Use Fig. 18.19 to state and explain the distance x at which the rate of change of potential with distance is:

maximum, **[2]**

minimum. **[2]**



▲ **Figure 18.19**

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4 A stationary α-particle, mass 6.6×10^{-27} kg and charge 3.2×10^{-19} C, is situated a distance of 4.7×10^{-14} m from the centre of a gold-197 ($^{197}_{79}Au$) nucleus.

a Calculate, for the α-particle and the gold nucleus:

i the electric force between them,

ii the electric potential energy.

b The α-particle is repelled from the stationary gold nucleus.

Use your answer in **a ii** to determine the maximum speed of the α-particle.

(The permittivity of free space ε_0 is 8.85×10^{-12} F m^{-1}.)

19 Capacitance

Learning outcomes

By the end of this topic, you will be able to:

19.1 Capacitors and capacitance

1. define capacitance, as applied to both isolated spherical conductors and to parallel plate capacitors
2. recall and use $C = Q/V$
3. derive, using $C = Q/V$, formulae for the combined capacitance for capacitors in series and in parallel
4. use the capacitance formulae for capacitors in series and in parallel

19.2 Energy stored in a capacitor

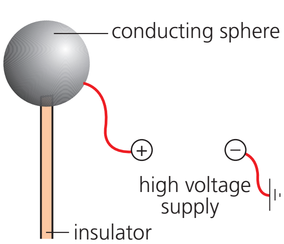
1. determine the electric potential energy stored in a capacitor from the area under the potential–charge graph
2. recall and use $W = \frac{1}{2}QV$ and hence $W = \frac{1}{2}CV^2$

19.3 Discharging a capacitor

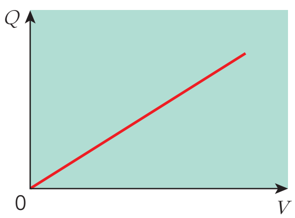
1. analyse graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor
2. recall and use $\tau = RC$ for the time constant for a capacitor discharging through a resistor
3. use equations of the form $x = x_0 e^{-(t/RC)}$ where x could represent current, charge or potential difference for a capacitor discharging through a resistor

Starting points

- ★ There is a uniform electric field between two charged parallel plates.
- ★ When charge is put on to an isolated conductor, the potential of the conductor rises.
- ★ Electric potential is the work done per unit positive charge in bringing a small positive test charge from infinity to the point.
- ★ The increase in potential of the conductor implies that more energy is being stored.



▲ **Figure 19.1** Charged spherical conductor



▲ **Figure 19.2** Relation between charge and potential

19.1 Capacitors and capacitance

Capacitance

Consider an isolated spherical conductor connected to a high voltage supply (Figure 19.1).

It is found that, as the potential of the sphere is increased, the charge stored on the sphere also increases. The graph showing the variation of charge Q on the conductor with potential V is shown in Figure 19.2.

It can be seen that charge Q is related to potential V by

$$Q \propto V$$

Hence,

$$Q = CV$$

where C is a constant which depends on the size and shape of the conductor. C is known as the **capacitance** of the conductor.

Capacitance C is the ratio of charge Q to potential V for a conductor.

That is

$$C = Q/V$$

Another chance for confusion! The letter C is used as an abbreviation for the unit of charge, the coulomb (see Topic 9.1). As an italic letter C it is used as the symbol for capacitance.

The unit of capacitance is the **farad** (symbol F). One farad is one coulomb per volt.

The farad is an inconveniently large unit. In electronic circuits and laboratory experiments, the range of useful values of capacitance is from about 10^{-12} F (1 picofarad, or 1 pF) to 10^{-3} F (1 millifarad, or 1 mF). (See Topic 1.2 for a list of decimal multiples and submultiples for use with units.)

Note that capacitance cannot apply to an insulator. When charge is placed on the sphere in Figure 19.1, the charge distributes itself so that there is one value of potential for the whole sphere. To define capacitance, the whole sphere must have the same value of potential. With an insulator, the charge would not be able to move, so that there would be different potentials at different points on the insulator. Thus, capacitance does not apply to an insulator.

Circuit components which store charge and, therefore, have capacitance are called **capacitors**.

WORKED EXAMPLE 19A

1 Show that the capacitance C of an isolated spherical conductor of radius r is given by $C = 4\pi\varepsilon_0 r$.

2 Calculate the charge stored on an isolated conductor of capacitance 280 μF when it is at a potential of 25 V.

Answers

1 Consider a charge $+Q$ on the surface of the sphere. The charge on an isolated conducting sphere may be considered to be a point charge at its centre. So, the potential V at the surface of the sphere is given by $V = Q/4\pi\varepsilon_0 r$.

Capacitance $C = Q/V = Q/(Q/4\pi\varepsilon_0 r) = 4\pi\varepsilon_0 r$

2 Using $C = Q/V$, we have $Q = CV = 280 \times 10^{-6} \times 25$

$= \mathbf{7.0 \times 10^{-3}\ C}$

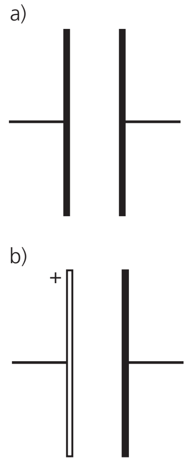
Question

1 The charge on an isolated sphere is 5.4 mC when its potential is 12 V. Calculate the capacitance, in microfarads, of the sphere.

Capacitors

The simplest capacitor in an electric circuit consists of two metal plates, with an air gap between them which acts as an insulator. This is called a parallel-plate capacitor. Figure 19.3a (overleaf) shows the circuit symbol for a capacitor. When the plates are connected to a battery, the battery transfers electrons from the plate connected to the positive terminal of the battery to the plate connected to the negative terminal. Thus the plates carry equal but opposite charges.

The capacitance of a parallel plate capacitor is defined as the charge stored on one plate per unit potential difference between the plates.



▲ **Figure 19.3** Circuit symbols for a) a capacitor and b) an electrolytic capacitor

Note that there are equal but opposite charges on the two plates. Thus, the capacitor does not store charge. We shall see later that a capacitor functions to store energy.

The capacitance of an air-filled capacitor can be increased by putting an insulating material, such as mica or waxed paper, between the plates. The material between the plates is called the **dielectric**. In a type of capacitor known as an **electrolytic capacitor** the dielectric is deposited by an electrochemical reaction. These capacitors must be connected with the correct polarity for their plates, or they will be damaged. The circuit symbol for an electrolytic capacitor is shown in Figure 19.3b. Electrolytic capacitors are available with capacitances up to about 1 mF.

EXTENSION

Factors affecting capacitance

As stated previously, the material used as a dielectric affects the capacitance of a capacitor. The other factors determining the capacitance are the area of the plates and the distance between them. Experiment shows that

The capacitance C is directly proportional to the area A of the plates, and inversely proportional to the distance d between them.

Putting these two factors together gives

$$C \propto A/d$$

where A is the area of one of the plates.

For a capacitor with air or a vacuum between the plates, the constant of proportionality is the permittivity of free space ε_0. Thus

$$C = \varepsilon_0 A/d$$

Since C is measured in farads, A in square metres and d in metres, we can see that the unit for ε_0 is farads per metre, $\mathrm{F\,m^{-1}}$ (see also Topic 18). The value of ε_0 is $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

We introduce a quantity called the **relative permittivity** ε_r of a dielectric to account for the fact that the use of a dielectric increases the capacitance.

The relative permittivity is defined as the capacitance of a parallel-plate capacitor with the dielectric between the plates divided by the capacitance of the same capacitor with a vacuum between the plates.

Relative permittivity ε_r is a ratio and has no units. Some values of relative permittivity are given in Table 19.1.

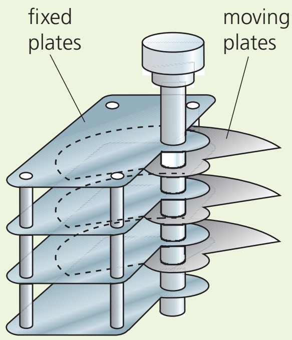
material	relative permittivity ε_r
air	1.0005
polyethylene (polythene)	2.3
sulfur	4
paraffin oil	4.7
mica	6
barium titanate	1200

▲ **Table 19.1** Relative permittivity of different dielectric materials

Including the relative permittivity factor, the full expression for the capacitance of a parallel-plate capacitor is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Variable capacitors (Figure 19.4) have one set of plates mounted on a spindle, so that the area of overlap can be changed.



▲ **Figure 19.4** Variable capacitor

Varying the capacitance in an electronic tuning circuit is one way of tuning in to different frequencies.

WORKED EXAMPLE 19B – EXTENSION

A parallel-plate, air-filled capacitor has square plates of side 30 cm that are a distance 1.0 mm apart. Calculate the capacitance of the capacitor.

Answer

Using $C = \varepsilon_0 \varepsilon_r A/d$

$C = 8.85 \times 10^{-12} \times 1 \times (30 \times 10^{-2})^2/1.0 \times 10^{-3}$

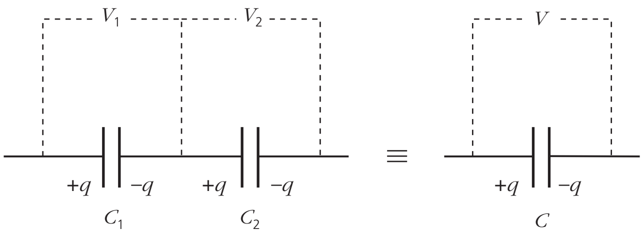
$= \mathbf{8.0 \times 10^{-10}\ F}$

Question

2 A capacitor consists of two metal discs of diameter 15 cm separated by a sheet of polythene 0.25 mm thick. The relative permittivity of polythene is 2.3. Calculate the capacitance of the capacitor. (ε_0 is $8.85 \times 10^{-12}\ F\ m^{-1}$.)

Capacitors in series and in parallel

In Figure 19.5, the two capacitors of capacitance C_1 and C_2 are connected in series.



▲ **Figure 19.5** Capacitors in series

We shall show that the combined capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

If the potential difference across the capacitor that is equivalent to these two capacitors is V and the charge stored on each plate is q, then $V = q/C$.

The potential difference across the combination is the sum of the potential differences across the individual capacitors, $V = V_1 + V_2$, and each capacitor has charge q on each plate. (Before applying any potential difference, the capacitors are uncharged. Then a charge of $+q$ induced on one plate of one capacitor will induce a charge of $-q$ on the other plate of the capacitor. This will, in turn, induce a charge of $+q$ on one plate of the second capacitor and $-q$ on its other plate. Remember that charge is always conserved.) Since $V_1 = q/C_1$ and $V_2 = q/C_2$, then

$$q/C = q/C_1 + q/C_2$$

Dividing each side of the equation by q, we have

$$1/C = 1/C_1 + 1/C_2$$

A similar result applies for any number of capacitors connected in series and

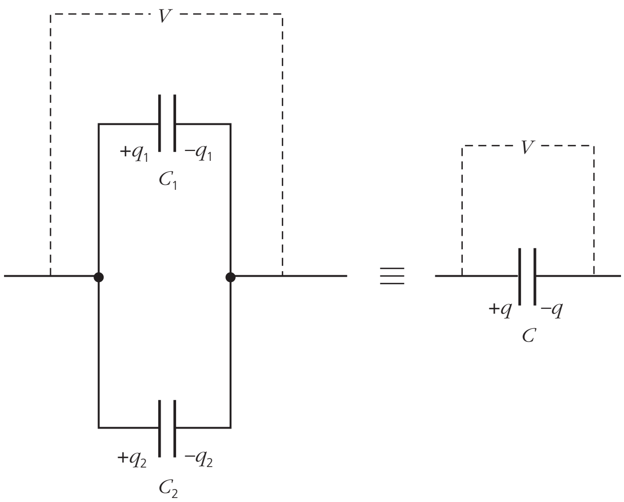
$$1/C = 1/C_1 + 1/C_2 + \ldots + 1/C_n$$

The reciprocal of the combined capacitance equals the sum of the reciprocals of the individual capacitances connected in series.

Note that:
- For two identical capacitors in series, the combined capacitance is equal to half of the value of each one.
- For capacitors in series, the combined capacitance is always less than the value of the smallest individual capacitance.

In Figure 19.6, the two capacitors of capacitance C_1 and C_2 are connected in parallel. We shall show that the combined capacitance C is given by

$$C = C_1 + C_2$$



▲ **Figure 19.6 Capacitors in parallel**

If the potential difference across the capacitor that is equivalent to the two capacitors in parallel is V and the charge stored on each plate is q, then $q = CV$. The total charge stored is the sum of the charges on the individual capacitors, $q = q_1 + q_2$, and there is the same potential difference V across each capacitor since they are connected in parallel. Since $q_1 = C_1V$ and $q_2 = C_2V$, then

$$CV = C_1V + C_2V$$

Dividing each side of the equation by V, we have

$$C = C_1 + C_2$$

The same result applies for any number of capacitors connected in parallel.

$$C = C_1 + C_2 + \ldots + C_n$$

The combined capacitance equals the sum of all the individual capacitances in parallel.

Note that the equation for capacitors in *series* is similar to the equation for resistors in *parallel*, and the equation for capacitors in parallel is similar to the equation for resistors in series (see Topic 10.2).

WORKED EXAMPLE 19C

1 A 50 μF capacitor, connected in parallel with a 10 μF capacitor, is connected to a 12 V supply. Calculate:
   - a the total capacitance
   - b the potential difference across each capacitor
   - c the charge stored on one plate of each capacitor.

2 A 12 μF capacitor, connected in series with a 6 μF capacitor is connected to a 15 V supply. Calculate:
   - a the combined capacitance
   - b the charge stored on one plate of each capacitor
   - c the potential difference across each capacitor.

Answers

1 a Using the equation for capacitors in parallel, $C = C_1 + C_2 = 50 + 10 =$ **60 μF**

b The potential difference across each capacitor is the same as the potential difference across the supply. This is **12 V**.

c Using $Q = CV$, the charge stored on the 50 μF capacitor is $50 \times 10^{-6} \times 12 =$ **6.0×10^{-4} C**

The charge stored on the 10 μF capacitor is $10 \times 10^{-6} \times 12 =$ **1.2×10^{-4} C**.

2 a Using the equation for capacitors in series,

$$1/C = 1/C_1 + 1/C_2 = 1/(12 \times 10^{-6}) + 1/(6 \times 10^{-6}) = 2.5 \times 10^5$$

Thus $C =$ **4 μF**.

b The charge stored by each capacitor is the same as the charge stored by the combination, so $Q = CV = 4 \times 10^{-6} \times 15 =$ **6.0×10^{-5} C**

c Using $V = Q/C$, the potential difference across the 12 μF capacitor is $(6.0 \times 10^{-5})/(12 \times 10^{-6})$ **= 5.0 V**

The potential difference across the 6 μF capacitor is $(6.0 \times 10^{-5})/(6 \times 10^{-6}) =$ **10.0 V**.

Note that the two potential differences add up to the supply voltage.

Question

3 a A 250 μF capacitor is connected to a 6.0 V supply. Calculate the charge stored on one plate of the capacitor.

b The capacitor in **a** is now disconnected from the supply and connected to an uncharged 250 μF capacitor.

i Explain why:

1 the capacitors are in parallel, rather than series

2 the total charge stored by the combination must be the same as the answer to **a**.

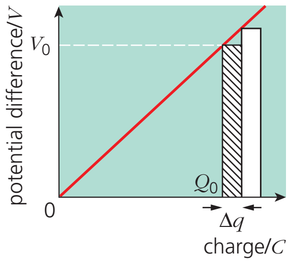
ii Calculate the capacitance of the combination.

iii Calculate the potential difference across each capacitor.

iv Calculate the charge stored on one plate of each capacitor.

19.2 Energy stored in a capacitor

When charging a capacitor, work is done by the battery to move charge on to the capacitor, separating positive and negative charges. Energy is transferred from the power supply and is stored as **electric potential energy** in the capacitor.



▲ **Figure 19.7 Graph of potential difference against charge for a capacitor**

There are many applications for the ability of capacitors to store energy. Camera flash units use a capacitor to store energy. The capacitor takes a few seconds to charge when connected to the battery in the camera. Then the energy is discharged very rapidly when the capacitor is connected to the flash-bulb to give a short but intense flash.

Since $Q = CV$, the charge stored on each plate of a capacitor is directly proportional to the potential difference across the capacitor (see Figure 19.7).

From the definition of potential difference (Topic 9), the work done to charge the capacitor (and therefore, the energy transferred to the capacitor) is the product of the potential difference and the charge. That is,

$$W \text{ (and } E_p) = VQ$$

However, while more and more charge is transferred to the capacitor, the potential difference is increasing. Suppose the potential difference is V_0 when the charge stored is Q_0. When a further small amount of charge Δq is supplied at an average potential difference V_0, the energy transferred is given by

$$\Delta E_p = V_0 \Delta q$$

which is equal to the area of the hatched strip in Figure 19.7. Similarly, the energy transferred when a further charge Δq is added is given by the area of the next strip, and so on. If the amount of charge Δq is very small, the strips become very thin and their combined areas are just equal to the area between the graph line and the horizontal axis. Thus

The energy transferred from the battery when a capacitor is charged is given by the area under the graph line when charge (x-axis) is plotted against potential difference (y-axis).

Because the graph is a straight line through the origin, this area is just the area of the right-angled triangle formed by the line and the charge axis. Thus

$$E_p = \tfrac{1}{2}QV$$

This is the expression for the energy transferred from the battery in charging the capacitor. This is electric potential energy, and it is released when the capacitor is discharged. Since $C = Q/V$, this expression can be written in different forms.

$$E_p = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(Q^2/C)$$

WORKED EXAMPLE 19D

1 Calculate the energy stored by a 280 μF capacitor charged to a potential difference of 12 V.

2 A camera flash-lamp uses a 5000 μF capacitor which is charged by a 9 V battery. The capacitor is then disconnected from the battery. Calculate the energy transferred when the capacitor is discharged through the lamp so that the final potential difference across its plates is 4.0 V.

Answers

1 Using $E_p = \tfrac{1}{2}CV^2$, $E_p = \tfrac{1}{2} \times 280 \times 10^{-6} \times 12^2 =$ **20 mJ**.

2 Energy change $= \tfrac{1}{2}CV_1^{\,2} - \tfrac{1}{2}CV_2^{\,2}$

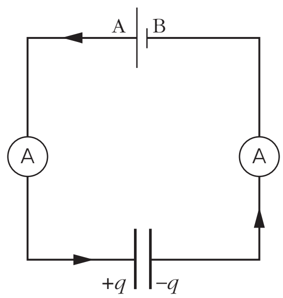
$= \tfrac{1}{2} \times 5000 \times 10^{-6} \times (9^2 - 4^2) =$ **0.163 J**

Note: $(V_1^{\,2} - V_2^{\,2})$ is not equal to $(V_1 - V_2)^2$ – a common mistake amongst students!

Question

4 A camera flash-lamp uses a 5000 μF capacitor which is charged by a battery of e.m.f. *E*. When the capacitor fully discharges, the energy transferred is 0.20 J. Calculate the charge stored on one plate of the fully charged capacitor.

19.3 Discharging a capacitor

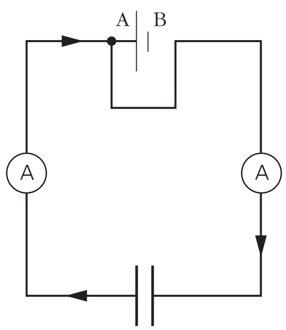


▲ **Figure 19.8** Charging a capacitor

When a battery is connected across a capacitor, as shown on Figure 19.8, electrons are transferred so that one plate becomes positively charged and the other, negatively charged. The capacitor has become charged.

Sensitive ammeters connected to each plate of the capacitor would indicate electrons moving onto the negative plate and at the same time, leaving the positive plate. The needles of the meters would both flick in the same direction and return to zero, indicating a momentary pulse of current.

The meters would show equal currents for equal short times, indicating that the charge on each plate has the same magnitude. The current and the charging process stop when the potential difference across the capacitor is equal to the e.m.f. of the battery.



▲ **Figure 19.9** Discharging a capacitor

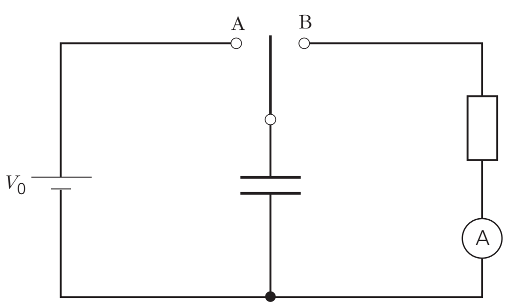
When the battery lead to terminal B is disconnected and joined to terminal A (Figure 19.9) so that the battery is no longer in the circuit, both meters give momentary equal flicks in the opposite direction to when the capacitor was being charged.

This time, a current in the opposite direction has moved the charge $-q$ from the right-hand plate to cancel the charge of $+q$ on the left-hand plate. The capacitor has become discharged.

Remember that in metal wires the current is carried by free electrons. These move in the opposite direction to that of the conventional current (see Topic 9). When the capacitor is charged, electrons move from the negative terminal of the battery to the right-hand plate of the capacitor, and from the left-hand plate to the positive terminal of the battery. When the capacitor is discharged, electrons flow from the negative right-hand plate of the capacitor to the positive left-hand plate.

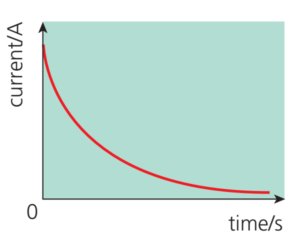
The experiment described using the circuit in Figure 19.9 showed that there is a momentary current when a capacitor discharges. A resistor connected in series with the capacitor will reduce the current, so that the capacitor discharges more slowly.

The circuit shown in Figure 19.10 can be used to investigate more precisely how a capacitor discharges.



▲ **Figure 19.10** Circuit for investigating capacitor discharge

When the two-way switch is connected to point A, the capacitor will charge until the potential difference between its plates is equal to the e.m.f. V_0 of the supply. When the switch is moved to B, the capacitor will discharge through the resistor. When the switch makes contact with B, the current can be recorded at regular intervals of time as the capacitor discharges.



▲ **Figure 19.11** Graph of current against time for capacitor discharge

A graph of the discharge current against time is shown in Figure 19.11.

The current is seen to change rapidly at first, and then more slowly. More detailed analysis shows that the decrease is **exponential** – *the current decreases by the same fraction over equal time intervals*. We shall meet exponential changes again when we deal with the decay of radioactive substances (Topic 23.2) and the attenuation of ultrasound and X-rays (Topic 24).

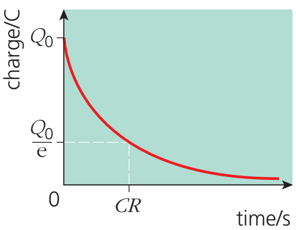
All exponential decay curves have an equation of the form

$$x = x_0 e^{-kt}$$

where x is the quantity that is decaying (and x_0 is the value of x at time $t = 0$), e to three decimal places is the number 2.718 (the base of natural logarithms) and k is a constant characteristic of the decay. A large value of k means that the decay is rapid, and a small value means a slow decay.

The solution for the discharge of a capacitor of capacitance C through a resistor of resistance R is of the form

$$Q = Q_0 e^{-t/CR}$$



▲ **Figure 19.12** Graph of charge against time for capacitor discharge

The graphs of Figures 19.11 and 19.12 have exactly the same shape, and thus the equation for the discharge current I in a capacitor may be written as

$$I = I_0 e^{-t/CR}$$

Furthermore, since for a capacitor Q is proportional to V, then the solution for the potential difference V as the capacitor discharges can be written as

$$V = V_0 e^{-t/CR}$$

WORKED EXAMPLE 19E

Calculate the time taken to discharge a capacitor of capacitance 2.5 μF through a resistance of 0.5 MΩ until the potential difference across it has been reduced by 50%.

Answer

Using $V = V_0 e^{-t/CR}$, $0.5 = e^{-t/(2.5 \times 10^{-6} \times 0.5 \times 10^{6})} = e^{-t/1.25}$.

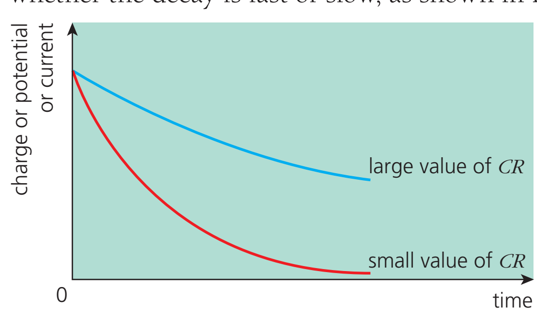
Taking natural logarithms of both sides, $\ln e^x = x$ so

$\ln 0.5 = -t/1.25$, $t = 0.693 \times 1.25 =$ **0.87 s**

Time constant

As time progresses, the exponential curve in Figure 19.12 gets closer and closer to the time axis, but never actually meets it. Thus, it is not possible to quote a time for the capacitor to discharge completely.

However, the quantity CR in the decay equation may be used to give an indication of whether the decay is fast or slow, as shown in Figure 19.13.



▲ **Figure 19.13** Decay curves for large and small time constants

CR is called the **time constant** of the capacitor–resistor circuit.

CR has the units of time, and is measured in seconds.

We can easily show that *CR* has units of time. From $C = Q/V$ and $R = V/I$, then $CR = Q/I$. Since charge Q is in coulombs and current I is in amperes, and one ampere is equal to one coulomb per second, Q/I is in seconds.

To find the charge Q on the capacitor plates after a time $t = CR$, we substitute in the exponential decay equation

$$Q = Q_0 e^{-CR/CR} = Q_0 e^{-1} = Q_0/e = Q_0/2.718$$

Thus

The time constant is the time for the charge to have decreased to 1/e (or 1/2.718) of its initial value.

Remember that the basic expression for the decay can be in terms of charge on one plate or current in the connecting leads or potential difference across the capacitor. So, time constant can be defined in terms of charge, current or potential difference.

In one time constant the charge stored by the capacitor drops to 1/e (roughly one-third) of its initial value. During the next time constant it will drop by the same ratio, to $1/e^2$, about one-ninth of the value at the beginning of the decay.

WORKED EXAMPLE 19F

A 250 μF capacitor is connected to a 12 V supply, and is then discharged through a 200 kΩ resistor. Calculate:

a the initial charge stored by the capacitor

b the initial discharge current

c the value of the time constant

d the charge on the plates after 100 s

e the time at which the remaining charge is 1.8×10^{-3} C.

Answers

a From $Q = CV$, we have $Q = 250 \times 10^{-6} \times 12 =$ $\mathbf{3.0 \times 10^{-3}\ C}$.

b From $I = V/R$, we have $I = 12/(200 \times 10^3) =$ $\mathbf{6.0 \times 10^{-5}\ A}$.

c $CR = 250 \times 10^{-6} \times 200 \times 10^3 = \mathbf{50\ s}$.

d After 50 s, the charge on the plates is $Q_0/e = 3.0 \times 10^{-3}/2.718 = 1.1 \times 10^{-3}$ C; after a further 50 s, the charge is $1.1 \times 10^{-3}/2.718 =$ $\mathbf{4.1 \times 10^{-4}\ C}$.

e Using $Q = Q_0 e^{-t/CR}$, $1.8 \times 10^{-3} = 3.0 \times 10^{-3} e^{-t/50}$, or $0.60 = e^{-t/50}$.
Taking natural logarithms of both sides, $\ln 0.60 = -0.510 = -t/50$, or $t = \mathbf{26\ s}$.

Question

5 A 12.0 μF capacitor is charged from a 20 V battery, and is then discharged through a 0.50 MΩ resistor. Calculate:

a the initial charge on the capacitor

b the charge on the capacitor 2.8 s after the discharge starts

c the potential difference across the capacitor at this time.

Uses of capacitors

Some uses of capacitors have already been outlined. As will be seen in Topic 21.2, capacitors may also be used in a circuit to give a smoother output p.d and reduce the 'ripple' on rectified current or voltage.

SUMMARY

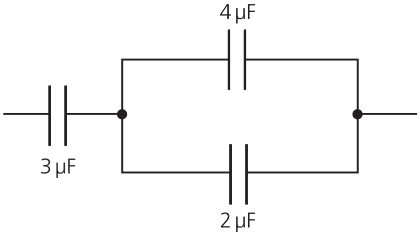
- A capacitor stores energy. A capacitor allows the storage of *separated* charges.
- The capacitance C of an isolated conductor is given by $C = Q/V$, where Q is the charge on the conductor and V is its potential.
- The capacitance C of a parallel plate capacitor is given by $C = Q/V$, where Q is the charge on one plate of the capacitor and V is the potential difference V between its plates.
- The unit of capacitance, the farad (F), is one coulomb per volt.
- The energy stored in a charged capacitor is given by $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C$.
- The area under a potential–charge graph is equal to the energy stored in a capacitor.
- The equivalent capacitance C of two or more capacitors connected in series is given by $1/C = 1/C_1 + 1/C_2 + \ldots$
- The equivalent capacitance C of two or more capacitors connected in parallel is given by $C = C_1 + C_2 + \ldots$
- When a charged capacitor discharges, the charge on the plates decays exponentially. The equation for the decay is $Q = Q_0e^{-t/CR}$ where Q_0 is the initial charge.
- The same decay equation $x = x_0e^{-t/CR}$ applies to the charge Q on one plate of the capacitor, to the potential difference V across the capacitor and to the discharge current I.
- The time constant τ of the circuit, given by $\tau = CR$, is the time for the quantity Q, V or I to decay to 1/e of its initial value.
- The time taken for the quantity Q, V or I to decay to any later value can be found by re-arranging the equation and taking logarithms to the base e (ln).

END OF TOPIC QUESTIONS

1 Fig. 19.14 shows an arrangement of capacitors.

a Calculate the total capacitance of this arrangement.

b The 4 μF capacitor is disconnected. Calculate the new capacitance.



▲ **Figure 19.14**

2 A 15 μF capacitor is charged from a 6.0 V battery.

a Calculate:

i the electric potential energy stored by the capacitor,

ii the charge stored on each plate of the capacitor.

b The charged capacitor is discharged through a 200 kΩ resistor. Calculate:

i the initial discharge current,

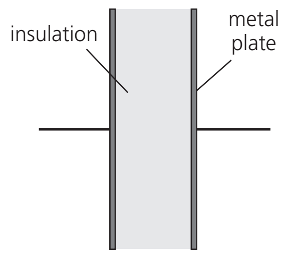
ii the time constant,

iii the potential difference across the capacitor after the capacitor has been discharging for 4.8 s.

3 **a** **i** Define *capacitance*. **[1]**

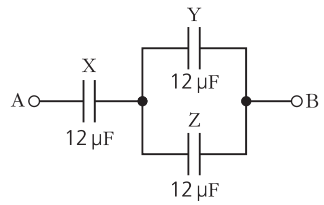
ii A capacitor is made of two metal plates, insulated from one another, as shown in Fig. 19.15.

Explain why the capacitor is said to store energy but not charge. **[4]**



▲ **Figure 19.15**

b Three uncharged capacitors X, Y and Z, each of capacitance 12 μF, are connected as shown in Fig. 19.16.



▲ **Figure 19.16**

A potential difference of 9.0 V is applied between points A and B.

i Calculate the combined capacitance of the capacitors X, Y and Z. [2]

ii Explain why, when the potential difference of 9.0 V is applied, the charge on one plate of capacitor X is 72 μC. [2]

iii Determine:

the potential difference across capacitor X, [1]

the charge on one plate of capacitor Y. [2]

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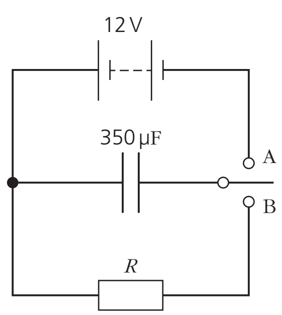
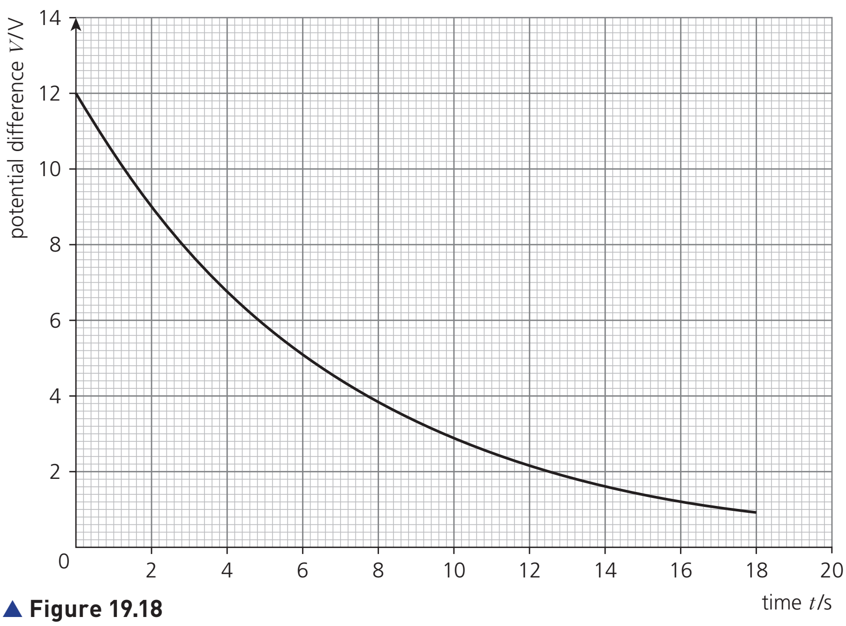
4 A capacitor of capacitance 350 μF and a resistor of resistance R are connected in to the circuit shown in Fig. 19.17. The variation with time of the potential difference V across the capacitor is shown in Fig. 19.18.

When the switch is in position A, the capacitor is charged to a potential difference of 12 V. The switch is then moved to position B so that, at time $t = 0$, the capacitor begins to discharge. The variation with time t of the potential difference V across the capacitor is shown in Fig. 19.18.



▲ **Figure 19.17**



▲ **Figure 19.18**

a For the discharge of the capacitor through the resistor:

i state what is meant by the *time constant,*

ii use data from Fig. 19.18 to determine the value of the time constant,

iii use your answer in **ii** to determine the resistance R.

b A second resistor, also of resistance R, is connected in series with the resistor in Fig. 19.17. The capacitor is re-charged and then allowed to discharge.

i By reference to your answer in **a ii**, state the new value of the time constant.

ii On a copy of Fig. 19.18, sketch for this second discharge, the variation with time t of the potential difference V across the capacitor for time $t = 0$ to time $t = 18$ s.

20 Magnetic fields

Learning outcomes

By the end of this topic, you will be able to:

20.1 Concept of a magnetic field

1 understand that a magnetic field is an example of a field of force produced either by moving charges or by permanent magnets
2 represent a magnetic field by field lines

20.2 Force on a current-carrying conductor

1 understand that a force might act on a current-carrying conductor placed in a magnetic field
2 recall and use the equation $F = BIL\sin\theta$, with directions as interpreted by Fleming's left-hand rule
3 define magnetic flux density as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field

20.3 Force on a moving charge

1 determine the direction of the force on a charge moving in a magnetic field
2 recall and use $F = BQv\sin\theta$
3 understand the origin of the Hall voltage and derive and use the expression $V_H = BI/(ntq)$ where t = thickness
4 understand the use of a Hall probe to measure magnetic flux density
5 describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle
6 explain how electric and magnetic fields can be used in velocity selection

20.4 Magnetic fields due to currents

1 sketch magnetic field patterns due to the currents in a long straight wire, a flat circular coil and a long solenoid
2 understand that the magnetic field due to the current in a solenoid is increased by a ferrous core
3 explain the origin of the forces between current-carrying conductors and determine the direction of the forces

20.5 Electromagnetic induction

1 define magnetic flux as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density
2 recall and use $\Phi = BA$
3 understand and use the concept of magnetic flux linkage
4 understand and explain experiments that demonstrate:
   - that a changing magnetic flux can induce an e.m.f. in a circuit
   - that the induced e.m.f. is in such a direction as to oppose the change producing it
   - the factors affecting the magnitude of the induced e.m.f.
5 recall and use Faraday's law and Lenz's law of electromagnetic induction

Starting points

★ Identify magnetic materials.
★ Magnets create a magnetic field around them.
★ Magnetic fields are produced by current-carrying conductors and coils.
★ The law of magnets – like poles repel, unlike poles attract.

20.1 Concept of a magnetic field

Some of the properties of magnets have been known for many centuries. The ancient Greeks discovered an iron ore called lodestone which, when hung from a thread, would come to rest always pointing in the same direction. This was the basis of the magnetic compass which has been in use since about 1500BCE as a means of navigation.

The magnetic compass is dependent on the fact that a freely suspended magnet will come to rest pointing north–south. The ends of the magnet are said to be poles. Th pole pointing to the north is referred to as the north-seeking pole (the north pole or N-pole) and the other, the south-seeking pole (the south pole or S-pole). It is now known that a compass behaves in this way because the Earth is itself a magnet.

Magnets exert forces on each other. These forces of either attraction or repulsion are used in many children's toys, in door catches and 'fridge magnets'. The effects of the forces may be summarised in the law of magnets.

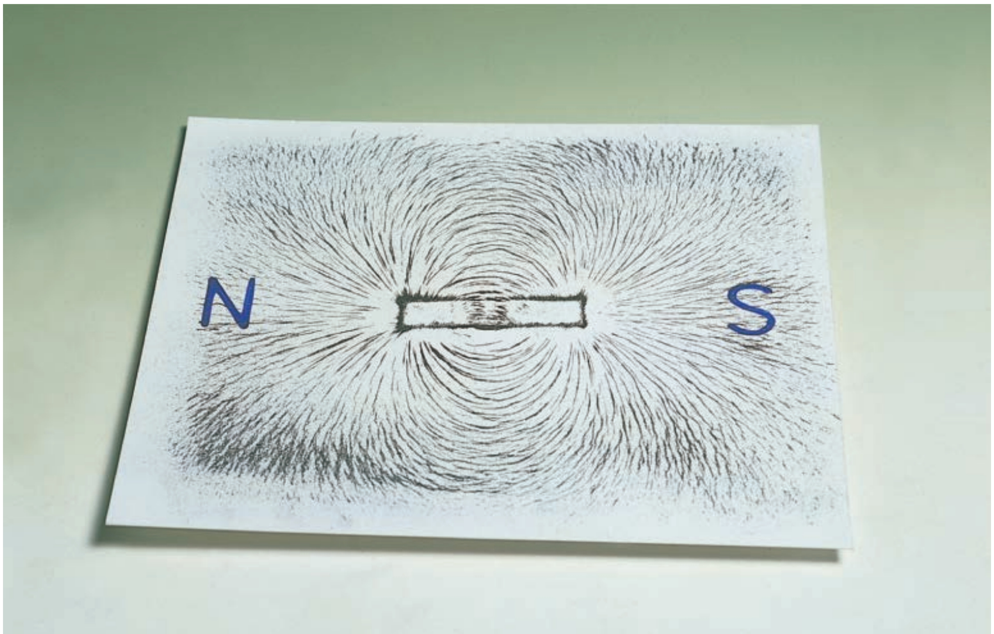
Like poles repel.

Unlike poles attract.

The law of magnets implies that around any magnet, there is a region where a magnetic pole will experience a force. This region is known as a **magnetic field**.

A magnetic field is a region of space where a magnetic pole experiences a force.

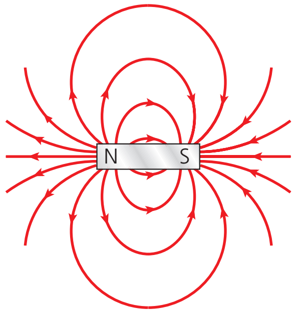
Magnetic fields are not visible but they may be represented by lines of magnetic force or **magnetic field lines**. We will return later in this topic to a fuller understanding of how the density or relative spacing of the field lines can be related to a quantity called magnetic 'flux'. A simple way of imagining magnetic field lines is to think of one such line as the direction in which a free magnetic north pole would move if placed in the field. Magnetic field lines may be plotted using a small compass (a plotting compass) or by the use of iron filings and a compass (Figure 20.1).



▲ **Figure 20.1** The iron filings line up with the magnetic field of the bar magnet which is under the sheet of paper. A plotting compass will give the direction of the field.

The magnetic field lines of a bar magnet are shown in Figure 20.2.

Effects due to the Earth's magnetic field have not been included since the Earth's field is relatively weak and would be of significance only at some distance from the magnet. It is important to realise that, although the magnetic field has been drawn in two dimensions, the actual magnetic field is three-dimensional.



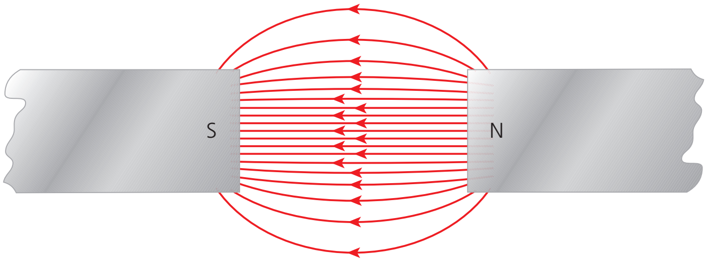
▲ **Figure 20.2** Magnetic field pattern of a bar magnet

For any magnetic field:

- the magnetic field lines start at a north pole and end at a south pole
- the arrow on each line shows the direction in which a free magnetic north pole would move if placed at that point on the line
- the magnetic field lines are smooth curves which never touch or cross
- the strength of the magnetic field is indicated by the closeness of the lines – the closer the lines, the stronger the magnetic field.

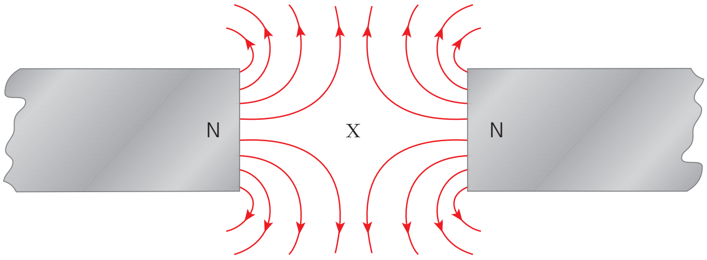
It can be seen that these properties are very similar to those for electric field lines (Topic 18.1).

Figure 20.3 illustrates the magnetic field pattern between the north pole of one magnet and the south pole of another. This pattern is similar to that produced between the poles of a horseshoe magnet.



▲ **Figure 20.3** Magnetic field pattern between a north and south pole

Figure 20.4 shows the magnetic field between the north poles of two magnets. The magnetic field due to one magnet opposes that due to the other. The field lines cannot cross and consequently there is a point X, known as a **neutral point**, where there is no resultant magnetic field because the two fields are equal in magnitude but opposite in direction.

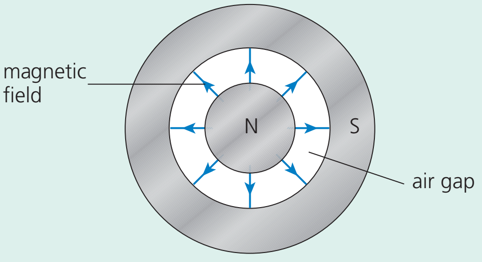


▲ **Figure 20.4** Magnetic field pattern between two north poles

WORKED EXAMPLE 20A

A circular magnet is made with its north pole at the centre, separated from the surrounding circular south pole by an air gap. Draw the magnetic field lines in the gap (Figure 20.5).

Answer



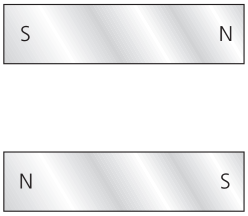
▲ **Figure 20.5**

Questions

1 Draw a diagram to illustrate the magnetic field between the south poles of two magnets.

2 Two bar magnets are placed on a horizontal surface (Figure 20.6).

Draw the two magnets, and sketch the magnetic field lines around them. On your diagram, mark the position of any neutral points (points where there is no resultant magnetic field).



▲ **Figure 20.6**

Magnetic effect of an electric current

▲ **Figure 20.7** Hans Christian Oersted

The earliest discovery of the magnetic effect of an electric current was made in 1820 by Oersted, a Danish physicist (Figure 20.7).

He noticed that a compass was deflected when brought near to a wire carrying an electric current. It is now known that all electric currents produce magnetic fields. The size and shape of the magnetic field depends on the size of the current and the shape (configuration) of the conductor through which the current is travelling. The shape of the fields will be considered in more detail in Topic 20.4. Hence, a fuller understanding of a magnetic field is that:

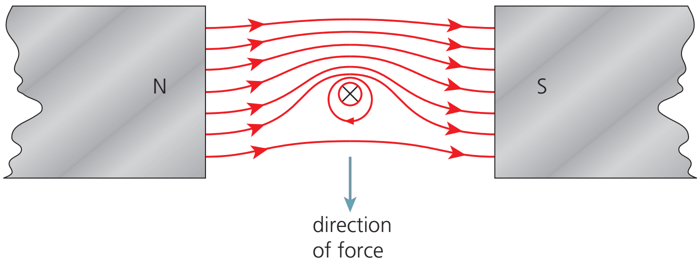
> A magnetic field is a region of space where a force is experienced either by moving charges or by permanent magnets.

20.2 Force on a current-carrying conductor

We have seen that the plotting of lines of magnetic force gives the direction and shape of the magnetic field. Also, the distance between the lines indicates the strength of the field. However, the strength of the magnetic field has not been defined. Physics is the science of measurement and, consequently, the topic would not be complete without defining and measuring magnetic field strength. Magnetic field strength is defined through a study of the force on a current-carrying conductor – the motor effect.

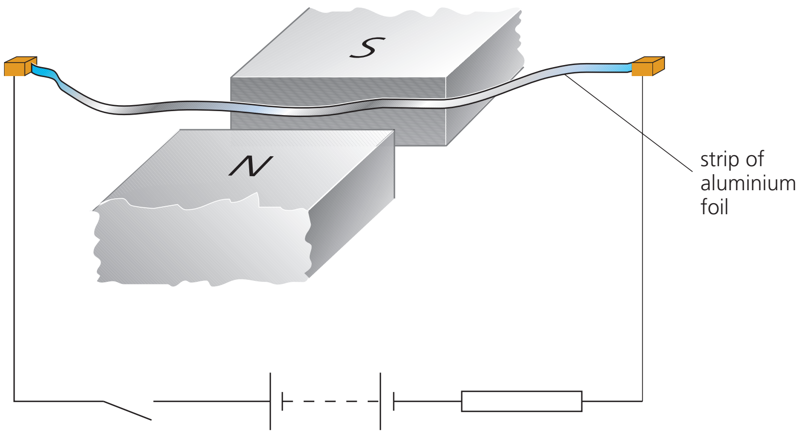
The motor effect

The interaction of the magnetic fields produced by two magnets causes forces of attraction or repulsion between the two. A current-carrying conductor produces a magnetic field around the conductor (see Topic 20.4). If a conductor is placed between the poles of a permanent magnet and a current is passed through the conductor, the magnetic fields of the current-carrying conductor and the magnet will interact, causing forces between them. This is illustrated in Figure 20.8.



▲ **Figure 20.8** The interacting magnetic fields of a current-carrying conductor and the poles of a magnet

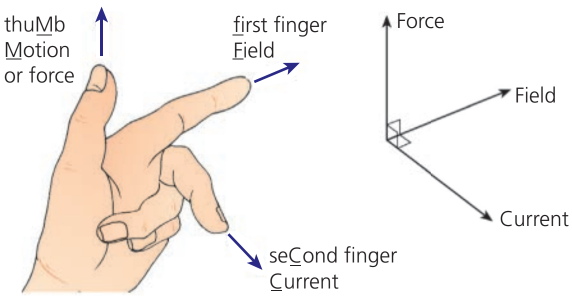
The existence of the force may be demonstrated with the apparatus shown in Figure 20.9.



▲ **Figure 20.9** Demonstrating the motor effect acting on a piece of aluminium

The strip of aluminium foil is held loosely between the poles of a horseshoe magnet so that the foil is at right angles to the magnetic field. When the current is switched on, the foil jumps and becomes taut, showing that a force is acting on it. The direction of the force, known as the **electromagnetic force**, depends on the directions of the magnetic field and of the current. This phenomenon, when a current-carrying conductor is at an angle to a magnetic field, is called the **motor effect** and is used in motors.

The direction of the force relative to the directions of the current and the magnetic field may be predicted using **Fleming's left-hand rule**. This is illustrated in Figure 20.10.



▲ **Figure 20.10** Fleming's left-hand rule

If the first two fingers and thumb of the left hand are held at right angles to one another with the **F**irst finger in the direction of the **F**ield and the se**C**ond finger in the direction of the **C**urrent, then the thu**M**b gives the direction of the force or **M**otion.

However, it must be remembered that the second finger is used to indicate the direction of the *conventional* current. Note that, if the conductor is held fixed, motion will not be seen but, nevertheless, there will be an electromagnetic force.

Experiments show that electromagnetic force F is proportional to the current I and proportional to the length L of conductor in the magnetic field. The force also depends on the angle θ between the conductor and the direction of the magnetic field. Hence the expression

$$F \propto IL\sin\theta$$

is derived.

If the wire and the field lines are parallel to each other, $\theta = 0$ and $\sin\theta$ is equal to 0. Hence there is no force exerted on the conductor if it is parallel to the magnetic field.

The expression can be re-written as

$$F = BIL\sin\theta$$

where B is a constant. If the magnetic field is uniform, the constant B depends on the strength of the magnet and, if stronger magnets are used, the constant has a greater value. The equation can, therefore, be used as the defining equation for magnetic field strength. Magnetic field strength is more correctly called **magnetic flux density**.

The equation can be rewritten as

$$B = \frac{F}{IL\sin\theta}$$

So,

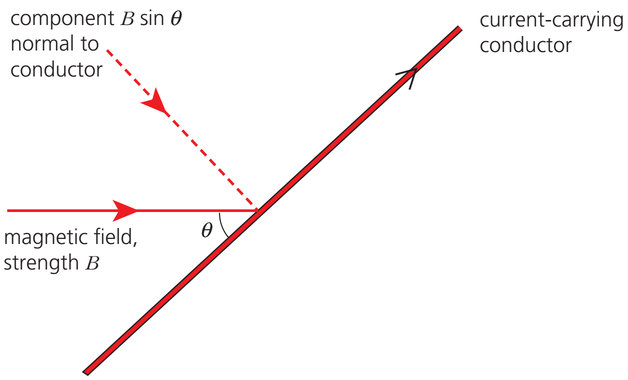
Magnetic flux density B is numerically equal to the force per unit current per unit length on a straight wire placed at right angles to a uniform magnetic field,

The magnetic flux density B is measured in tesla (T). As we shall see in Topic 20.5, an alternative name for this unit is weber per square metre ($\mathrm{Wb\,m^{-2}}$).

One tesla is the uniform magnetic flux density which, acting normally to a long straight wire carrying a current of 1 ampere, causes a force per unit length of $1\,\mathrm{N\,m^{-1}}$ on the conductor.

Since force is measured in newtons, length in metres and current in amperes, it can be derived from the defining equation for magnetic flux density that the tesla may also be expressed as $\mathrm{N\,m^{-1}\,A^{-1}}$. The unit involves force which is a vector quantity and thus magnetic flux density is also a vector.

When using the equation $F = BIL\sin\theta$, it is sometimes helpful to think of the term $B\sin\theta$ as being the component of the magnetic flux density which is at right angles (or normal) to the conductor (see Figure 20.11).



▲ **Figure 20.11** $B\sin\theta$ **is the component of the magnetic field which is normal to the conductor.**

The tesla is a large unit for the measurement of flux density. A very strong magnet may have a flux density between its poles of a few teslas. The magnetic flux density due to the Earth in the UK is about $44\,\mu\mathrm{T}$ at an angle of 66° to the horizontal.

WORKED EXAMPLE 20B

The horizontal component of the Earth's magnetic flux density is 1.8×10^{-5} T. The current in a horizontal cable is 120 A. Calculate, for this cable:

a the maximum force per unit length

b the minimum force per unit length.

In each case, state the angle between the cable and the magnetic field.

Answers

a Force per unit length = $F/L = BI \sin\theta$

Force per unit length is a maximum when $\theta = 90°$ and $\sin\theta = 1$.

Force per unit length = $1.8 \times 10^{-5} \times 120 = 2.16 \times 10^{-3}\,\text{N m}^{-1}$

Maximum force per unit length = $\mathbf{2.16 \times 10^{-3}\,N\,m^{-1}}$ when the cable is at right angles to the field.

b Force per unit length is a minimum when $\theta = 0$ and $\sin\theta = 0$. Minimum force per unit length = **0** when the cable is along the direction of the field.

Questions

3 The effective length of the filament in a light bulb is 3.0 cm and, for normal brightness, the current in the filament is 0.40 A. Calculate the maximum electromagnetic force on the filament when in the Earth's magnetic field of flux density 42 μT.

4 A straight conductor carrying a current of 7.5 A is situated in a uniform magnetic field of flux density 4.0 mT. Calculate the electromagnetic force per unit length of the conductor when the angle between the conductor and the field is:

a 90°

b 60°.

20.3 Force on a moving charge

An electric current is charge in motion. Since charge is always associated with particles, then the current in a conductor is a movement of charged particles. If a current-carrying conductor is placed in a magnetic field, it may experience a force depending on the angle between the field and the conductor. The force arises from the force on the individual moving charged particles in the conductor.

It has been shown that a conductor of length L carrying a current I at an angle θ to a uniform magnetic field of flux density B experiences a force F given by

$$F = BIL \sin\theta$$

If there are n charged particles in a length L of the conductor, each carrying a charge q, that pass a point in the conductor in time t, then the current I in the conductor is given by

$$I = \frac{nq}{t}$$

and the speed v of charged particles is given by $v = \frac{L}{t}$. Hence,

$$F = B \times \left(\frac{nq}{t}\right) \times L \sin\theta$$

and

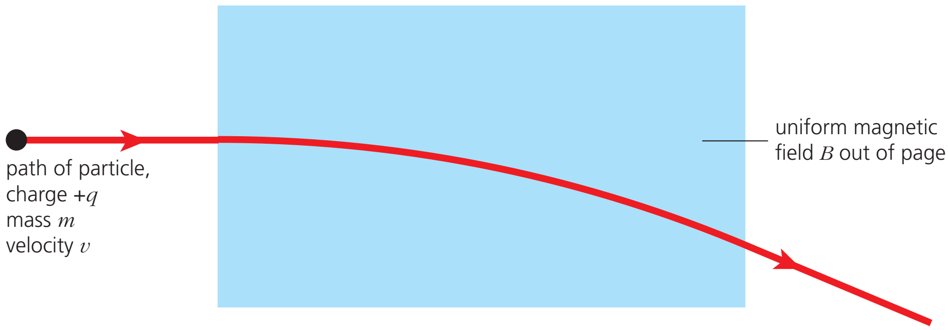
$$F = Bnqv \sin\theta$$

This force is the force on n charged particles. Therefore,

The force on a particle of charge q moving at speed v at an angle θ to a uniform magnetic field of flux density B is given by $F = Bqv \sin\theta$.

The direction of the force will be given by Fleming's left-hand rule (see Figure 20.10). However, it must be remembered that the second finger is used to indicate the direction of the *conventional current*. If the particles are positively charged, then the second finger is placed in the same direction as the velocity. However, if the particles are negatively charged (e.g. electrons), the finger must point in the opposite direction to the velocity.

Consider a positively charged particle of mass m carrying charge q and moving with velocity v as shown in Figure 20.12.



▲ **Figure 20.12** Path of a charged particle in a magnetic field

The particle enters a uniform magnetic field of flux density B which is normal to the direction of motion of the particle. As the particle enters the field, it will experience a force normal to its direction. This force will not change the speed of the particle but it will change its direction of motion. As the particle moves through the field, the force will remain constant, since the speed has not changed, and it will always be normal to the direction of motion. The particle will, therefore, move in an arc of a circle of radius r (see Topic 12.2).

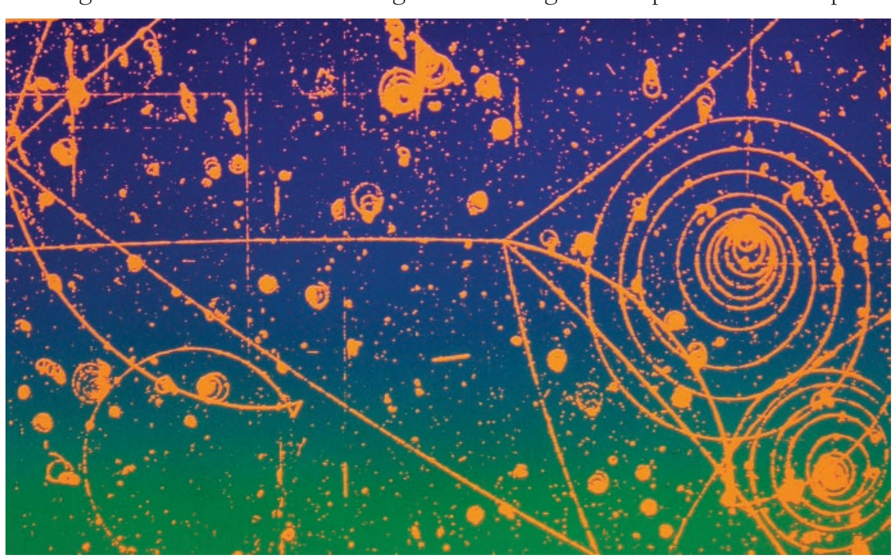
The force $F = Bqv\sin\theta$ (in this case, $\sin\theta = 1$), provides the centripetal force for the circular motion. Hence,

$$\text{centripetal force} = mv^2/r = Bqv$$

Re-arranging,

$$\text{radius of path} = r = \frac{(mv)}{(Bq)}$$

The importance of this equation is that, if the speed of the particle and the radius of its path are known, then the specific charge, i.e. the ratio of charge to mass q/m, can be found. Then, if the charge on the particle is known, its mass may be calculated. The technique is also used in nuclear research to identify some of the fundamental particles. The tracks of charged particles are made visible in a bubble chamber (Figure 20.13). Analysing these tracks gives information as to the sign of the charge on the particle and its specific charge.



▲ **Figure 20.13** Tracks of particles produced in a bubble chamber

WORKED EXAMPLE 20C

Electrons are accelerated to a speed of $9.1 \times 10^6\,\text{m}\,\text{s}^{-1}$. They then pass into a region of uniform magnetic flux of flux density 0.50 mT. The path of the electrons in the field is a circle with a radius 10.2 cm. Calculate:

a the specific charge of the electron

b the mass of the electron, assuming the charge on the electron is $1.6 \times 10^{-19}\,\text{C}$.

Answers

a $e/m = v/Br$

$= 9.1 \times 10^6/(0.50 \times 10^{-3} \times 0.102)$

$= \mathbf{1.8 \times 10^{11}\,C\,kg^{-1}}$

b $e/m = 1.8 \times 10^{11} = 1.6 \times 10 \times^{-19}/m$

$m = \mathbf{9.0 \times 10^{-31}\,kg}$

Question

5 Electrons are accelerated through a potential difference of 250 V. They then pass into a region of uniform magnetic flux of flux density 0.58 mT. The path of the electrons is normal to the magnetic field. Given that the charge on the electron is $1.6 \times 10^{-19}\,\text{C}$ and its mass is $9.1 \times 10^{-31}\,\text{kg}$, calculate:

a the speed of the accelerated electron

b the radius of the circular path in the magnetic field.

EXTENSION

Specific charge of the electron: the fine-beam tube

Specific charge is the name given to the ratio of the charge *q* on a particle and its mass *m*.

specific charge = q/m

As already mentioned, specific charge can give us information about a particle and, if the charge on the particle is known, then the mass of the particle can be determined.

The charge on the electron is $1.60 \times 10^{-19}\,\text{C}$. Determination of the specific charge on the electron enables us to obtain a value for its mass.

We have seen above that a particle of mass *m* and charge *q* moving with speed *v* at right angles to a uniform magnetic field of flux density *B* experiences a force *F* given by

$F = Bqv$

The direction of this force is given by Fleming's left-hand rule and is always normal to the velocity, giving rise to circular motion

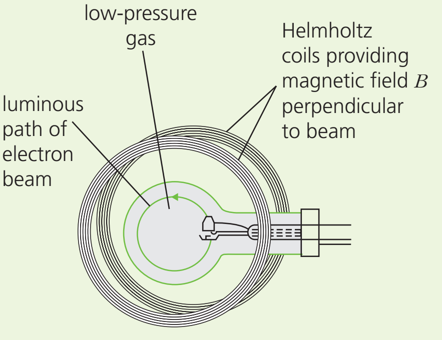
$Bqv = mv^2/r$

Re-arranging the terms,

$q/m = v/Br$

The ratio charge/mass (e/m_e) for an electron – its specific charge – may be determined using a fine-beam tube, as shown in Figure 20.14.

The path of electrons is made visible by having low-pressure gas in the tube and, thus, the radius of the orbit may be



▲ **Figure 20.14 Fine-beam tube**

measured. By accelerating the electrons through a known potential difference V, their speed v on entry into the region of the magnetic field may be calculated (see Motion of a charged particle in an electric field in Topic 18.2) using

$$eV = \frac{1}{2} m_e v^2$$

The magnetic field is provided by a pair of current-carrying coils (Helmholtz coils, see Figure 20.27).

Combining the equations $e/m_e = v/Br$ and $eV = \frac{1}{2} m_e v^2$

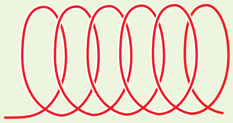
then

specific charge on electron $e/m_e = 2V/B^2r^2$.

Values for the charge e and mass m_e are usually given as

charge $e = 1.60 \times 10^{-19}$ C
mass $m_e = 9.11 \times 10^{-31}$ kg

It is of interest to rotate the fine-beam tube slightly, so that the velocity of the electrons is not normal to the magnetic field. In this case, the path of the electrons is seen to be a helix (rather like the coils of a spring). The component of the velocity normal to the field gives rise to circular motion. However, there is also a component of velocity along the direction of the field. There is no force on the electrons resulting from this component of velocity. Consequently, the electrons execute circular motion *and* move in a direction normal to the plane of the circle. The circle is 'pulled out' into a helix (Figure 20.15). The helical path is an important aspect of the focusing of electron beams by magnetic fields in an electron microscope (Figure 20.16).



▲ **Figure 20.15** Electrons moving in a helix

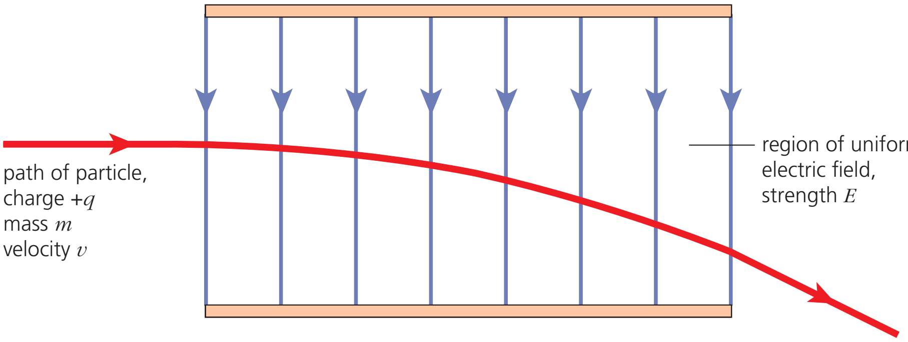
▲ **Figure 20.16** Laboratory technician using an electron microscope

Velocity selection of charged particles

We have seen (Topic 18.2) that when particles of mass m and charge $+q$ enter an electric field of field strength E, there is a force F_E on the particle given by

$$F_E = qE$$

If the velocity of the particles before entry into the field is v and is at right angles to the field lines (Figure 20.17), the particles will follow a parabolic path as they pass through the field.



▲ **Figure 20.17**

Now suppose that a uniform magnetic field acts in the same region as the electric field. If this field acts downwards into the plane of the page, then, by Fleming's left-hand rule, a force will act on the charged particle in the direction opposite to the force due to the electric field. The magnitude F_B of this force is given by

$$F_B = Bqv$$

where B is the flux density of the magnetic field.

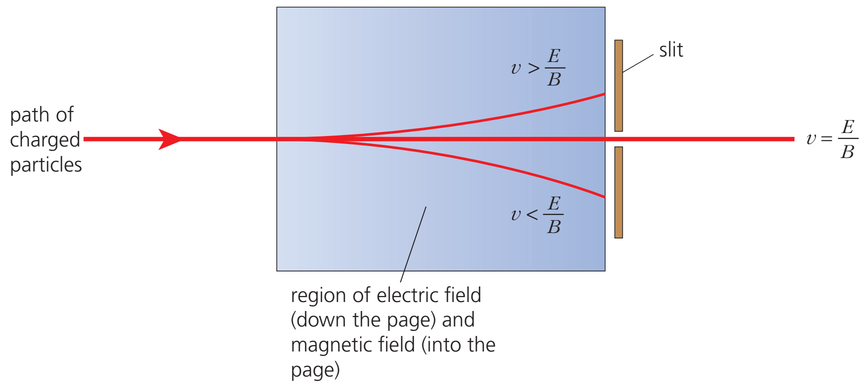
If the magnitude of one of the two fields is adjusted, then a situation can arise where the two forces, F_E and F_B, are equal in magnitude but opposite in direction. Thus

$$Bqv = qE$$

and

$$v = E/B$$

For the value of the velocity given by E/B, the particles will not be deflected, as shown in Figure 20.18.



▲ **Figure 20.18 Velocity selector**

However since the magnetic force is speed-dependent, charged particles with any other velocities will be deflected. If a parallel beam of particles enters the field then all the particles passing undeviated through the slit will have the same velocity. Note that the mass does not come into the equation for F_B or F_E and so, particles with a different mass (but the same charge) will all pass undeviated through the region of the fields if they satisfy the condition $v = E/B$.

The arrangement shown in Figure 20.18 is known as a velocity selector. Velocity selectors are very important in the study of ions. Frequently, the production of the ions gives rise to different speeds but to carry out investigations on the ions, the ions must have one speed only.

If some ions all have the same speed – achieved using a velocity selector – then the radius of the path of an ion in a magnetic field is dependent on the ratio of charge and mass of the ion. Hence, ions can be identified according to their charge-to-mass ratios, and importantly, knowing the charge on the ion, the mass of the ion can be determined. Instruments using these principles to determine mass are referred to as *mass spectrometers*.

WORKED EXAMPLE 20D

It is required to select charged ions which have a speed of $6.3 \times 10^6\,\text{m s}^{-1}$. The electric field strength in the velocity selector is $4.8 \times 10^4\,\text{V m}^{-1}$. Calculate the magnetic flux density required.

Answer

$$v = E/B$$

$$B = (4.8 \times 10^4)/(6.3 \times 10^6)$$

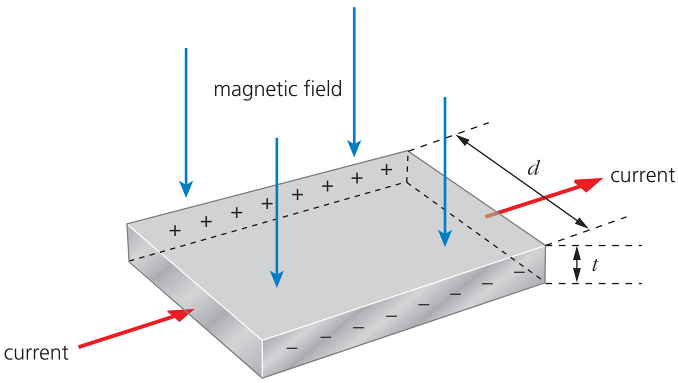
$$= \mathbf{7.6 \times 10^{-3}\,T}$$

Question

6 Singly charged ions pass undeviated through a velocity selector. The electric field strength in the selector is $4.2 \times 10^4\,\text{V m}^{-1}$ and the magnetic flux density is $8.4 \times 10^{-3}\,\text{T}$. Calculate the selected speed of the ions.

The Hall effect

Consider a thin slice of a conductor which is normal to a magnetic field, as illustrated in Figure 20.19.



▲ **Figure 20.19 The Hall effect**

When there is a current in the conductor in the direction shown, charge carriers (electrons in a metal) will be moving at right angles to the magnetic field. They will experience a force which will tend to make them move to one side of the conductor. A potential difference known as the **Hall voltage** V_H will develop across the conductor. The Hall voltage does not increase indefinitely but reaches a constant value when the force on the charge carrier due to the magnetic field is equal to the force due to the electric field set up as a result of the Hall voltage.

If the distance between the two faces with the potential difference V_H is d (see Figure 20.19), then the electric field strength E between these two faces will be given by $E = V_H/d$ (see Topic 18.2). The force F_E acting on each charge carrier will be

$$F_E = q \times (V_H/d)$$

where q is the charge on the charge carrier.

The force F_B on the charge carrier due to the magnetic field of flux density B is given by $F_B = Bqv$ where v is the drift speed of the charge carriers.

When the electric field has been established, charge carriers will pass undeviated through the slice and $F_E = F_B$,

$$q \times (V_H/d) = Bqv$$

$$V_H/d = Bv$$

Now, the drift speed v of the charge carriers is given, in terms of the current I in the slice, by the expression (see Topic 9.1)

$$I = Anvq$$

where A is the area of cross-section of the slice and n is the number density of the charge carriers (number of charge carriers per unit volume).

Since the area A is equal to td where t is the thickness of the slice (see Figure 20.19), then

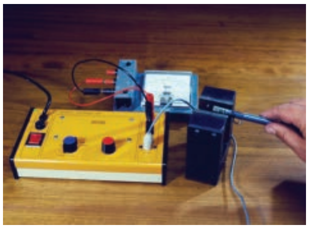
$$V_H/d = B \times (I/tdnq)$$

and

$$\text{Hall voltage } V_H = \frac{BI}{(ntq)}$$

The size of the Hall voltage depends on the material and thickness of the conductor, the current in the sample and the magnetic flux density. The number density of charge carriers is very large in metals and, consequently, the Hall voltage is very small. However, with semiconductors, the number density is very much reduced and, therefore, detectable and measurable Hall voltages are possible. In fact, the Hall effect is used to study semiconductor materials and identify (from the sign of the Hall voltage) whether the charge carriers are positive or negative.

If the current is kept constant, then the Hall voltage across a sample will be proportional to the magnetic flux density. The Hall effect thus provides a means by which flux densities may be measured, using a Hall probe.



▲ **Figure 20.20** Hall probe apparatus

Measuring magnetic flux density

The Hall probe apparatus used in school or college laboratories consists of a thin slice of a semiconductor material which is placed with its plane at right angles to the direction of the magnetic field. The control unit is arranged to pass a certain current through the semiconductor slice. The Hall voltage, which is proportional to the magnetic flux density, is read off on an analogue or digital meter, which is calibrated in units of magnetic flux density (tesla). The arrangement is illustrated in Figure 20.20.

Note that the Hall voltage is dependent on the angle between the magnetic field and the plane of the Hall probe. Before commencing the taking of readings, the probe should be placed in the field and rotated to obtain the maximum reading on the meter. The plane of the Hall probe is then at right angles to the magnetic field.

WORKED EXAMPLE 20E

A Hall probe is placed at right angles to a uniform magnetic field. A Hall voltage of 82 mV is measured.

The probe is adjusted so that the angle its plane makes with the magnetic field is 35°.

Calculate the value of the Hall voltage.

Answer

The component of the magnetic flux density B that is normal to the plane is $B\sin 35°$.

Since Hall voltage is proportional to the magnetic flux density normal to the plane of the probe, then

Hall voltage = 82 sin 35° = **47 mV**

Question

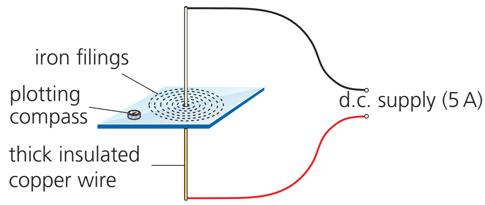
7 A piece of aluminium foil is 0.10 mm thick. The current in the foil is 4.5 A. Aluminium has 6.0×10^{28} free electrons per cubic metre.

A uniform magnetic field of flux density 85 mT is normal to the slice. Calculate the Hall voltage that is generated. (Electronic charge = 1.6×10^{-19} C.)

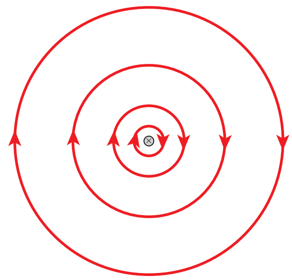
20.4 Magnetic fields due to currents

The magnetic field due to a long straight wire may be plotted using the apparatus illustrated in Figure 20.21. Note that the current must be quite large (about 5 A). Iron filings are sprinkled on to the horizontal board and a plotting compass is used to determine the direction of the field.

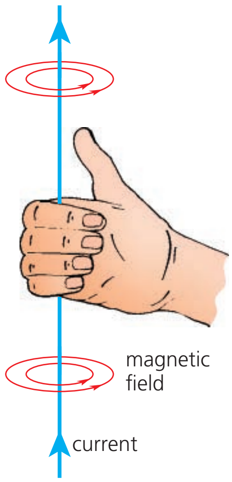
Figure 20.22 shows the field pattern due to a long straight current-carrying wire. The lines are concentric circles centred on the middle of the wire. The separation of the lines increases with distance from the wire, indicating that the field is decreasing in strength. The direction of the magnetic field may be found using the right-hand rule as illustrated in Figure 20.23.



▲ **Figure 20.21** Apparatus to plot the magnetic field due to a long wire



▲ **Figure 20.22** Magnetic field pattern due to a long straight wire

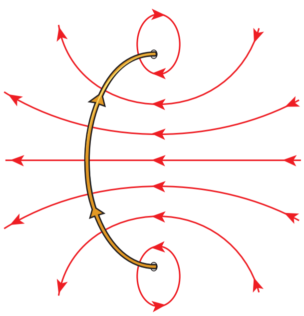


▲ **Figure 20.23** The right-hand rule

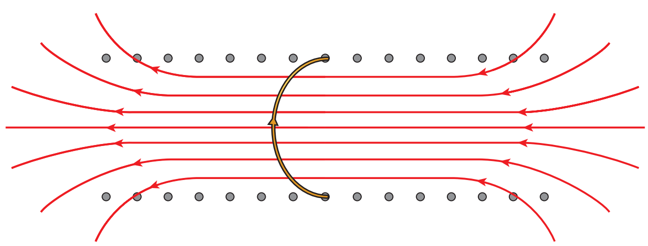
Imagine holding the conductor in the right hand with the thumb pointing in the direction of the conventional current. The direction of the fingers gives the direction of the magnetic field.

Similar apparatus to that in Figure 20.21 may be used to investigate the shapes of the magnetic field due to a flat coil and to a solenoid (a long coil). Figure 20.24 illustrates the magnetic field pattern due to a flat coil. The field has been drawn in a plane normal to the coil and through its centre.

A **solenoid** may be thought of as being made up of many flat coils placed side-by-side. The magnetic field pattern of a long solenoid (that is, a coil which is long in comparison with its diameter) is shown in Figure 20.25.

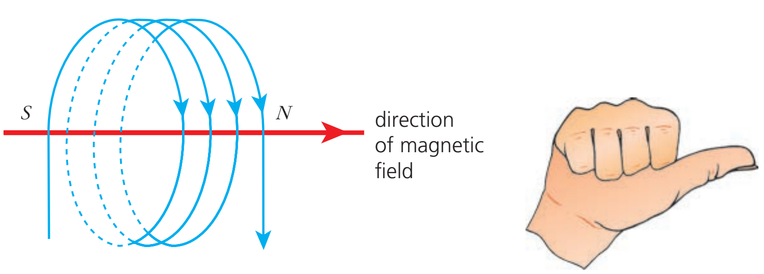


▲ **Figure 20.24** Magnetic field pattern due to a flat coil



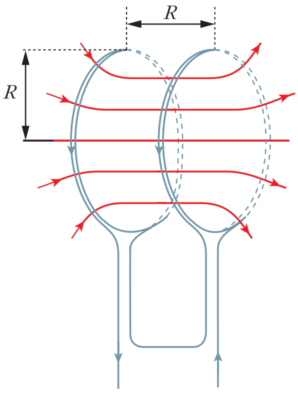
▲ **Figure 20.25** Magnetic field pattern due to a solenoid

The field lines are parallel and equally spaced over the centre section of the solenoid, indicating that the field is uniform. The field lines spread out towards the ends. The strength of the magnetic field at each end is one-half that at the centre. The direction of the magnetic field in a flat coil and in a solenoid may be found using the **right-hand grip rule**, as illustrated in Figure 20.26.



▲ **Figure 20.26** The right-hand grip rule

Grasp the coil or solenoid in the right hand with the fingers pointing in the direction of the conventional current. The thumb gives the direction of the magnetic field.



▲ **Figure 20.27** Magnetic field in Helmholtz coils

The magnetic north end of the coil or solenoid is the end from which the lines of magnetic force are emerging. Note the similarities and, more importantly, the differences between this rule and the right-hand rule for the long straight wire (Figure 20.23).

Uniform magnetic fields are of importance in the study of charged particles, such as when using a velocity selector (see Topic 20.3) or when using a fine-beam tube to determine the ratio e/m.

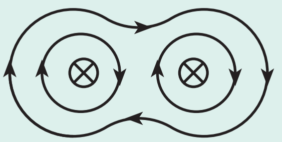
A uniform field is produced in a solenoid but this field is inside the solenoid and consequently, it may be difficult to make observations and to take measurements. This problem is overcome by using **Helmholtz coils**. These are two identical flat coils, with the same current in each. The coils are positioned so that their planes are parallel and separated by a distance equal to the radius of either coil. The coils and their resultant magnetic field are illustrated in Figure 20.27.

WORKED EXAMPLE 20F

Two long straight wires, of circular cross-section, are each carrying the same current directly away from you, down into the page. Draw the magnetic field due to the two current-carrying wires.

Answer

The solution is shown in Figure 20.28.



▲ **Figure 20.28**

Questions

8 Draw magnetic field patterns, one in each case to represent:
   a a uniform field
   b a field which is decreasing in strength in the direction of the field
   c a field which is increasing in strength along the direction of the field.

9 Draw a diagram of the magnetic field due to two long straight wires when the currents in the two wires are in opposite directions.

Electromagnets and their uses

The strength of the magnetic field due to a flat coil or a solenoid may be increased by winding the coil on a bar of soft iron. The bar is said to be the **core** of the coil. The iron is referred to as being 'soft' because it can be magnetised and demagnetised easily. With such a core (ferrous core), the strength of the magnetic field may be increased by up to 1000 times. With ferrous alloys (iron alloyed with cobalt or nickel), the field may be 10^4 times stronger. Magnets such as these are called **electromagnets**. Electromagnets have many uses because, unlike a permanent magnet, the magnetic field can be switched off by switching off the current in the coil.

Comparing the effects of fields

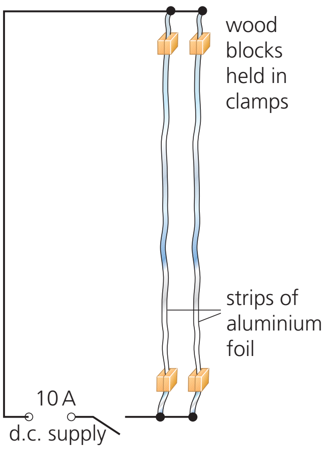
We can summarise the effects of the different sorts of fields on masses, charges and current-carrying conductors. Although, as we have seen, there are close analogies between magnetic, gravitational and electric fields, there are some ways in which they behave very differently.

Start with the effect of a gravitational field on a mass. Because masses always attract each other, a mass placed in a gravitational field will always move in the direction of the field, from a position of higher potential to lower potential. For a field produced by a point mass, the field strength obeys an inverse square law relationship, and the potential obeys a reciprocal relationship with distance from the source of the field.

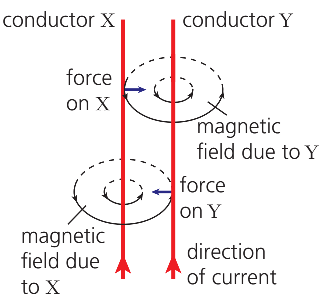
Electric fields are like gravitational fields in that, for a field produced by a point charge, the field strength is also given by the inverse square law, and the potential by a reciprocal relationship. However, we can have both positive charges and negative charges. A positive electric charge moves in the direction of the field, from a position of higher potential to a lower potential (just like a mass in a gravitational field). But a negative charge does just the opposite, against the direction of the field and from a low potential to a high potential.

What about electric charges in a magnetic field? A stationary charge is unaffected, but a moving charge experiences a force F given by $F = Bqv\sin\theta$. The direction of the force is given by Fleming's left-hand rule (for positive charges).

Finally, a current-carrying conductor in a magnetic field does not experience a force if the conductor is parallel to the field direction, but for all other directions it experiences a force given by $F = BIL\sin\theta$. The direction of the force is again given by Fleming's left-hand rule.



▲ **Figure 20.29** Apparatus to demonstrate the force between parallel current-carrying conductors



▲ **Figure 20.30** Diagram to illustrate the force between parallel current-carrying conductors

Force between parallel conductors

A current-carrying conductor has a magnetic field around it. If a second current-carrying conductor is placed parallel to the first, this second conductor will be in the magnetic field of the first and, by the motor effect, will experience a force.

By similar reasoning, the first conductor will also experience a force. By Newton's third law these two forces will be equal in magnitude and opposite in direction. The effect can be demonstrated using the apparatus in Figure 20.29.

It can be seen that, if the currents are in the same direction, the pieces of foil move towards one another (the pinch effect). If the currents are in opposite directions, the pieces of foil move apart. An explanation for the effect can be found by reference to Figure 20.30.

The current in conductor X causes a magnetic field and the field lines are concentric circles (see Figure 20.22). These field lines will be at right angles to conductor Y and so, using Fleming's left-hand rule, there will be a force on Y in the direction of X. Using similar reasoning, the force on X due to the magnetic field of Y is towards Y. Reversing the direction of the current in one conductor will reverse the directions of the two forces and, thus, when the currents are in opposite directions, the conductors tend to move apart.

The force per unit length on each of the conductors depends on the magnitude of the current in each conductor and also their separation. Until 2019, the ampere was defined in terms of the force per metre length acting between two long straight parallel current-carrying conductors of negligible area of cross-section, situated in a vacuum.

WORKED EXAMPLE 20G

A charged particle has mass 6.7×10^{-27} kg and charge $+3.2 \times 10^{-19}$ C. It is travelling at a speed of $3.4 \times 10^{8}\,\text{m s}^{-1}$ when it enters a region of space where there is a uniform magnetic field of flux density 1.8 T at right angles to its direction of motion. Calculate:

a the gravitational force on the particle
b the force on the particle due to the magnetic field
c the radius of its orbit in the field.

Answers

a Gravitational force = $mg = 6.7 \times 10^{-27} \times 9.81$
$= \mathbf{6.6 \times 10^{-26}\,N}$ (negligible when compared with the force due to the magnetic field)

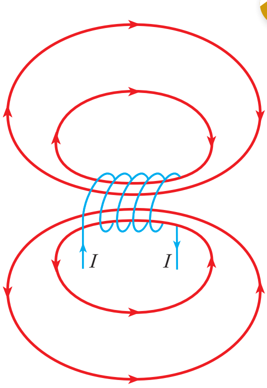
b Force = $Bqv\sin\theta$
$= 1.8 \times 3.2 \times 10^{-19} \times 3.4 \times 10^{8} \times 1$
$= \mathbf{2.0 \times 10^{-10}\,N\ (1.96 \times 10^{-10}\,N)}$

c Centripetal force is provided by the electromagnetic force
$mv^2/r = Bqv$
$6.7 \times 10^{-27} \times (3.4 \times 10^{8})^2/r = 1.96 \times 10^{-10}$
$r = \mathbf{4.0\,m\ (3.95\,m)}$

Question

10 An electron and an α-particle travelling at the same speed both enter the same region of a uniform magnetic field which is at right angles to their direction of motion. State and explain any differences between the two paths in the field.

20.5 Electromagnetic induction



▲ **Figure 20.31** Continuous magnetic field lines

In Topic 20.4 we examined the pattern of the magnetic field in the region of a straight wire and various coils. However, the patterns shown in Figures 20.24 and 20.25 are not complete. All magnetic field lines should be continuous, as illustrated in Figure 20.31.

Early experimenters thought that there was a flow of something along these lines and this gave rise to the idea of a magnetic flux, since 'flux' means 'flow'. Referring to Figure 20.3, we can see that the magnetic field lines are closer together between the centres of the poles than outside the region of the poles. Thus the closer together the lines the stronger the magnetic field and the greater the magnetic flux density. If an area is drawn parallel to the faces of the poles, then the number of lines passing normally through unit area gives a measure of the magnetic flux density. The **magnetic flux**, symbol Φ, can be thought of as the total number of lines passing through the whole area at right angles to the lines.

Magnetic flux is the product of the magnetic flux density and the area normal to the lines of flux.

$$\Phi = BA$$

The angle a surface makes with the magnetic field affects the value for Φ. The flux will be maximum when the surface is at right angles to the field lines and zero when the surface is parallel to the field lines.

For a uniform magnetic field of flux density B which makes an angle θ with an area A, the magnetic flux Φ is given by the expression $\Phi = BA\sin\theta$.

The unit of magnetic flux is the weber (Wb). Since the unit of magnetic flux density is the tesla, from $\Phi = BA$, one weber is equal to one tesla metre-squared, i.e. T m^2.

The magnetic flux density in a coil of N turns having an area of cross-section A is B. The flux Φ through this coil is given by $\Phi = BA$. This magnetic flux passes through the N turns of the coil and results in **magnetic flux linkage** $N\Phi$.

$$\text{magnetic flux linkage} = N\Phi = BAN$$

WORKED EXAMPLE 20H

A coil is constructed by winding 400 turns of wire on to a cylindrical iron core. The mean radius of the coil is 3.0 cm. It is found that the flux density B in the core due to a current in the coil is 1.4 T.

Calculate:

a the magnetic flux in the core

b the flux linkage of the coil.

Answers

a $\Phi = BA$, so $\Phi = 1.4 \times \pi(0.03)^2 =$ **4.0 mWb**

b flux linkage $= N\Phi = 4.0 \times 400 =$ **1.6 Wb**

Question

11 A flat coil contains 250 turns of insulated wire and has a mean radius of 1.5 cm. The coil is placed in a region of uniform magnetic flux of flux density 85 mT such that there is an angle between the plane of the coil and the flux lines.

Calculate the flux linkage in the coil for an angle of:

a zero

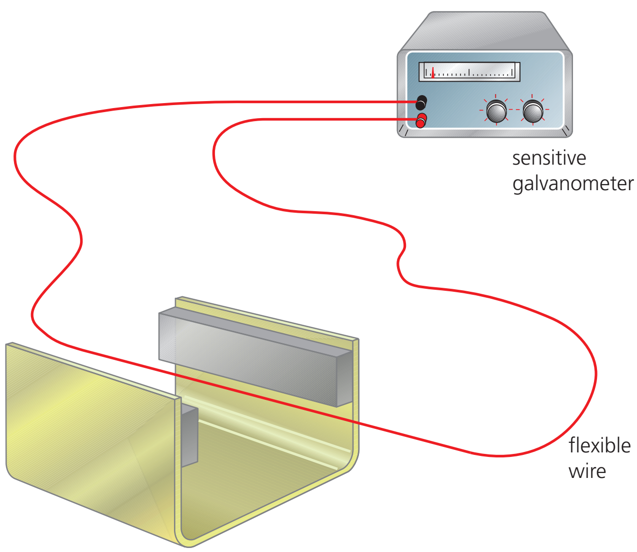
b 90°

c 35°.

Electromagnetic induction

As described earlier, the link between electric current and magnetic field was discovered by Oersted in 1820. In 1831, Joseph Henry in the United States and Michael Faraday in England demonstrated that an e.m.f. could be induced by a magnetic field. The effect was called **electromagnetic induction**.

Electromagnetic induction is now easy to demonstrate in the laboratory because sensitive meters are available. Figure 20.32 illustrates apparatus which may be used for this purpose.



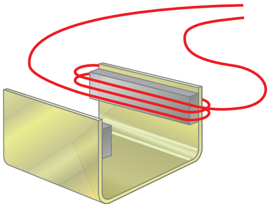
▲ **Figure 20.32 Apparatus to demonstrate electromagnetic induction**

The galvanometer detects very small currents but it is important to realise that what is being detected are small electromotive forces (e.m.f.s). The current arises because there is a complete circuit incorporating an e.m.f. The following observations can be made.

- An e.m.f. is induced when:
  - the wire is moved through the magnetic field, across the face of the pole-pieces
  - the magnet is moved so that the wire passes across the face of the pole-pieces.
- An e.m.f. is *not* induced when:
  - the wire is held stationary between the pole-pieces
  - the magnet is moved so that the pole-pieces move along the length of the wire
  - the wire moves lengthways so that it does not change its position between the poles of the magnet.

These observations lead to the conclusion that an e.m.f. is induced whenever lines of magnetic flux are cut. The cutting may be caused by a movement of either the wire or the magnet. The magnitude of the e.m.f. is also observed to vary.

- The magnitude of the e.m.f.:
  - increases as the speed at which the wire is moved increases
  - increases as the speed at which the magnet is moved increases
  - increases if the wire is made into a loop with several turns (see Figure 20.33)
  - increases as the number of turns on the loop increases.



▲ **Figure 20.33** Wire wound to form a loop of several turns

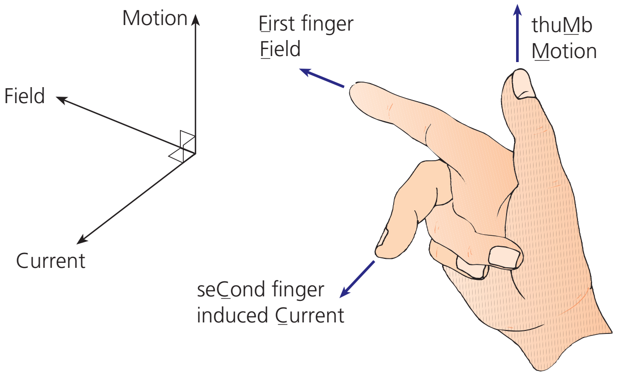
It can be concluded that the magnitude of the induced e.m.f. depends on the rate at which magnetic flux lines are cut. The rate may be changed by varying the rate at which the flux lines are cut by a single wire or by using different numbers of turns of wire. The two factors are taken into account by using the **magnetic flux linkage** ($N\Phi$). Change in magnetic flux linkage $\Delta(N\Phi)$ is equal to the product of the change in magnetic flux $\Delta\Phi$ and the number of turns N of a conductor involved in the change in flux.

change in magnetic flux linkage $\Delta(N\Phi) = N\Delta\Phi$

The experimental observations are summarised in **Faraday's law of electromagnetic induction**.

The e.m.f. induced is proportional to the rate of change of magnetic flux linkage.

The experimental observations made with the apparatus of Figure 20.32 and Figure 20.33 have been concerned with the magnitude of the e.m.f. However, it is noticed that the direction of the induced e.m.f. changes and that the direction is dependent on the direction in which the magnetic flux lines are being cut. The direction of the induced e.m.f. or current in a wire moving through a magnetic field at right angles to the field may be determined using **Fleming's right-hand rule**. This rule is illustrated in Figure 20.34.

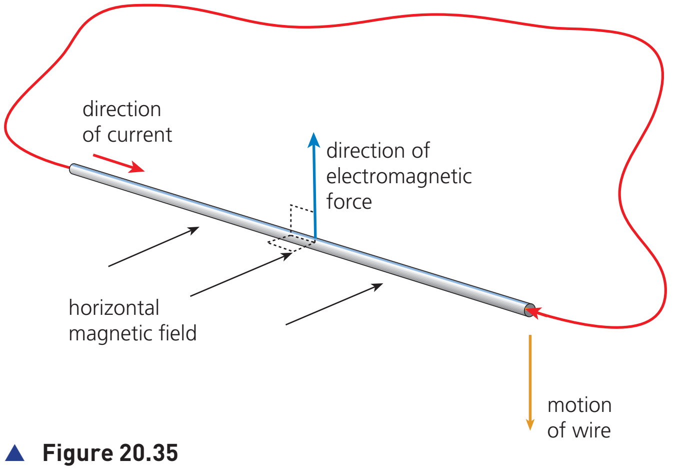


▲ **Figure 20.34** Fleming's right-hand rule

If the first two fingers and thumb of the right hand are held at right angles to one another, the First finger in the direction of the magnetic Field and the thuMb in the direction of Motion, then the seCond finger gives the direction of the induced e.m.f. or Current.

An explanation for the direction of the induced e.m.f. can be found by reference to the motor effect and conservation of energy.

Figure 20.35 shows a wire being moved downwards through a magnetic field.



▲ **Figure 20.35**

Since the wire is in the form of a continuous loop, the induced e.m.f. gives rise to a current, and the direction of this current can be found using Fleming's right-hand rule. This current is at right angles to the magnetic flux and, by the motor effect, there will be a force on the wire. Using Fleming's left-hand rule (Figure 20.10), the force is upwards when the wire is moving downwards. Reversing the direction of motion of the wire causes a current in the opposite direction and, hence, the electromagnetic force would once again oppose the motion. This conclusion is not surprising when conservation of energy is considered. An electric current transfers energy and this energy must have been transferred from a source. Movement of the wire against the electromagnetic force means that work has been done on the wire in overcoming this force and it is this work which transfers energy to the charges in the wire. Anyone who has ridden a bicycle with a dynamo will realise that work has to be done to light the lamp!

This application of conservation of energy is summarised in **Lenz's law**.

The direction of the induced e.m.f. is such as to cause effects to oppose the change producing it.

Faraday's law of electromagnetic induction and Lenz's law may be summarised using the equation

$$E = \frac{-\,\mathrm{d}(N\Phi)}{\mathrm{d}t}$$

(see the Maths Note below) where E is the e.m.f. induced by a rate of change of flux linkage of $\mathrm{d}(N\Phi)/\mathrm{d}t$. The minus sign indicates that the induced e.m.f. causes effects to oppose the change producing it.

For a small change $\Delta(N\Phi)$ in flux linkage that occurs in time Δt (or where the flux linkage changes linearly with time), then the induced e.m.f. E is given by

$$E = \frac{-\,\Delta(N\Phi)}{\Delta t}$$

MATHS NOTE

The shorthand way of expressing the rate of change of a quantity x with time t is rate of change of x with $t = \mathrm{d}x/\mathrm{d}t$.

This represents a mathematical operation known as *differentiation*. It is achieved by finding the gradient of the graph of x against t.

You will come across this notation here, in connection with the rate of change of magnetic flux linkage, and in one of the equations for radioactive decay (Topic 23.2). This mathematics is beyond the scope of Cambridge International AS & A Level Physics. However, you may come across it if you are studying Cambridge International AS & A Level Mathematics.

WORKED EXAMPLE 20I

The uniform flux density between the poles of a magnet is 0.075 T. A small coil of area of cross-section 4.8 cm^2 has 200 turns and is placed with its plane at right angles to the magnetic field. The coil is withdrawn from the field in a time of 0.24 s. Determine:

a the magnetic flux through the coil when it is between the poles of the magnet
b the change in magnetic flux linkage when the coil is removed from the field
c the average e.m.f. induced in the coil whilst it is being withdrawn.

Answers

a magnetic flux $\Phi = BA \sin\theta$
$= 0.075 \times 4.8 \times 10^{-4}$
$= \mathbf{3.6 \times 10^{-5}\,Wb}$

b change in flux linkage $= (N\Phi)_{\text{FINAL}} - (N\Phi)_{\text{INITIAL}}$
$= 0 - (200 \times 3.6 \times 10^{-5})$
$= \mathbf{-7.2 \times 10^{-3}\,Wb}$

(the sign indicates that the flux linkage is decreasing)

c induced e.m.f. $= \dfrac{\text{change in flux linkage}}{\text{time taken}}$
$= \dfrac{(7.2 \times 10^{-3})}{0.24}$
$= \mathbf{30\,mV}$

Questions

12 An aircraft has a wingspan of 16 m and is flying horizontally in a northerly direction at a speed of 85 m s^{-1}. The vertical component of the Earth's magnetic field is 40 μT in a downward direction.

a Calculate:

i the area swept out per second by the wings
ii the magnetic flux cut per second by the wings
iii the e.m.f. induced between the wingtips.

b State which wing-tip will be at the higher potential.

13 A current-carrying solenoid produces a uniform magnetic flux of density 4.6×10^{-2} T along its axis. A small circular coil of radius 1.2 cm has 350 turns of wire and is placed on the axis of the solenoid with its plane normal to the axis. Calculate the average e.m.f. induced in the coil when the current in the solenoid is reversed in a time of 85 ms.

14 A metal disc is made to spin at 16 revolutions per second about an axis through its centre normal to the plane of the disc. The disc has radius 20 cm and spins in a uniform magnetic field of flux density 0.15 T, parallel to the axis of rotation. Calculate:

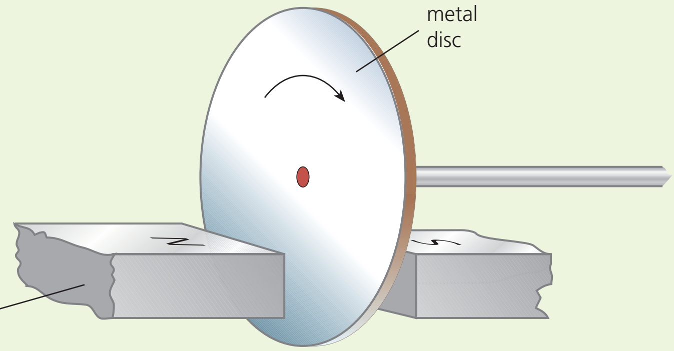
a the area swept out each second by a radius of the disc
b the flux cut each second by a radius of the disc
c the e.m.f. induced in the disc.

EXTENSION

Applications of electromagnetic induction

Eddy current damping

The generation of an electric current in a conductor by doing mechanical work may be shown by spinning a metal disc in a magnetic field, as illustrated in Figure 20.36.



▲ **Figure 20.36** Apparatus to demonstrate eddy current damping

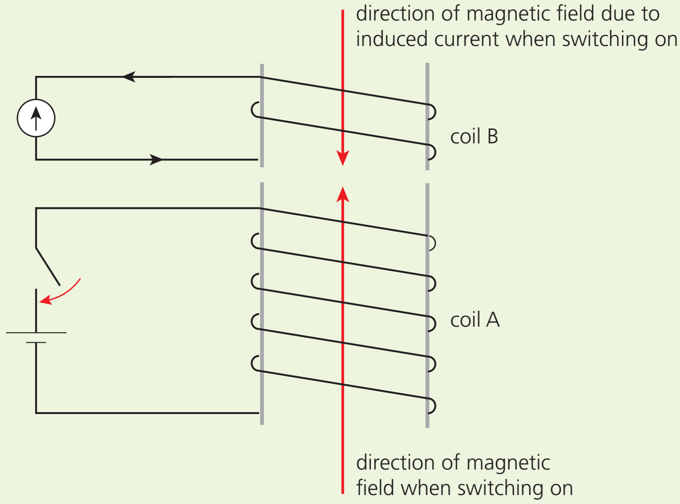
An e.m.f. is induced between the rim of the disc and the axle. The apparatus illustrated is the basis of a means by which a direct e.m.f. may be generated (see question 14 opposite).

The disc is seen to slow down much more rapidly with the magnet in place than when it has been removed. As the disc spins, it cuts through the flux lines of the magnet. This cutting becomes more obvious if the radius of the disc is considered. As the radius rotates, it will cut flux. An e.m.f. will be induced in the disc but, because the rate of cutting of flux varies from one part of the disc to another, the e.m.f. will have different magnitudes in different regions of the disc. The disc is metal and, therefore, electrons will move between regions within the disc that have different e.m.f. values. Currents are induced in the disc. Since these currents vary in magnitude and direction, they are called **eddy currents**. The eddy currents cause heating in the disc and the dissipation of the energy of rotation of the disc is referred to as **eddy current damping**.

If the permanent magnet in Figure 20.36 is replaced by an electromagnet, the spinning disc will be slowed down whenever there is a current in the electromagnet. This is the principle behind electromagnetic braking. The advantage over conventional brakes is that there is no physical contact with the spinning disc. This makes such brakes very useful for trains travelling at high speeds. However, the disadvantage is that, as the disc slows down, the induced eddy currents will be smaller and, therefore, the braking will be less efficient. This system would be useless as the parking brake on a car!

e.m.f. induced between two coils

A current-carrying solenoid or coil is known to have a magnetic field. Consider the apparatus illustrated in Figure 20.37.



▲ **Figure 20.37**

As the current in coil A is being switched on, the magnetic field in this coil grows. The magnetic field links with the turns on coil B and, as a result, there is a change in flux linkage in coil B and an e.m.f. is induced in this coil. Coil B forms part of a complete circuit and hence there is a current in coil B. The direction of this current can be determined using Lenz's law.

The change which brought about the induction of a current was a *growth* in the magnetic flux in coil A. The induced current in coil B will give rise to a magnetic field in coil B and this field will, by Lenz's law, try to oppose the growth of the field in coil A. Consequently, since the field in coil A is vertically upwards (the right-hand grip rule), the field in coil B will be vertically downwards and the induced current will be in an anticlockwise direction through the meter.

When the current in coil A is switched off, the magnetic field in coil A will *decay*. The magnetic field in coil B due to the induced current must try to prevent this decay and hence it will be vertically upwards. The induced current has changed in direction.

The magnitude of the induced e.m.f. can be increased by inserting a soft-iron core into the coil (but be careful not to damage the meter as any induced e.m.f. will be very much greater) or by increasing the number of turns on the coils or by switching a larger current in coil A.

It is important to realise that an e.m.f. is induced only when the magnetic flux in coil A is changing; that is, when the current in coil A is changing. A steady current in coil A will not give rise to an induced e.m.f. An e.m.f. may be induced continuously in coil B if an alternating current is provided for coil A. This is the principle of the **transformer**.

SUMMARY

- A magnetic field is a region of space where either a magnet or moving charge will experience a force.
- A magnetic field can be represented by magnetic field lines. Magnetic field lines never touch or cross.
- The closeness of magnetic field lines indicates the strength of the magnetic field.
- The direction of the magnetic field is given by the direction in which a free magnetic north pole would move, if placed in the field.
- There is a force on a current-carrying conductor whenever it is at an angle to a magnetic field.
- The direction of the force is given by Fleming's left-hand rule – place the first two fingers and thumb of the left hand at right angles to each other, first finger in the direction of the magnetic field, second finger in the direction of the current, then the thumb gives the direction of the force.
- The magnitude of the force F on a conductor of length L carrying a current I at an angle θ to a magnetic field of flux density B is given by the expression $F = BIL\sin\theta$.
- The magnitude of the flux density is defined as the force acting per unit current per unit length on a straight wire placed at right angles to the magnetic field.
- Magnetic flux density (field strength) is measured in tesla (T). $1\,\mathrm{T} = 1\,\mathrm{Wb\,m^{-2}}$.
- The force F on a particle with charge q moving at speed v at an angle θ to a magnetic field of flux density B is given by the expression: $F = Bqv\sin\theta$.
- The direction of the force is given by Fleming's left-hand rule.
- The path of a charged particle, moving at constant speed in a plane at right angles to a uniform magnetic field, is circular.
- The force on a charge carrier moving through a conductor placed in a magnetic field results in the production of a potential difference, the Hall voltage, across the conductor.
- The Hall voltage V_H is given by the expression $V_H = BI/ntq$. Since the Hall voltage is proportional to the magnetic flux density, a calibrated Hall probe can be used to measure flux densities.
- Electric and magnetic fields, placed mutually perpendicular to each other and normal to the direction of motion of a charged particle, may be used for the selection of the velocity of charged particles.
- An electric current gives rise to a magnetic field, the strength and direction of which depends on the size of the current and the shape of the current-carrying conductor.
- The direction of the field due to a straight wire is given by the right-hand rule.
- The direction of the field in a solenoid is given by the right-hand grip rule.
- The field of a solenoid may be increased in strength by a ferrous core; this is the principle of an electromagnet.
- Two wires carrying current in the same direction attract each other, and repel if the currents are opposite in directions.
- Magnetic flux is the product of flux density and area normal to the flux: $\Phi = BA$.
- Magnetic flux linkage is the product of the magnetic flux through the coil and the number of turns on the coil, $N\Phi$.
- The direction of the induced current in a conductor moving through a magnetic field is given by Fleming's right-hand rule. That is, if the first two fingers and thumb of the right hand are held at right angles to each other, the first finger in the direction of the magnetic field and the thumb in the direction of motion, then the second finger gives the direction of the induced e.m.f. or current.
- Faraday's law of electromagnetic induction states that the e.m.f. induced is proportional to the rate of change of magnetic flux linkage.
- Lenz's law states that the direction of the induced e.m.f. is such as to cause effects to oppose the change producing it.
- Faraday's law of electromagnetic induction and Lenz's law may be summarised using the equation $E = -\mathrm{d}(N\Phi)/\mathrm{d}t$ where E is the e.m.f. induced by a rate of change of flux linkage of $\mathrm{d}(N\Phi)/\mathrm{d}t$. The sign indicates the relative direction of the e.m.f. and the change in flux linkage.

END OF TOPIC QUESTIONS

1 A stiff straight wire has a mass per unit length of 55 g m^{-1}. The wire is laid on a horizontal bench and a student passes a current through it to try to make it lift off the bench. The horizontal component of the Earth's magnetic field is 18 μT in a direction from south to north and the acceleration of free fall is 10 m s^{-2}.

a **i** State the direction in which the wire should be laid on the bench.

ii Calculate the minimum current required.

b Suggest whether the student is likely to be successful with this experiment.

2 The magnetic flux density B at a distance r from a long straight wire carrying a current I is given by the expression $B = (2.0 \times 10^{-7}) \times I/r$, where r is in metres and I is in amperes.

a Calculate:

i the magnetic flux density at a point distance 4.0 cm from a wire carrying a current of 16 A,

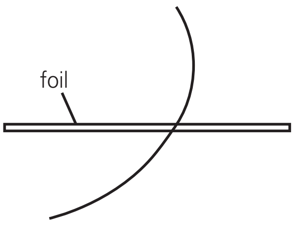
ii the force per unit length on a second wire, also carrying a current of 16 A, which is parallel to, and 4.0 cm from, the first wire.

b Suggest why the force between two wires is demonstrated in the laboratory using aluminium foil rather than copper wires.

3 A small horseshoe magnet is placed on a balance and a stiff wire is clamped in the space between its poles. The length of wire between the poles is 5.0 cm. When a current of 3.5 A is passed through the wire, the reading on the balance increases by 0.027 N.

a Calculate the magnetic flux density between the poles of the magnet.

b State three assumptions which you have made in your calculation.



▲ **Figure 20.38**

4 Fig. 20.38 shows the track of a particle in a bubble chamber as it passes through a thin sheet of metal foil. A uniform magnetic field is applied into the plane of the paper.

State with a reason:

a in which direction the particle is moving,

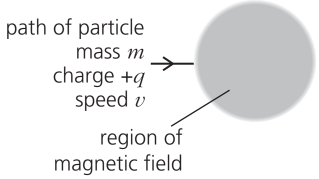
b whether the particle is positively or negatively charged.

5 **a** Explain what is meant by a *magnetic field*. **[2]**

b A particle has mass m, charge $+q$ and speed v.

The particle enters a uniform magnetic field of flux density B such that, on entry, it is moving normal to the magnetic field, as shown in Fig. 20.39.

The direction of the magnetic field is perpendicular to, and into the plane of the paper.



▲ **Figure 20.39**

i On a copy of Fig. 20.39, draw the path of the particle through, and beyond, the region of the magnetic field. **[3]**

ii There is a force acting on the particle, causing it to accelerate.

Explain why the speed of the particle on leaving the magnetic field is v. **[1]**

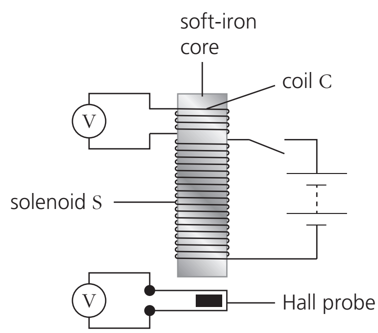
c The particle in **b** loses an electron so that its charge becomes $+2q$. Its change in mass is negligible.

Determine, in terms of v, the initial speed of the particle such that its path through the magnetic field is unchanged. Explain your working. **[3]**

Cambridge International AS and A Level Physics (9702) Paper 43 Q8 Oct/Nov 2018

6 **a** State Faraday's law of electromagnetic induction. **[2]**

b A solenoid S is wound on a soft-iron core, as shown in Fig. 20.40.



▲ **Figure 20.40**

A coil C having 120 turns of wire is wound on to one end of the iron core.
The area of cross-section of the coil C is 1.5 cm^2.

A Hall probe is close to the other end of the core.

When there is a constant current in the solenoid S, the flux density in the soft-iron core is 0.19 T.

The reading on the voltmeter connected to the Hall probe is 0.20 V.

The current in solenoid S is now reversed in a time of 0.13 s at a constant rate.

i Calculate the reading on the voltmeter connected to coil C during the time that the current is changing. **[2]**

ii Complete a copy of Fig. 20.41 for the voltmeter readings for the times before, during and after the direction of the current is reversed.

	before current changes	**during current change when current is zero**	**after current change**
reading on voltmeter connected to coil C / V			
reading on voltmeter connected to Hall probe / V	0.20		

▲ **Figure 20.41** **[4]**

Cambridge International AS and A Level Physics (9702) Paper 43 Q9 Oct/Nov 2018

21 Alternating currents

Learning outcomes

By the end of this topic, you will be able to:

21.1 Characteristics of alternating currents

1. understand and use the terms period, frequency and peak value as applied to an alternating current or voltage
2. use equations of the form $x = x_0 \sin\omega t$ representing a sinusoidally alternating current or voltage
3. recall and use the fact that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current
4. distinguish between root-mean-square (r.m.s.) and peak values and recall and use $I_{r.m.s} = I_0/\sqrt{2}$ and $V_{r.m.s} = V_0/\sqrt{2}$ for a sinusoidal alternating current

21.2 Rectification and smoothing

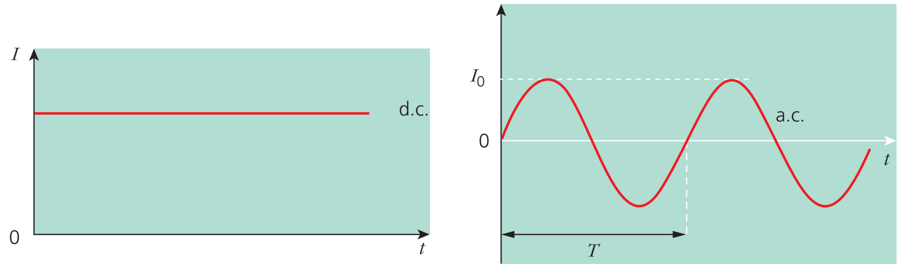
1. distinguish graphically between half-wave and full-wave rectification
2. explain the use of a single diode for the half-wave rectification of an alternating current
3. explain the use of four diodes (bridge rectifier) for the full-wave rectification of an alternating current
4. analyse the effect of a single capacitor in smoothing, including the effect of the value of capacitance and the load resistance

Starting points

- ★ Frequency f is related to period T by the expression $f = 1/T$.
- ★ Power is dissipated in a resistor and the magnitude of the power is given by the expressions $P = I^2R$ or $P = VI$ or $P = V^2/R$.
- ★ A diode is a device that allows current to move in one direction only.

21.1 Characteristics of alternating currents

Up to this point in our studies, we have dealt with systems in which a battery is connected to a circuit and there is a steady current in one direction. You will know this sort of current as a **direct current**, abbreviated to d.c. However, the domestic electricity supply, produced by generators, is one which uses **alternating current** (a.c.). An alternating current or voltage reverses its direction regularly and is usually sinusoidal, as shown in Figure 21.1.



▲ **Figure 21.1** Direct and alternating currents

The time T taken for one complete cycle of the alternating current is the period of the current.

We can represent the current and the voltage by the equations

$$I = I_0 \sin \omega t$$
$$V = V_0 \sin \omega t$$

The graphs of $I = I_0 \sin \omega t$ and $V = V_0 \sin \omega t$ have the same sinusoidal shape as the graphs used to represent simple harmonic motion and so the electrons in a wire carrying a.c. move backwards and forwards with s.h.m. Hence we can describe the angular frequency of the current in rad s^{-1} as ω where $T = 2\pi/\omega$. The reciprocal of the period is the frequency f. That is, $f = 1/T$ and $f = \omega/2\pi$. The frequency is the number of complete cycles per unit time. The unit of frequency is the hertz (Hz) where 1 Hz = 1 cycle per second.

The **peak value** of the current or voltage is I_0 or V_0, the amplitude of the oscillating current or voltage. Sometimes the term peak-to-peak value is used; this means $2I_0$ or $2V_0$, or twice the amplitude.

WORKED EXAMPLE 21A

The variation with time t (in seconds) of the potential difference V (in volts) of an alternating supply is given by $V = 24 \sin 380t$.
Determine, for this supply:

a the peak potential difference
b the frequency.

Answers

a peak potential difference = **24 V**
b $\omega = 2\pi f = 380$
frequency = $380/2\pi$ = **60 Hz**

Question

1 The peak value of the mains potential difference delivered to homes in India is 325 V and the frequency is 50 Hz. Write an equation for the variation in the mains potential difference in terms of time t.

Measuring period and frequency using a cathode-ray oscilloscope

A **cathode-ray oscilloscope (CRO)** has a calibrated time-base, so that measurements from the screen of the CRO can be used to give values of time intervals. One application is to measure the frequency of a sinusoidally varying current or voltage. The signal is connected to the Y-input of the CRO, and the y-gain and time-base controls are adjusted until a trace of at least one, but fewer than about five, complete cycles of the signal is obtained on the screen. The distance L on the graticule (the scale on the screen) corresponding to one complete cycle is measured (Figure 21.2).

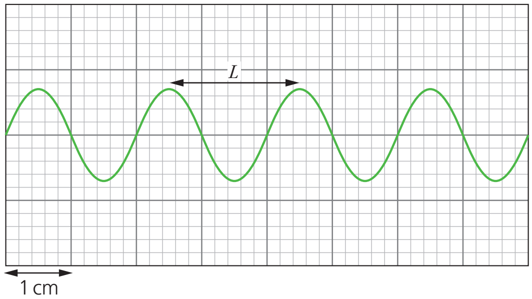


Figure 21.2 The use of a CRO to measure frequency

It is good practice to measure the length of, say, four cycles, and then divide by 4 so as to obtain an average value of L. The graticule will probably be divided into centimetre and perhaps millimetre or two-millimetre divisions. If the time-base setting is x (which will be in units of seconds, milliseconds or microseconds per centimetre), the time T for one cycle is given by $T = Lx$. The frequency f of the signal is then obtained from $f = 1/T$.

The uncertainty of the determination will depend on how well you can estimate the measurement of the length of the cycle from the graticule. Remembering that the trace has a finite width, you can probably measure this length to an uncertainty of about ±2 mm.

WORKED EXAMPLE 21B

An alternating current is connected to the Y-input of a CRO. When the time-base control is set at 0.50 milliseconds per centimetre, the trace shown in Figure 21.3 is obtained. What is the frequency of the current?

Answer

Two complete cycles of the trace occupy 6.0 cm on the graticule. The length of one cycle is therefore 3.0 cm. The time-base setting is $0.50\,\text{ms cm}^{-1}$, so 3.0 cm is equivalent to $3.0 \times 0.50 = 1.5$ ms. The frequency is thus $1/1.5 \times 10^{-3}$ = **670 Hz**.

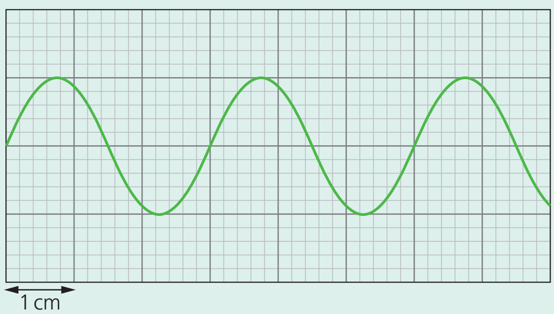


Figure 21.3

Question

2 The same signal is applied to the Y-input of the CRO as in Worked Example 21B, but the time-base control is changed to 2.0 milliseconds per centimetre. How many complete cycles of the trace will appear on the screen, which is 8.0 cm wide?

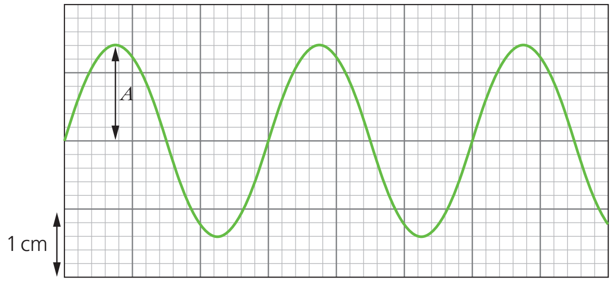
Measuring voltage or current using a cathode-ray oscilloscope

The cathode-ray oscilloscope, with its calibrated y-gain, may be used to measure the amplitude of an alternating voltage signal.

(We have already seen how the time-base of the CRO may be used to measure time.) The signal is connected to the Y-input, and the y-gain and time-base settings are adjusted until a suitable trace is obtained (Figure 21.4). The amplitude A of the trace is measured. If the y-gain setting is Q (in units of volts per centimetre), the peak value V_0 of the signal is given by $V_0 = AQ$. The peak-to-peak value is $2V_0$.

If the time base is switched off a straight vertical line will be displayed on the screen. The peak-to-peak value of this trace from bottom to top can be measured. The peak value V_0 is half the peak-to-peak voltage.

Once we know the peak voltage (V_0) and the resistance (R) of the component the oscilloscope is connected across, we can calculate the peak current (I_0) using the equation $V = IR$.



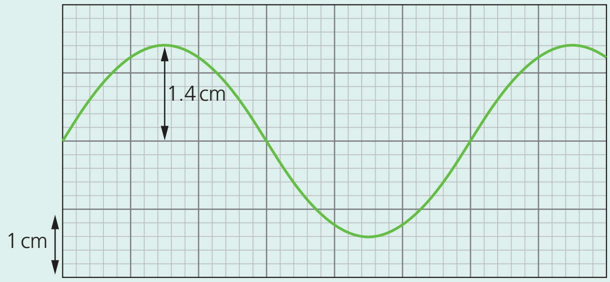
▲ **Figure 21.4** Measurement of alternating voltage

WORKED EXAMPLE 21C

An alternating voltage is connected to the Y-input of a CRO. When the Y-amplifier control is set to 5.0 millivolts per centimetre, the trace shown in Figure 21.5 is obtained. Find the peak voltage of the signal.

Answer

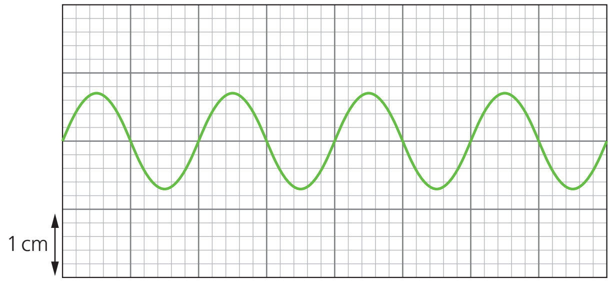
Measure the amplitude of the trace on the graticule: this is 1.4 cm. The *y*-gain setting is 5.0 mV cm^{-1}. 1.4 cm is thus equivalent to 1.4 × 5.0 = **7.0 mV**. The peak voltage of the signal is 7.0 mV.



▲ **Figure 21.5**

Question

3 The Y-input of a CRO is connected to an alternating voltage source. When the *y*-gain control is set to 20 millivolts per centimetre, the trace shown in Figure 21.6 is obtained. Find the peak-to-peak voltage of the signal.



▲ **Figure 21.6**

Power in an a.c. circuit

It is clear from Figure 21.1 that the average value of an alternating current is zero. However, this does not mean that when an a.c. source is connected to a resistor, no power is generated in the resistor. An alternating current in a wire can be thought of as electrons moving backwards and forwards, and passing on their energy by collision.

The power generated in a resistance R is given by the usual formula

$$P = I^2 R$$

but here the current I must be written as

$$I = I_0 \sin \omega t$$

Thus

$$P = I_0^{\,2} R \sin^2 \omega t$$

Since $I_0^{\,2}$ and $\sin^2 \omega t$ are always positive, we see that the power P is also always positive.

The expression we have just derived for P gives the power at any instant. What is much more useful is the average or mean power. This is the quantity which must be used in assessing the power generated in the resistor. Because I_0 and R are constants, the average value of P will depend on the average value of $\sin^2 \omega t$, which is $\frac{1}{2}$. So the average power $<P>$ delivered to the resistor is

$$<P> = \tfrac{1}{2} I_0^{\,2} R = \tfrac{1}{2} V_0^{\,2}/R$$

or

For a sinusoidal alternating current the mean power is half the maximum power.

We could use the average value of the square of the current or the voltage in these relations, since

$$<I^2> = \tfrac{1}{2} I_0^{\,2} \text{ and } <V^2> = \tfrac{1}{2} V_0^{\,2}$$

The square root of $<I^2>$ is called the **root-mean-square**, or **r.m.s.** value of the current I_{rms} and similarly the square root of $<V^2>$ the root-mean-square, or r.m.s. value of the voltage V_{rms}. The numerical relations are

$$I_{rms} = \sqrt{<I^2>} = I_0/\sqrt{2} = 0.707 I_0$$
$$V_{rms} = \sqrt{<V^2>} = V_0/\sqrt{2} = 0.707 V_0$$

The r.m.s. values are useful because they represent the *effective* values of current and voltage in an a.c. circuit. A direct current with a value of I, equal to the r.m.s current I_{rms} of an a.c. circuit, will produce exactly the same heating effect in a resistor. In specifying a domestic supply voltage, it is the r.m.s. value that is quoted, not the peak value.

The r.m.s. value of the alternating current or voltage is that value of the direct current or direct voltage that would produce thermal energy at the same rate in a resistor.

WORKED EXAMPLE 21D

A 1.5 kW heater is connected to the domestic supply, which is quoted as 230 V. Calculate the peak current in the heater, and its resistance.

Answer

The r.m.s version of the power/current/voltage equation is $I_{rms} V_{rms}$ = mean power.

This gives $I_{rms} = (1.5 \times 10^3)/230 = 6.5\,\text{A}$.

The peak current $I_0 = \sqrt{2} I_{rms} = \mathbf{9.2\,A}$

The resistance $R = V_{rms}/I_{rms} = 230/6.5 = \mathbf{35\,\Omega}$

Questions

4 The peak voltage of an alternating signal is 28 mV. Find the r.m.s. voltage.

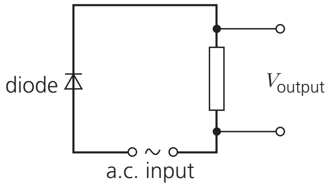
5 For the alternating supply given by $V = 24 \sin 380t$ where V is in volts, determine the r.m.s. voltage.

6 A heater of resistance 35 Ω is connected to a domestic supply, quoted as 230 V.
   a Calculate:
      i the peak voltage of the supply
      ii the average power in the resistor.
   b What are the maximum and minimum values of the instantaneous power in the resistor?

21.2 Rectification and smoothing

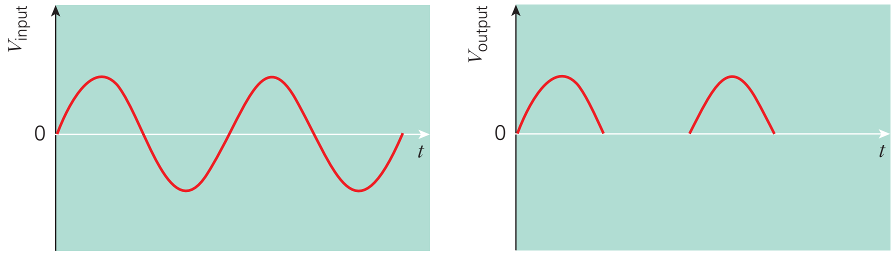
It is sometimes necessary to convert an alternating current into a direct current. This is because most electronic devices require direct current, whereas the domestic supply is alternating. The conversion can be done by a process known as **rectification**.

Suppose a single diode (see Topic 9.3) is connected into the a.c. circuit of Figure 21.7.



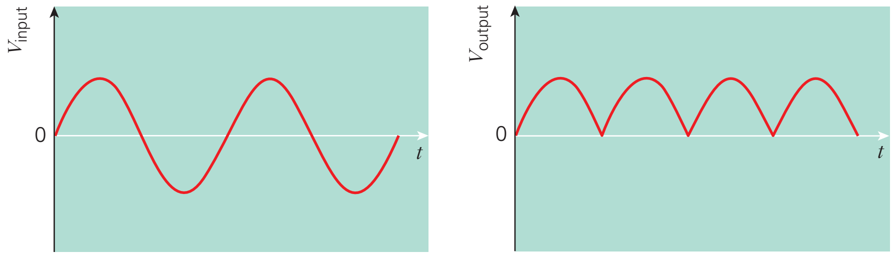
▲ **Figure 21.7** Single-diode circuit for half-wave rectification

We know that the diode allows current to flow in one direction only. This means that the output voltage across the resistor will consist only of the positive half-cycles of the input voltage, as shown in Figure 21.8. The diode has rejected the negative part of the input, producing a unidirectional voltage across the output resistor which fluctuates considerably, rather than a constant direct voltage. Nevertheless, we have achieved what is called **half-wave rectification**.



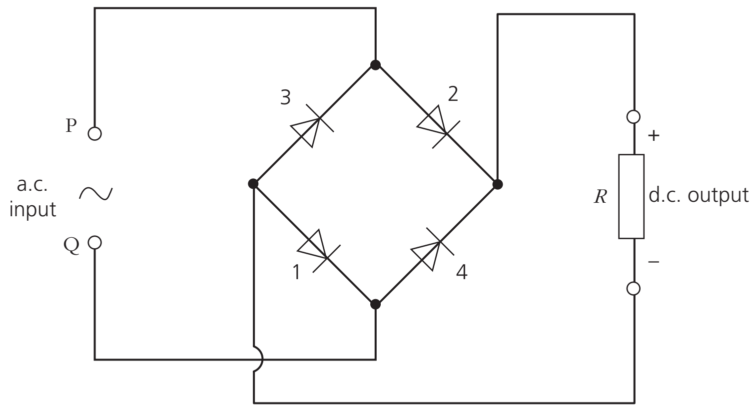
▲ **Figure 21.8** Half-wave rectification

It is more satisfactory, and energy efficient, to make use of the negative half-cycles of the input and reverse their polarity, as shown in Figure 21.9. This process is called **full-wave rectification**.



▲ **Figure 21.9** Full-wave rectification

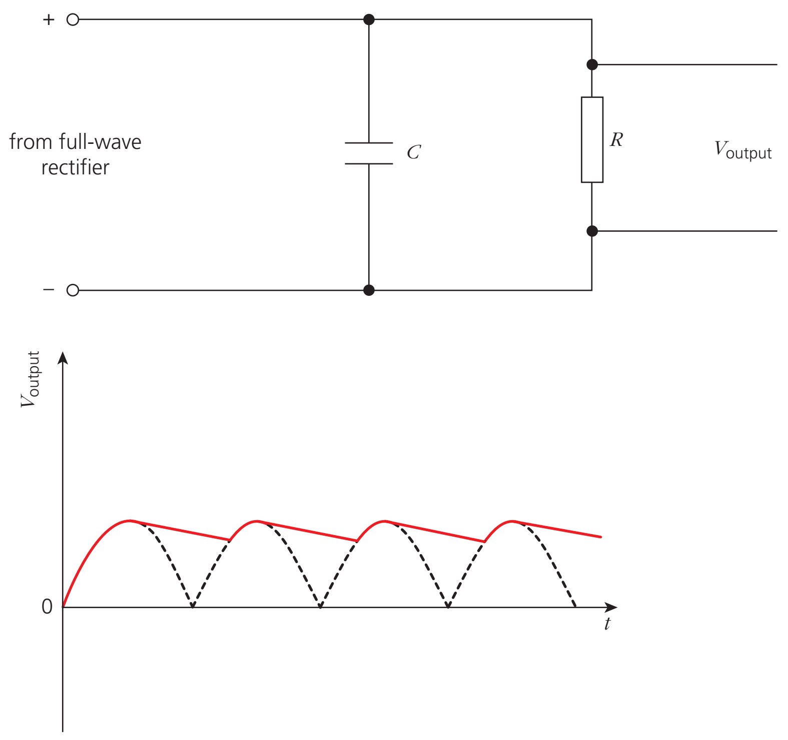
One circuit used for full-wave rectification is illustrated in Figure 21.10.



▲ **Figure 21.10** Four-diode (bridge) circuit for full-wave rectification

It uses four diodes arranged in a diamond pattern and is referred to as a **bridge rectifier** circuit. The input terminals are P and Q. If P is positive during the first half-cycle, diodes 1 and 2 on opposite sides of the diamond will conduct. In the next half-cycle Q is positive, and diodes 3 and 4 conduct. Thus the resistor acting as a load will always have its upper terminal positive and its lower terminal negative.

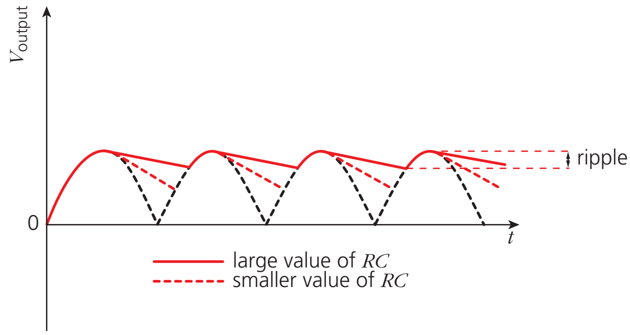
The circuit has produced a unidirectional voltage, but the output is still not a good approximation to steady direct voltage, as required by most electronic equipment. We can improve the situation by inserting a capacitor across the output terminals of the bridge circuit as in Figure 21.11.



▲ **Figure 21.11** Smoothing by capacitor

The capacitor charges up on the rising part of the half-cycle, and then discharges through the resistor as the output voltage falls. The effect is to reduce the fluctuations in the unidirectional output. This process is called **smoothing**.

The discharge of a capacitor was discussed in Topic 19.3. The important factor is the time constant of the resistor–capacitor circuit. If the product of the capacitance C and the load resistance R is much larger than the half-period of the original supply to the rectifier circuit, the ripple on the direct voltage or current will be small. Ripple is the magnitude of the variation of the voltage or current that is superimposed on the direct voltage or current. Reducing the time constant will increase the ripple, as illustrated in Figure 21.12.



▲ **Figure 21.12** Magnitude of the ripple

SUMMARY

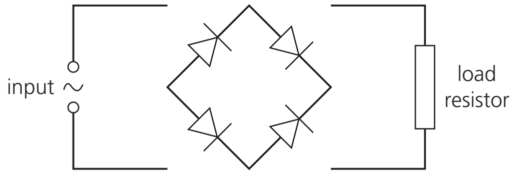
- Alternating current or voltage is represented by an equation of the form $x = x_0 \sin\omega t$.
- The r.m.s. value of the alternating current or voltage is that value of the direct current or direct voltage that would produce thermal energy at the same rate in a resistor.
- Peak and root-mean-square (r.m.s.) values of sinusoidal current or voltage are related by an equation of the form $x_0 = \sqrt{2}x_{rms}$.
- For a sinusoidal input, mean power in a resistive load is one half of the peak power.
- A single diode gives half-wave rectification: negative half-cycles are blocked.
- A bridge circuit of four diodes can give full-wave rectification.
- A capacitor connected across the output reduces the fluctuations of the rectified output voltage applied to the load resistor.
- The degree of ripple depends on the time constant RC of the discharge of the smoothing capacitor through the load resistor. Larger RC gives smaller ripple.

END OF TOPIC QUESTIONS

1 A stereo system has two output channels, each connected to a loudspeaker of effective resistance 8.0 Ω. Each channel can deliver a maximum average power output of 48 W to its speaker. Calculate the r.m.s. voltage and the r.m.s. current fed to one speaker at this maximum power. Assume that the loudspeaker can be treated as a simple resistance and that the voltage is sinusoidal.

2 The full-wave rectifier circuit of Fig. 21.10 is used to rectify an a.c. input voltage of 240 V r.m.s. The output resistor has resistance 17 kΩ.
  a Calculate the peak value of the input voltage.
  b Estimate the average current in the output resistor.

3 a Complete a copy of Fig. 21.13 to show four diodes connected to form a full-wave rectifier.



▲ **Figure 21.13**

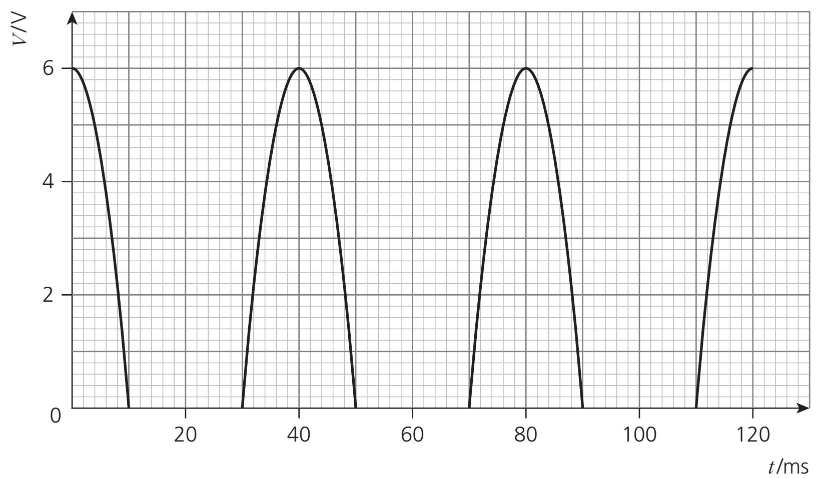
  b The root-mean-square input potential difference to the completed full-wave rectifier is 12 V.
  The frequency of the input potential difference is 50 Hz.

On the axes of Fig. 21.14, sketch the variation with time t of the potential difference V across the load resistor for 3 periods of the input potential difference.

▲ **Figure 21.14**

c On the completed circuit diagram of Fig. 21.13, draw the symbol for a capacitor, connected so as to produce smoothing of the output potential difference.

d The output load resistor in a circuit providing half-wave rectification is 23 kΩ. The variation with time t of the output voltage V across the load is shown in Fig. 21.15.



▲ **Figure 21.15**

A capacitor of capacitance 2 μF is now connected in parallel with the load resistor.

On a copy of Fig. 21.15, sketch a line to show the variation with time t of the smoothed output voltage.

22 Quantum physics

Learning outcomes

By the end of this section you will be able to:

22.1 Energy and momentum of a photon

1. understand that electromagnetic radiation has a particulate nature
2. understand that a photon is a quantum of electromagnetic energy
3. recall and use $E = hf$
4. use the electronvolt (eV) as a unit of energy
5. understand that a photon has momentum and that the momentum is given by $p = E/c$

22.2 Photoelectric effect

1. understand that photoelectrons may be emitted from a metal surface when it is illuminated by electromagnetic radiation
2. understand and use the terms threshold frequency and threshold wavelength
3. explain photoelectric emission in terms of photon energy and work function energy
4. recall and use $hf = \Phi + \frac{1}{2}mv_{max}^2$
5. explain why the maximum kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity

22.3 Wave-particle duality

1. understand that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature
2. describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles
3. understand the de Broglie wavelength as the wavelength associated with a moving particle
4. recall and use $\lambda = h/p$

22.4 Energy levels in atoms and line spectra

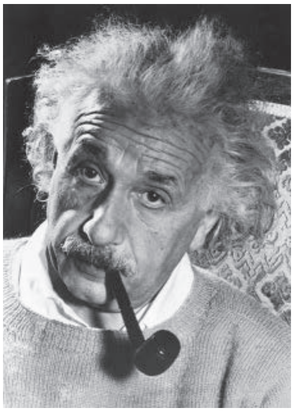
1. understand that there are discrete electron energy levels in isolated atoms (e.g. atomic hydrogen)
2. understand the appearance and formation of emission and absorption line spectra
3. recall and use the relation $hf = E_1 - E_2$

Starting points

- ★ A simple model of an atom involves a small massive positively charged nucleus around which orbit negatively charged electrons.
- ★ The electrons orbit in shells at different distances from the nucleus.
- ★ The speed of electromagnetic waves in free space is $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$.
- ★ The speed c, frequency f and wavelength λ of electromagnetic radiation are related by the expression $c = f\lambda$.
- ★ Electromagnetic waves may undergo interference and diffraction.
- ★ Momentum of a particle is the product of its mass and velocity.

22.1 Energy and momentum of a photon

At the end of the nineteenth century, it was fully accepted that light is a wave motion. Evidence for this came from observed interference and diffraction effects (Topic 8). The conclusions of experiments on the photoelectric effect first observed in 1887 (and described later in this topic) produced doubt as to whether light is a continuous wave.



▲ **Figure 22.1** Albert Einstein

Quantised energy

In 1901, the German physicist Max Planck suggested that the energy carried by electromagnetic radiation might exist as discrete packets or quantities called **quanta**. The energy E carried in each quantum is given by

$$E = hf = \frac{hc}{\lambda}$$

where f is the frequency of the radiation, λ is the wavelength and h is a constant called the Planck constant. The value of the Planck constant is 6.63×10^{-34} J s.

In 1905, Albert Einstein developed the theory of quantised energy to explain all the observations associated with photoelectric emission. He proposed that light radiation consists of a stream of energy packets called **photons**.

A photon is the special name given to a quantum of energy when the energy is in the form of electromagnetic radiation.

In the wave theory of light, the flow of energy in the wave is continuous. The concept of quanta, or 'packets' of energy, gives rise to a particulate nature of electromagnetic waves.

A particulate nature of electromagnetic radiation means that each photon will have not only energy but also momentum. The momentum p of each photon is related to its energy E and the speed c of electromagnetic waves by the expression

$$p = \frac{E}{c}$$

The energy of a photon of visible light varies between approximately 5.7×10^{-19} J and 2.7×10^{-19} J. It can be seen that when dealing with photons, expressing the energy in joules leads to numerical values that are not easy to appreciate. Consequently a smaller unit of energy, the **electronvolt (eV)** is frequently used.

The electronvolt (eV) is the energy gained by an electron when it is accelerated from rest in a vacuum through a potential difference of one volt.

Since the work done in moving a charge Q through a potential difference V is QV (see Topic 9.2) and the charge on the electron is 1.60×10^{-19} C, then $1\,\text{eV} = 1.60 \times 10^{-19}$ J.

The electronvolt is frequently a convenient unit for the measurement of photon energies and electron energy levels in atoms (as we shall see in Topic 22.4). Furthermore, we shall see that, when studying nuclear physics (Topic 23), a multiple of the electronvolt, the mega-electronvolt (MeV), is a convenient unit for the measurement of nuclear energies. One mega-electronvolt (MeV) is 10^6 eV or 1.60×10^{-13} J.

WORKED EXAMPLE 22A

An electromagnetic wave has speed $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$ and wavelength 550 nm. For one photon in this wave, calculate:

a the energy in electronvolts

b the momentum.

(The Planck constant = 6.63×10^{-34} J s.)

Answers

a energy = $hc/\lambda = (6.63 \times 10^{-34} \times 3.00 \times 10^8)/(550 \times 10^{-9})$

$= 3.6 \times 10^{-19}\,\mathrm{J} = (3.6 \times 10^{-19})/(1.6 \times 10^{-19})$

$= \mathbf{2.3\,eV}$

b momentum $= E/c$

$= (3.6 \times 10^{-19})/(3.0 \times 10^8)$

$= \mathbf{1.2 \times 10^{-27}\,N\,s}$

Question

1 A photon has an energy of 4.3 eV. Calculate, for this photon:

a the wavelength of electromagnetic radiation

b its momentum.

(The Planck constant = 6.63×10^{-34} J s.)

22.2 Photoelectric effect

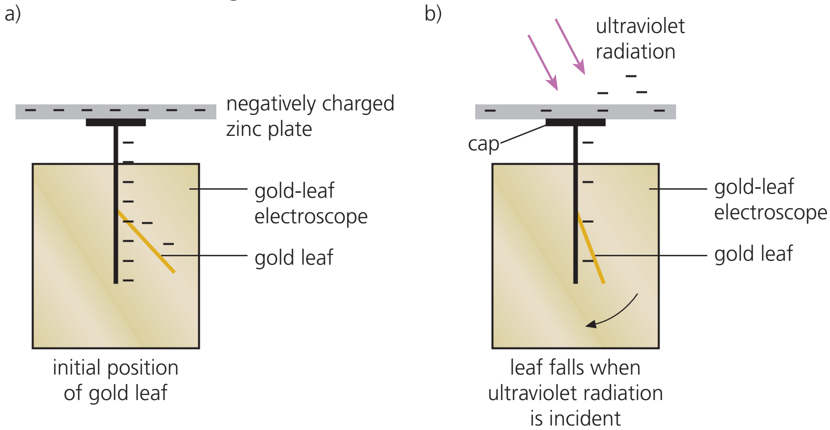
Some of the electrons in a metal are free to move around in it. (It is these electrons that form the electric current when a potential difference is applied across the ends of a metal wire.) However, to remove electrons from a metal surface requires energy, because they are held in the metal by the electrostatic attraction of the positively charged nuclei. If an electron is to escape from the surface of a metal, work must be done on it. The electron must be given energy. When this energy is in the form of light energy, the phenomenon is called **photoelectric emission**.

Photoelectric emission is the release of electrons from the surface of a metal when electromagnetic radiation is incident on its surface.

The electrons emitted are referred to as **photoelectrons**.

Demonstration of photoelectric emission

A clean zinc plate is placed on the cap of a gold-leaf electroscope. The electroscope is then charged negatively, and the gold leaf deflects, proving that the zinc plate is charged. This is illustrated in Figure 22.2.



▲ **Figure 22.2** Demonstration of photoelectric emission

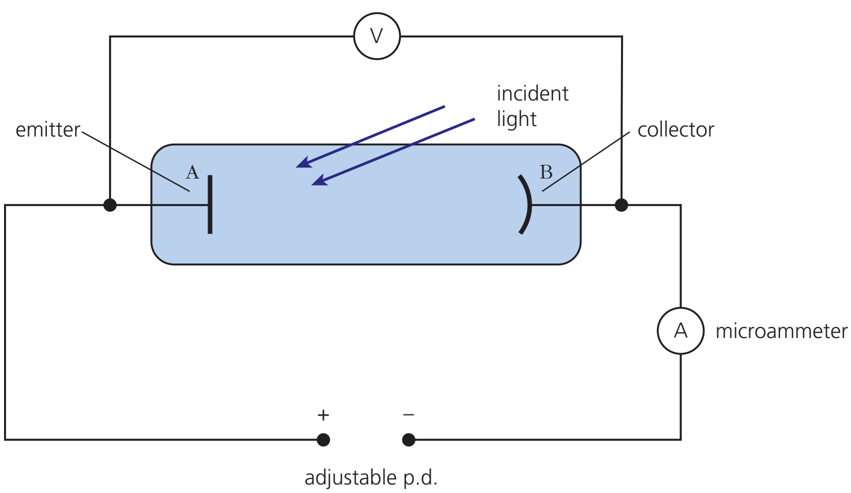
If visible light of any colour is shone on to the plate, the leaf does not move. Even when the intensity (the brightness) of the light is increased, the leaf remains in its deflected position. However, when ultraviolet radiation is shone on the plate, the leaf begins to fall immediately, showing that it is losing negative charge. This means that electrons are being emitted from the zinc plate. These electrons are **photoelectrons**. If the intensity of the ultraviolet radiation is increased, the leaf falls more quickly, showing that the rate of emission of electrons has increased.

The difference between ultraviolet radiation and visible light is that ultraviolet radiation has a shorter wavelength and a higher frequency than visible light.

Further investigations with apparatus like this lead to the following conclusions:

- A positively charged electroscope cannot be discharged, indicating that only electrons are emitted.
- If photoemission takes place, it does so instantaneously. There is no delay between illumination and emission.
- Photoemission takes place only if the frequency of the incident radiation is above a certain minimum value called the **threshold frequency** f_0.
- The wavelength corresponding to the threshold frequency is known as the **threshold wavelength** λ_0.
- Different metals have different threshold frequencies.
- Whether or not emission takes place depends only on whether the frequency of the radiation used is above the threshold for that surface. It does not depend on the intensity of the radiation.
- For a given frequency, the rate of emission of photoelectrons (the photoelectric current) is proportional to the intensity of the radiation.

Another experiment, using the apparatus shown in Figure 22.3, can be carried out to investigate the energies of the photoelectrons.



▲ **Figure 22.3 Experiment to measure the maximum kinetic energy of photoelectrons**

If ultraviolet radiation of a fixed frequency (above the threshold) is shone on to the metal surface A, it emits photoelectrons. Some of these electrons travel from A to B. Current is detected using the microammeter. If a potential difference is applied between A and B, with B negative with respect to A, any electron going from A to B will gain potential energy as it moves against the electric field. The gain in potential energy is at the expense of the kinetic energy of the electron. That is,

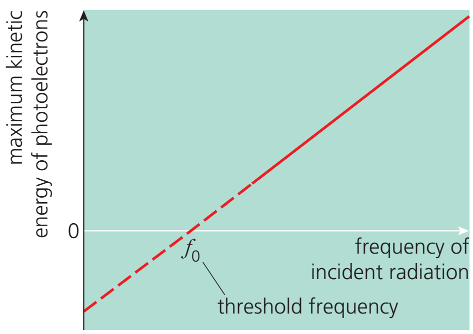
loss in kinetic energy = gain in potential energy

= charge of electron × potential difference

If the voltage between A and B is gradually increased, the current registered on the microammeter decreases and eventually falls to zero. The minimum value of the potential difference necessary to stop the electron flow is known as the **stopping potential**. It measures the maximum kinetic energy with which the photoelectrons are emitted. The fact that there is a current in the microammeter at voltages less than the stopping potential indicates that there is range of kinetic energies for these electrons.

If the experiment is repeated with radiation of greater intensity but the same frequency, the maximum current in the microammeter increases, but the value of the stopping potential is unchanged.

The experiment can be repeated using ultraviolet radiation of different frequencies, measuring the stopping potential for each frequency. When the maximum kinetic energy of the photoelectrons is plotted against the frequency of the radiation, the graph of Figure 22.4 is obtained.



▲ **Figure 22.4** Graph of maximum kinetic energy of photoelectrons against frequency of radiation

The following conclusions are drawn from this experiment:

- The photoelectrons have a range of kinetic energies, from zero up to some maximum value. If the frequency of the incident radiation is increased, the maximum kinetic energy of the photoelectrons also increases.
- For constant frequency of the incident radiation, the maximum kinetic energy is unaffected by the intensity of the radiation.
- When the graph of Figure 22.4 is extrapolated to the point where the maximum kinetic energy of the photoelectrons is zero, the minimum frequency required to cause emission from the surface (the threshold frequency) may be found.

The conclusions of experiments on photoemission produced doubt as to whether light is a continuous wave. One of the main problems concerns the existence of a threshold frequency.

Classical wave theory predicts that when an electromagnetic wave (that is, light) interacts with an electron, the electron will absorb energy from it. So, if an electron absorbs enough energy, it should be able to escape from the metal. Remember that the energy carried by a wave depends on its amplitude (and its frequency) (see Topic 7.1). Thus, even if we have a low-frequency wave, its energy can be boosted by increasing the amplitude (that is, by increasing the brightness of the light). So, according to wave theory, we ought to be able to cause photoemission using any frequency of light, provided we make it bright enough. Alternatively, we could use less bright light and shine it on the metal for a longer time, until enough energy to cause emission has been delivered. But this does not happen – emission is instantaneous, if at all.

The experiments we have described above showed conclusively that radiation of frequency below the threshold, no matter how intense or for how long it is used, does not produce photoelectrons. Furthermore, emission is instantaneous. The classical wave theory of electromagnetic radiation leads to the following predictions:

1 Whether an electron is emitted or not should depend on the power of the incident wave; that is, on its intensity. A very intense wave, of any frequency, should cause photoemission.
2 The maximum kinetic energy of the photoelectrons should be greater if the radiation intensity is greater.
3 There is no reason why photoemission should be instantaneous.

These predictions, based on wave theory, do not match the observations. A new approach, based on an entirely new concept, the **quantum theory**, was used to explain these findings.

Einstein's theory of photoelectric emission

In 1905, Albert Einstein developed the theory of quantised energy, proposed by Planck, to explain all the observations associated with photoelectric emission. He proposed that light radiation consists of a stream of energy packets called **photons**. Remember that a photon is the special name given to a quantum of energy when the energy is in the form of electromagnetic radiation.

When a photon interacts with an electron, it transfers all its energy to the electron. It is only possible for a single photon to interact with a single electron; the photon cannot share its energy between several electrons. This transfer of energy is instantaneous.

The photon theory of photoelectric emission is as follows. If the frequency of the incident radiation is less than the threshold frequency for the metal, the energy carried by each photon is insufficient for an electron to escape the surface of the metal. If the photon energy is insufficient for an electron to escape, it is converted to thermal energy in the metal.

The minimum amount of energy necessary for an electron to escape from the surface is called the **work function energy** Φ.

Some values for the work function energy Φ and threshold frequency f_0 of different metals are given in Table 22.1.

metal	Φ/J	Φ/eV	f_0/Hz
sodium	3.8×10^{-19}	2.4	5.8×10^{14}
calcium	4.6×10^{-19}	2.9	7.0×10^{14}
zinc	5.8×10^{-19}	3.6	8.8×10^{14}
silver	6.8×10^{-19}	4.3	1.0×10^{15}
platinum	9.0×10^{-19}	5.6	1.4×10^{15}

▲ **Table 22.1** Work function energies and threshold frequencies

Remember: $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

If the frequency of the incident radiation is equal to the threshold frequency, the energy carried by each photon is just sufficient for electrons at the surface to escape. If the frequency of the incident radiation is greater than the threshold frequency, surface electrons will escape and have surplus energy in the form of kinetic energy. These electrons will have the maximum kinetic energy. If a photon interacts with an electron below the surface, some energy is used to take the electron to the surface, so that it is emitted with less than the maximum kinetic energy. This gives rise to a range of values of kinetic energy.

Einstein used the principle of conservation of energy to derive the photoelectric equation

photon energy = work function energy + maximum kinetic energy of photoelectron

or

$$hf = \Phi + \frac{1}{2}m_e v_{max}^2$$

For radiation incident at the threshold frequency f_0, then $\frac{1}{2}m_e v_{max}^2 = 0$ so that $hf_0 = \Phi$. The photoelectric equation can then be written

$$hf = hf_0 + \frac{1}{2}m_e v_{max}^2$$

Note

Experimental evidence indicates that the photoelectric current, i.e. the rate of emission of photoelectrons, depends on the intensity of the radiation when the frequency is constant. Increasing intensity gives rise to increasing rate of emission of photoelectrons.

This observation has led many students to believe that rate of emission is independent of frequency. This is incorrect. A beam of radiation has an intensity. This intensity is numerically equal to the power incident normally on unit area of the surface. Since the beam consists of a stream of photons, the intensity is the product of the rate of arrival of photons and the energy of each photon. At constant intensity, the rate at which photons arrive at the metal surface depends on the energy of each photon. So, if the frequency of the radiation increases, the energy of each photon increases and, therefore, for constant intensity, the rate of arrival of photons decreases. Fewer photons per unit time mean a smaller rate of emission of electrons.

WORKED EXAMPLE 22B

The work function energy of platinum is 9.0×10^{-19} J. Calculate:

a the threshold frequency for the emission of photoelectrons from platinum

b the maximum kinetic energy of a photoelectron when radiation of frequency 2.0×10^{15} Hz is incident on a platinum surface.
(The Planck constant = 6.63×10^{-34} J s.)

Answers

a Using $hf_0 = \Phi$, $f_0 = \Phi/h$, so
$f_0 = 9.0 \times 10^{-19}/6.63 \times 10^{-34} =$ **1.4×10^{15} Hz (1.36×10^{15} Hz)**

b Using $hf = hf_0 + \frac{1}{2}m_e v_{max}^2$, $hf - hf_0 = \frac{1}{2}m_e v_{max}^2$ and so
$\frac{1}{2}m_e v_{max}^2 = 6.63 \times 10^{-34}\,(2.0 \times 10^{15} - 1.4 \times 10^{15}) =$ **4.0×10^{-19} J**

Questions

2 The work function energy of zinc is 3.6 eV. Calculate the threshold wavelength for electromagnetic radiation incident on the surface of zinc.

3 Electromagnetic radiation of frequency 3.0×10^{15} Hz is incident on the surface of sodium metal. The emitted photoelectrons have a maximum kinetic energy of 10 eV. Calculate the threshold frequency for photoemission from sodium.

4 Data for the threshold frequency f_0 and the work function energy Φ of some metal surfaces are shown in Table 22.2.

metal	$f_0/10^{14}$ Hz	$\Phi/10^{-19}$ J
platinum		9.0
sodium	5.8	3.8
zinc	8.8	5.8

▲ **Table 22.2**

a Calculate the threshold frequency for platinum.

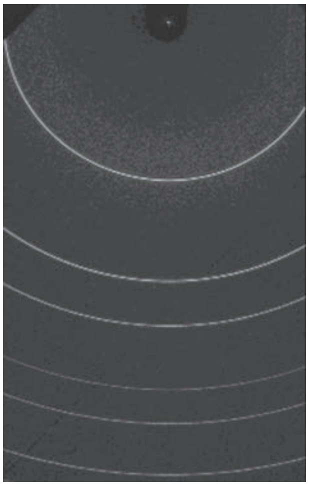
b A beam of electromagnetic radiation having a continuous range of wavelengths between 320 nm and 550 nm is incident, in turn, on each of the above metals. Determine which metals will emit photoelectrons.

c When light of frequency f and intensity I is incident on a certain metal surface, electrons are emitted. State and explain the effect, if any, on the emission of photoelectrons for light of frequency $2f$ and intensity I.

22.3 Wave–particle duality

We have seen that the photoelectric effect provides evidence for a particulate nature (photons) of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature. There is a 'wave–particle duality'. The question is, if light waves can behave like particles (photons), perhaps moving particles can behave like waves?

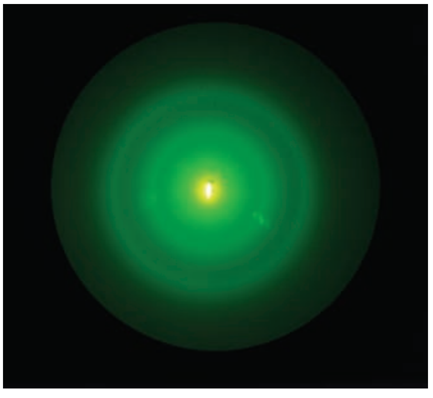
When a beam of X-rays of a single wavelength is directed at a thin metal foil, a diffraction pattern is produced as shown in Figure 22.5.



▲ **Figure 22.5** X-ray diffraction pattern of a metal foil

This is a similar effect to the diffraction pattern produced when light passes through a diffraction grating (see Topic 8). The metal foil contains many tiny crystals. The gaps between neighbouring planes of atoms in the crystals act as slits, creating a diffraction pattern. Note that the spot in the centre is comparable to the central (or zero order) maximum from a diffraction grating and the rings, going outward from the central spot, are analogous to the 1st, 2nd and 3rd order maxima.

In 1927 experiments showed that if a beam of electrons is directed at a graphite film, a similar diffraction pattern is produced, as shown in Figure 22.6.



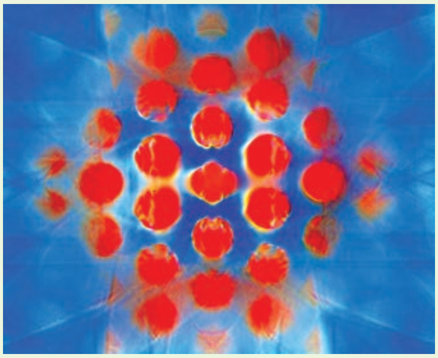
▲ **Figure 22.6** Electron diffraction pattern of graphite

EXTENSION

Probing matter using electrons

Electrons are not affected by the strong nuclear force. It was suggested that they might, therefore, be a more effective tool with which to investigate the structure of the atom.

If a beam of electrons is directed at a sample of powdered crystal and the electron wavelength is comparable with the interatomic spacing in the crystal, the electron waves are scattered from planes of atoms in the tiny crystals, creating a diffraction pattern (Figure 22.7). The fact that a diffraction pattern is obtained confirms the regular arrangement of the atoms in a crystalline solid. Measurements of the angles at which strong scattering is obtained can be used to calculate the distances between planes of atoms.



▲ **Figure 22.7** Electron diffraction pattern of a sample of pure titanium

If the energy of the electron beam is increased, the wavelength decreases. Eventually, the electron wavelength may be of the same order of magnitude as the diameter of the nucleus. Probing the nucleus with high-energy electrons, rather than α-particles, gives a further insight into the dimensions of the nucleus, and also gives information about the distribution of charge in the nucleus itself.

The electrons, which we normally consider to be particles, are exhibiting a property we would normally associate with waves. Remember that, to observe diffraction, the wavelength of the radiation should be comparable with the size of the gap or 'aperture'. The separation of planes of atoms in crystals is of the order of 10^{-10} m. The fact that diffraction is observed with electrons suggests that they have a wavelength of about the same magnitude as the spacing of the atoms in the graphite. Measurements taken from the electron diffraction pattern also allow the value of wavelength to be determined from the diffraction grating formula (see Topic 8.3).

In 1924 the French physicist Louis de Broglie had suggested that all moving particles have a wave-like nature. Using ideas based on quantum theory and on Einstein's theory of relativity, he suggested that the momentum p of a particle and its associated wavelength λ are related by the equation

$$\lambda = h/p$$

where h is the Planck constant. λ is known as the **de Broglie wavelength**.

Measurements from the electron diffraction experiments made three years after de Broglie's prediction confirmed not only that electrons have wave-like properties, but that their wavelength as given by the de Broglie equation $\lambda = h/p$.

WORKED EXAMPLE 22C

Calculate the de Broglie wavelength of an electron travelling with a speed of $2.6 \times 10^7\,\mathrm{m\,s^{-1}}$.

(Planck constant $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$; electron mass $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$.)

Answer

Using $\lambda = h/p$ and $p = mv$,

$\lambda = 6.63 \times 10^{-34}/9.11 \times 10^{-31} \times 2.6 \times 10^7 = \mathbf{2.8 \times 10^{-11}\,m}$

Questions

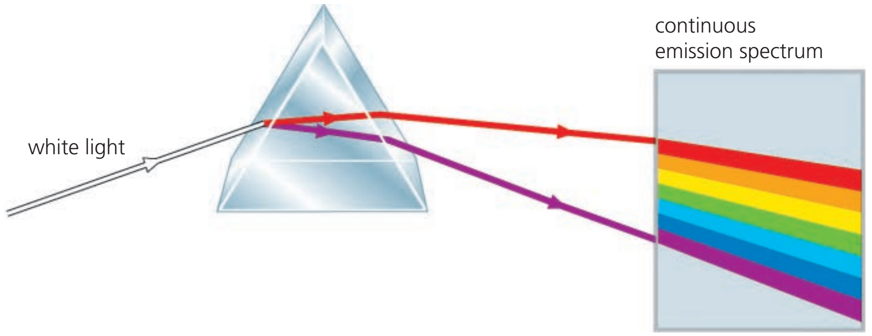
5 Calculate the de Broglie wavelength of an electron travelling with a speed of $7.5 \times 10^7\,\mathrm{m\,s^{-1}}$.
(Planck constant $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$; electron mass $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$.)

6 Calculate the de Broglie wavelength of an electron which has been accelerated from rest through a potential difference of 650 V.
(Planck constant $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$; electron mass $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$; electron charge $e = -1.60 \times 10^{-19}\,\mathrm{C}$.)

7 Calculate the speed of a neutron with de Broglie wavelength $1.6 \times 10^{-10}\,\mathrm{m}$.
(Planck constant $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$; neutron mass $m_n = 1.7 \times 10^{-27}\,\mathrm{kg}$.)

22.4 Energy levels in atoms and line spectra

When white light from a tungsten filament lamp is passed through a prism, the light is dispersed into its component colours, as illustrated in Figure 22.8.



▲ **Figure 22.8** Continuous spectrum of white light from a tungsten filament lamp

The band of different colours is called a **continuous spectrum**. A continuous spectrum has all colours (and wavelengths) between two limits. In the case of white light, the colour and wavelength limits are violet (about 400 nm) and red (about 700 nm). Since this spectrum has been produced by the emission of light from the heated tungsten filament, it is referred to as an **emission spectrum**. Finer detail of emission spectra than is obtained using a prism may be achieved using a diffraction grating.

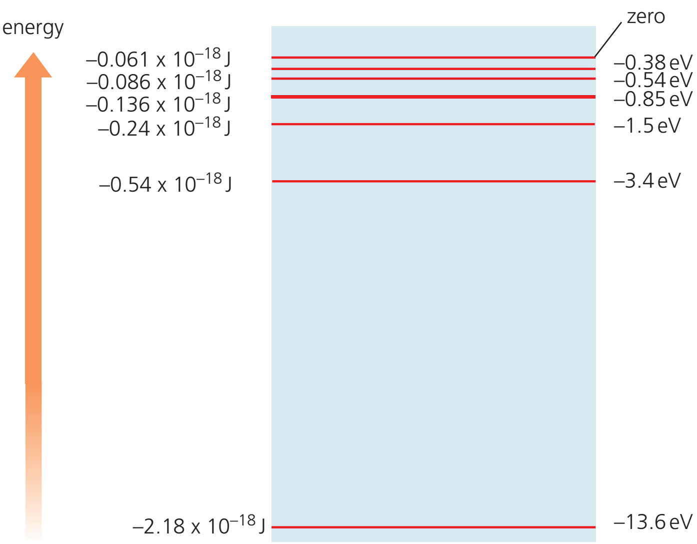
A discharge tube is a transparent tube containing a gas at low pressure. When a high potential difference is applied across two electrodes in the tube, light is emitted. Examination of the light with a diffraction grating shows that the emitted spectrum is no longer continuous, but consists of a number of bright lines (Figure 22.9).

▲ **Figure 22.9** Line spectrum of hydrogen from a discharge tube

Such a spectrum is known as an **emission line spectrum**. It consists of a number of separate colours, each colour being seen as a line which is the image of the slit in front of the source of light. The wavelengths corresponding to the lines of the spectrum are characteristic of the gas which is in the discharge tube. Note the gas could be a vaporised solid or liquid.

Electron energy levels in atoms

To explain how line spectra are produced we need to understand how electrons in atoms behave. Electrons in an atom can have only certain specific energies. These energies are called the **electron energy levels** of the atom. The energy levels may be represented as a series of lines against a vertical scale of energy, as illustrated in Figure 22.10.

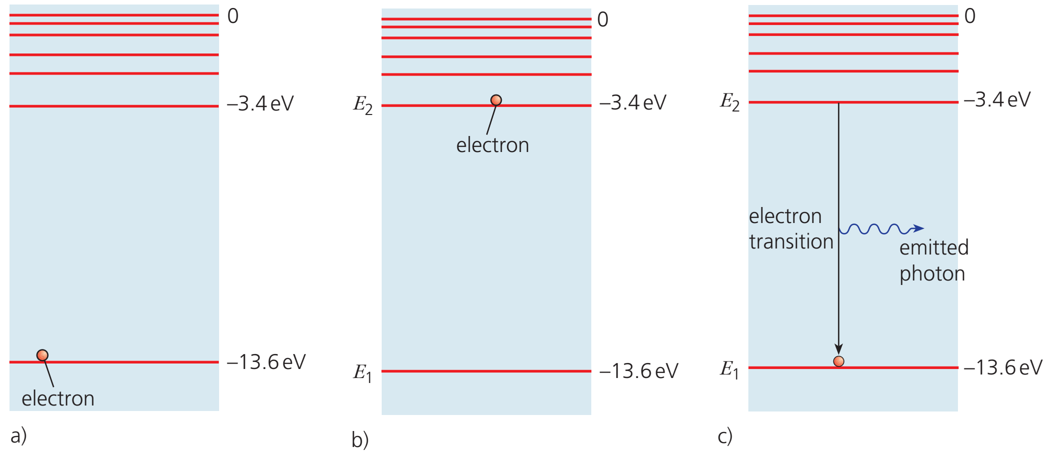


▲ **Figure 22.10** Electron energy levels for the hydrogen atom

The electron in the hydrogen atom can have any of these energy values, but cannot have energies between them. The energy levels are referred to as **discrete energy levels**.

Normally electrons in an atom occupy the lowest energy levels available. Under these conditions the atom and its electrons are said to be in the **ground state**. Figure 22.11a (overleaf) represents a hydrogen atom with its single electron in the lowest energy a state.

If, however, the electron absorbs energy, perhaps by being heated, or by collision with another electron, it may be promoted to a higher energy level. The energy absorbed is exactly equal to the difference in energy of the two levels. Under these conditions the atom is described as being in an **excited state**. This is illustrated in Figure 22.11b.



▲ **Figure 22.11** Electron in a hydrogen atom a) in its ground state, b) in an excited state and c) returning to its ground state with photon emission

An excited atom is unstable. After a short time, the excited electron will return to a lower level. To achieve this, the electron must lose energy. It does so by emitting a photon of electromagnetic radiation, as illustrated in Figure 22.11c.

The energy hf of the photon is given by

$$hf = E_2 - E_1$$

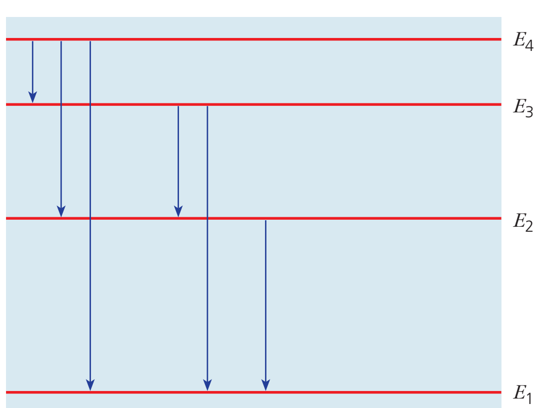
where E_2 is the energy of the higher level and E_1 is that of the lower, and h is the Planck constant. Using the relation between the speed c of light, frequency f and wavelength λ, the wavelength of the emitted radiation is given by

$$\lambda = \frac{hc}{\Delta E}$$

where $\Delta E = E_2 - E_1$. This movement of an electron between energy levels is called an **electron transition**. Note that, the larger the energy of the transition, the higher the frequency (and the shorter the wavelength) of the emitted radiation.

Note that this downward transition results in the **emission** of a photon. The atom can be raised to an excited state by the **absorption** of a photon, but the photon must have just the right energy, corresponding to the difference in energy of the excited state and the initial state. So, a downward transition corresponds to photon emission, and an upward transition to photon absorption.

Figure 22.12 shows some of the possible transitions that might take place when electrons in an excited atom return to lower energy levels.



▲ **Figure 22.12** Some possible electron transitions

Each of the transitions results in the emission of a photon with a particular wavelength. For example, the transition from E_4 to E_1 results in light with the highest frequency and shortest wavelength. On the other hand, the transition from E_4 to E_3 gives the lowest frequency and longest wavelength.

Because the atoms of all elements have different energy levels, the energy differences are unique to each element. Consequently, each element produces a different and characteristic line spectrum. Spectra can be used to identify the presence of a particular element. The emission line spectrum of mercury is shown in Figure 22.13.

▲ **Figure 22.13** Mercury line spectrum from a mercury-vapour lamp

The study of spectra is called **spectroscopy**, and instruments used to measure the wavelengths of spectra are **spectrometers**. Spectrometers for accurate measurement of wavelength make use of diffraction gratings to disperse the light.

Continuous spectra

While the light emitted by isolated atoms such as those in low-pressure gases produces line spectra, the light emitted by atoms in a solid, a liquid, or a gas at high pressure produces a continuous spectrum. This happens because of the proximity of the atoms to each other. Interaction between the atoms results in a broadening of the electron energy levels. Consequently, transitions of a wide range of magnitudes of energy are possible, and light of a broad spread of wavelengths may be emitted to form a continuous spectrum with no gaps. Most continuous spectra are from hot, dense objects like stars, or the hot filament of an electric lamp as in Figure 22.8.

Absorption spectra

When light with a continuous spectrum (such as white light) passes through a low-pressure gas, such as the outer layers of a star, and the spectrum of the light is then analysed, it is found that light of certain wavelengths is missing. In their place are dark lines. This type of spectrum is called an **absorption spectrum**; one is shown in Figure 22.14.

▲ **Figure 22.14** Spectrum of the Sun showing absorption lines

As the light passes through the gas, some electrons absorb energy and make transitions to higher energy levels. Only photons of certain energies are absorbed. The wavelengths of the light absorbed correspond exactly to the energies needed to make particular upward transitions. When these excited electrons return to lower levels, the photons are emitted in all directions, rather than in the original direction of the light. Thus, the intensity of specific wavelengths is reduced and these wavelengths appear as dark lines. It follows that the wavelengths missing from an absorption line spectrum are those present in the emission line spectrum of the same element. This is illustrated in Figure 22.15. Thus lines in an absorption line spectrum can be used to identify the presence of a particular element in the gaseous or vaporised substance that the light has passed through.

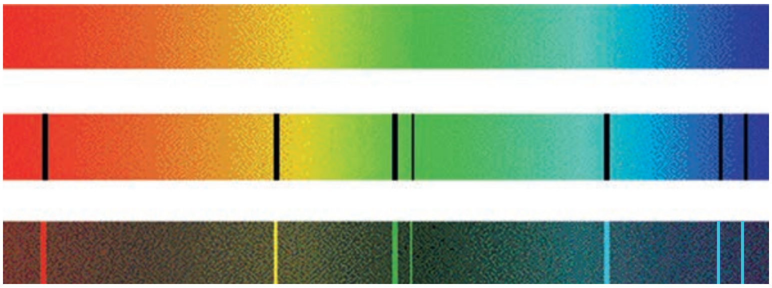


Figure 22.15 Relation between an absorption spectrum and the emission spectrum of the same element: a) spectrum of white light, b) absorption spectrum of element and c) emission spectrum of the same element

WORKED EXAMPLE 22D

Calculate the wavelength of the radiation emitted when the electron in a hydrogen atom makes a transition from the energy level at -0.54×10^{-18} J to the level at -2.18×10^{-18} J.

(Planck constant $h = 6.63 \times 10^{-34}$ J s; speed of light $c = 3.00 \times 10^{8}$ m s^{-1}.)

Answer

Here $\Delta E = E_2 - E_1 = -0.54 \times 10^{-18} - (-2.18 \times 10^{-18}) = 1.64 \times 10^{-18}$ J

Using $\lambda = hc/\Delta E$,

$\lambda = 6.63 \times 10^{-34} \times 3.00 \times 10^{8}/1.64 \times 10^{-18} = 1.21 \times 10^{-7}$ m = **121 nm**

Questions

8 Calculate the wavelength of the radiation emitted when the electron in a hydrogen atom makes a transition from the energy level at -0.85 eV to the level at -3.4 eV.

(Planck constant $h = 6.63 \times 10^{-34}$ J s; speed of light $c = 3.00 \times 10^{8}$ m s^{-1}.)

9 The electron in a hydrogen atom makes a transition from the energy level at -13.58 eV to the level at -0.38 eV when a photon is absorbed. Calculate the frequency of the radiation absorbed.

(Planck constant $h = 6.63 \times 10^{-34}$ J s; 1 eV = 1.60×10^{-19} J.)

10 The energy required to completely remove an electron in the ground state from an atom is called the ionisation energy. This energy may be supplied by the absorption of a photon, in which case the process is called photo-ionisation. Use information from Figure 22.10 to deduce the wavelength of radiation required to achieve photo-ionisation of hydrogen.

(Planck constant $h = 6.63 \times 10^{-34}$ J s; speed of light $c = 3.00 \times 10^{8}$ m s^{-1}.)

SUMMARY

- A photon is a quantum (or packet) of energy of electromagnetic radiation having energy equal to the product of the Planck constant and the frequency of the radiation, $E = hf$.
- The energy of subatomic particles is often measured in electronvolts (eV) or mega-electronvolts (MeV).
- A photon has momentum given by $p = E/c$.
- Electrons may be emitted from metal surfaces if the metal is illuminated by electromagnetic radiation. This phenomenon is called photoelectric emission.
- Photoelectric emission cannot be explained by the wave theory of light. It is necessary to use the quantum theory, in which electromagnetic radiation is thought of as consisting of packets of energy called photons.
- The work function energy Φ of a metal is the minimum energy needed to free an electron from the surface of the metal.
- The Einstein photoelectric equation is: $hf_0 = \Phi + \frac{1}{2}m_e v_{max}^2$.
- The threshold frequency f_0 is the minimum frequency of incident radiation required to cause photoelectron emission from the surface of a particular metal and is given by: $hf_0 = \Phi$.
- The threshold wavelength is the corresponding maximum wavelength to give rise to photoelectric emission.
- Increasing intensity of the incident radiation increases the photoelectric current (rate of emission of photoelectrons) but does not affect the kinetic energy of emitted electrons.
- Interference and diffraction provide evidence for a wave nature of electromagnetic radiation while the photoelectric effect provides evidence for a particulate nature (photons).
- Electron diffraction provides evidence that moving electrons have a wave-like property.
- Moving particles show wave-like properties.
- The de Broglie wavelength of a moving particle is given by $\lambda = h/p$, where p is the momentum of the particle and h is the Planck constant.
- Electrons in isolated atoms can have only certain energies. These energies may be represented in an energy level diagram.
- Electrons in a given energy level may absorb energy and make a transition to a higher energy level.
- Excited electrons may return to a lower level with the emission of a photon, producing an emission line spectrum.
- The frequency f of the emitted radiation is given by $E_2 - E_1 = hf$, where E_2 and E_1 are the energies of the upper and lower levels and h is the Planck constant; the wavelength λ is given by $\lambda = c/f$, where c is the speed of light.
- When an electron absorbs energy from a continuous spectrum of electromagnetic radiation and moves to a higher energy level, an absorption line spectrum may be observed.

END OF TOPIC QUESTIONS

1 A zinc plate is placed on the cap of a gold-leaf electroscope and charged negatively. The gold leaf is seen to deflect. Explain fully the following observations.

a When the zinc plate is illuminated with red light, the gold leaf remains deflected.

b When the zinc plate is irradiated with ultraviolet radiation, the leaf collapses.

c When the intensity of the ultraviolet radiation is increased, the leaf collapses more quickly.

d If the zinc plate is initially charged positively, the gold leaf remains deflected regardless of the nature of the incident radiation.

2 A parallel beam of monochromatic light of wavelength 660 nm has an intensity of $0.25\,\text{mW}\,\text{m}^{-2}$.

The beam is incident normally on a surface where the light is absorbed totally.

Calculate, for a cross-sectional area of $1.2\,\text{cm}^2$ of the beam:

a the number of photons passing per second through the area,

b the momentum of a photon of the light,

c the force exerted on the surface by the light.

(Planck constant $h = 6.63 \times 10^{-34}\,\text{J s}$; speed of light $c = 3.00 \times 10^8\,\text{m s}^{-1}$.)

3 The work function energy of the surface of a certain metal is 3.9×10^{-19} J.

a Calculate the longest wavelength for which photoemission is obtained.

b This metal is irradiated with ultraviolet radiation of wavelength 250 nm. Calculate, for the emitted electrons:

i the maximum kinetic energy,

ii the maximum speed.

(Planck constant $h = 6.63 \times 10^{-34}$ J s; speed of light $c = 3.00 \times 10^{8}$ m s^{-1}; electron mass $m_e = 9.11 \times 10^{-31}$ kg.)

4 Calculate the de Broglie wavelengths of:

a a ball of mass 0.30 g moving at 50 m s^{-1},

b a bullet of mass 50 g moving at 500 m s^{-1},

c an electron of mass 9.1×10^{-31} kg moving at 3.0×10^{7} m s^{-1},

d a proton of mass 1.7×10^{-27} kg moving at 3.0×10^{6} m s^{-1}.

(Planck constant $h = 6.63 \times 10^{-34}$ J s.)

5 When the visible spectrum emitted by the Sun is observed closely it is noted that light of certain frequencies is missing and in their place are dark lines.

a Explain how the gaseous outer atmosphere of the Sun could be responsible for the absence of these frequencies.

b Suggest how an analysis of this spectrum could be used to determine which gases are present in the Sun's atmosphere.

6 **a** Explain what is meant by the *photoelectric effect*. **[2]**

b One wavelength of electromagnetic radiation emitted from a mercury vapour lamp is 436 nm.

Calculate the photon energy corresponding to this wavelength. **[2]**

c Light from the lamp in **b** is incident, separately, on the surfaces of caesium and tungsten metal.

Data for the work function energies of caesium and tungsten metal is given in Fig. 22.16.

metal	work function energy/eV
caesium	1.4
tungsten	4.5

▲ **Figure 22.16**

Calculate the threshold wavelength for photoelectric emission from:

i caesium, **[2]**

ii tungsten. **[1]**

d Use your answers in **c** to state and explain whether the radiation from the mercury lamp of wavelength 436 nm will give rise to photoelectric emission from each of the metals. **[2]**

Cambridge International AS and A Level Physics (9702) Paper 41 Q10 Oct/Nov 2016

23 Nuclear physics

Learning outcomes

By the end of this topic you will be able to:

23.1 Mass defect and nuclear binding energy

1 understand the equivalence between energy and mass as represented by $E = mc^2$ and recall and use this equation

2 represent simple nuclear reactions by nuclear equations of the form

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

3 define and use the terms mass defect and binding energy

4 sketch the variation of binding energy per nucleon with nucleon number

5 explain what is meant by nuclear fusion and nuclear fission

6 explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission

7 calculate the energy released in nuclear reactions using $E = c^2\Delta m$

23.2 Radioactive decay

1 understand that fluctuations in count rate provide evidence for random nature of radioactive decay

2 understand that radioactive decay is both spontaneous and random

3 define activity and decay constant, and recall and use $A = \lambda N$

4 define half-life

5 use $\lambda = 0.693/t_{1/2}$

6 understand the exponential nature of radioactive decay, and sketch and use the relationship $x = x_0 e^{-\lambda t}$, where x could represent activity, number of undecayed nuclei or received count rate

Starting points

- An atom may be modelled as a massive, but very small, positively charged nucleus surrounded by negatively charged electrons.
- A nucleus contains protons and neutrons.
- Use the notation ${}^{A}_{Z}X$ for the representation of nuclides and to write nuclear reaction equations.
- The masses of atoms and their constituents are usually expressed in unified atomic mass units (u).
- The electronvolt (eV) is a unit of energy.
- During radioactive decay an unstable nucleus emits particles and/or electromagnetic radiation.

23.1 Mass defect and nuclear binding energy

Mass defect

At a nuclear level, the masses we deal with are so small that it would be very clumsy to measure them in kilograms. Instead, we measure the masses of nuclei and nucleons in **unified atomic mass units (u)**.

One unified atomic mass unit (1u) is defined as being equal to one-twelfth of the mass of a carbon-12 atom. 1u is equal to 1.66×10^{-27} kg.

Using this scale of measurement, to six decimal places, we have

proton mass $m_p = 1.007276\,u$

neutron mass $m_n = 1.008665\,u$

electron mass $m_e = 0.000549\,u$

Because all atoms and nuclei are made up of protons, neutrons and electrons, we should be able to use these figures to calculate the mass of any atom or nucleus. For example, the mass of a helium-4 nucleus, consisting of two protons and two neutrons, should be

$$(2 \times 1.007276)\,u + (2 \times 1.008665)\,u = 4.031882\,u$$

However, the actual mass of a helium nucleus is 4.001508 u.

The difference between the expected mass and the actual mass of a nucleus is called the **mass defect** of the nucleus. In the case of the helium-4 nucleus, the mass defect is $4.031882 - 4.001508 = 0.030374\,u$.

The mass defect of a nucleus is the difference between the total mass of the separate nucleons and the combined mass of the nucleus.

WORKED EXAMPLE 23A

Calculate the mass defect for a carbon-14 ($^{14}_{6}C$) nucleus. The measured mass is 14.003240 u.

Answer

The nucleus contains six protons and eight neutrons, of total mass $(6 \times 1.007276) + (8 \times 1.008665) = 14.112976\,u$.
The mass defect is $14.112976 - 14.003240 =$ **0.109736 u**.

Question

1 Calculate the mass defect for a nitrogen-14 ($^{14}_{7}N$) nucleus. The measured mass is 14.003070 u.

Mass–energy equivalence

In 1905, Albert Einstein proposed that there is an equivalence between mass and energy. The relationship between energy E and mass, or change in mass m, is

$$E = mc^2$$

where c is the speed of light. E is measured in joules, m in kilograms and c in metres per second.

Using this relation, we can calculate that 1.0 kg of matter is equivalent to $1.0 \times (3.0 \times 10^8)^2 = 9.0 \times 10^{16}\,J$.

The mass defect of the helium nucleus, calculated previously as 0.030374 u, is equivalent to

$$0.030374 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 = 4.54 \times 10^{-12}\,J.$$

(Note that the mass in u must be converted to kg by multiplying by 1.66×10^{-27}.)

The joule is an inconveniently large unit to use for nuclear calculations. A more convenient energy unit is the electronvolt (eV) or mega-electronvolt (MeV) as many energy changes that take place in the nucleus are of the order of several MeV.

Since one electronvolt is the energy gained by one electron when it is accelerated through a potential difference of one volt (see Topic 22.1)

$$1\,\text{MeV} = 1.60 \times 10^{-19} \times 1.0 \times 10^{6}$$

or

$$1\,\text{MeV} = 1.60 \times 10^{-13}\,\text{J}$$

The energy equivalent of the mass defect of the helium nucleus is thus $4.54 \times 10^{-12}/1.60 \times 10^{-13} = 28.4\,\text{MeV}$.

If mass is measured in u and energy in MeV, 1 u is the equivalent of 934 MeV.

Binding energy

Within the nucleus there are strong forces which bind the protons and neutrons together. To completely separate all these nucleons requires energy. This energy is referred to as the **binding energy** of the nucleus. Stable nuclei, those which have little or no tendency to disintegrate, have large binding energies. Less stable nuclei have smaller binding energies.

Similarly, when protons and neutrons are joined together to form a nucleus, this binding energy must be released. The binding energy is the energy equivalent of the mass defect.

We have seen that energy equivalent of the mass defect of the helium-4 nucleus is 28.4 MeV. Therefore, 28.4 MeV is the binding energy of the nucleus and is the energy required to separate, to infinity, the two protons and the two neutrons of this nucleus.

Binding energy is the energy equivalent of the mass defect of a nucleus. It is the energy required to separate to infinity all the nucleons of a nucleus.

WORKED EXAMPLE 23B

Calculate the binding energy, in MeV, of a carbon-14 nucleus with a mass defect of 0.109736 u.

Answer

Either Using the equivalence 1 u = 934 MeV, 0.109736 u is equivalent to 102 MeV. Since the binding energy is the energy equivalent of the mass defect, the binding energy = **102 MeV**.

Or

$$\text{binding energy} = 0.109736 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^{2} = 1.64 \times 10^{-11}\,\text{J}$$

$$= (1.64 \times 10^{-11})/(1.60 \times 10^{-13}) = \mathbf{102\,MeV}$$

Question

2 Calculate the binding energy, in MeV, of a nitrogen-14 nucleus with a mass defect of 0.1085 u.

Nuclear equations

The nucleus of any nuclide can be represented using nuclide notation (see Topic 11.1). This notation is useful when we wish to consider a nuclear reaction. For example, when a helium nucleus bombards a nitrogen nucleus, the reaction can be represented by

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

Remember that nucleon number and charge are conserved in nuclear processes (Topic 11.1). For this reaction to take place, then *three* conditions must be met.

1 Conservation of proton number (7 + 2 = 8 + 1)
2 Conservation of nucleon number (14 + 4 = 17 + 1)
3 Conservation of mass–energy.

mass of $^{14}_{7}N$ = 14.003074 u

mass of $^{4}_{2}He$ = 4.002604 u

mass of $^{17}_{8}O$ = 16.99913 u

mass of $^{1}_{1}H$ = 1.007825 u

The change in mass is 18.005678 u – 18.006955 u = (–)1.277×10^{-3} u.

This change in mass is equivalent to 1.2 MeV.

Note that the mass of the products is greater than the mass of the reacting nuclei. There is a mass excess.

For this reaction to take place then, by conservation of mass–energy, the helium nucleus must have kinetic energy of at least 1.2 MeV when it bombards the nitrogen nucleus.

For a reaction to occur spontaneously, there must be a mass defect so that the products of the reaction have some kinetic energy and thus mass–energy is conserved.

WORKED EXAMPLE 23C

A nucleus of uranium-234 decays by emission of an α-particle to form a nucleus of thorium-90. The total energy released during this decay is 4.77 MeV. Calculate the total change in mass during the decay.

Answer

4.77 MeV = $4.77 \times 1.60 \times 10^{-13} = 7.63 \times 10^{-13}$ J

$E = c^2\Delta m$

7.63×10^{-13} J = $(3.0 \times 10^8)^2 \times \Delta m$

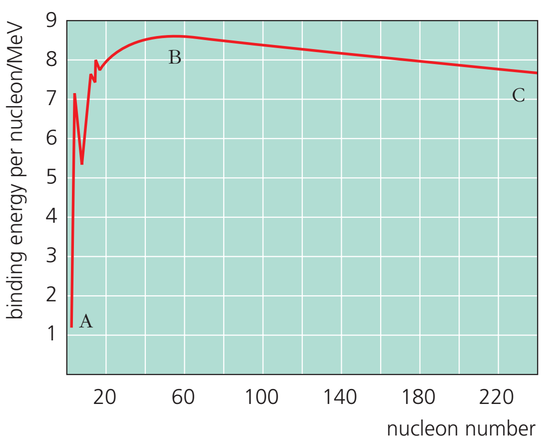
mass change = **8.48×10^{-30} kg**

Stability of nuclei

A stable nucleus is one which has a very low probability of decay. Less stable nuclei are more likely to disintegrate. A useful measure of stability is the **binding energy per nucleon** of the nucleons in the nucleus.

Binding energy per nucleon is defined as the total energy needed to completely separate all the nucleons in a nucleus divided by the number of nucleons in the nucleus.

Figure 23.1 shows the variation with nucleon number of the binding energy per nucleon for different nuclides.



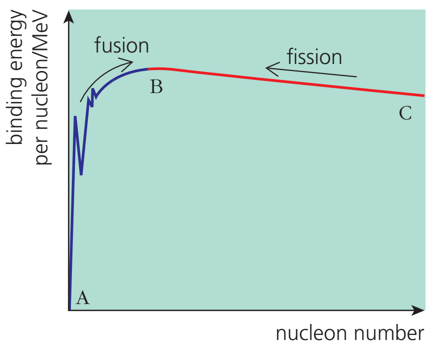
▲ **Figure 23.1** Binding energy per nucleon against nucleon number

The most stable nuclides are those with the highest binding energy per nucleon. That is, those near point B on the graph. Iron-56 is one of these most stable nuclides. Typically, very stable nuclides have binding energies per nucleon of about 8 MeV.

Light nuclei, between A and B on the graph, may combine or fuse to form larger nuclei with larger binding energies per nucleon. This process is called **nuclear fusion** and is described in more detail below. For the process to take place, conditions of very high temperature and pressure are required, such as in stars like the Sun.

Heavy nuclei, between B and C on the graph, when bombarded with neutrons, may break into two smaller nuclei, again with larger binding energy per nucleon values. This process is called **nuclear fission**. This process is also described in more detail below.

Figure 23.2 highlights nuclides which may undergo fusion (blue part of curve) or fission (red part of curve) in order to increase their binding energy per nucleon.



▲ **Figure 23.2**

When nuclear fusion or fission takes place, there is a change in the nucleon numbers of the nuclei involved in the reaction. A higher binding energy per nucleon is achieved, and this is accompanied by a release of energy. This release of energy during fission reactions is how the present generation of nuclear power stations produce electrical energy.

WORKED EXAMPLE 23D

1 The binding energy of a helium-4 nucleus is 28.4 MeV. Calculate the binding energy per nucleon.

2 The masses of a uranium-238 nucleus ($^{238}_{92}U$), a proton and a neutron are 237.9997 u, 1.00728 u and 1.00867 u respectively.
Calculate the binding energy per nucleon of uranium-238.

Answers

1 The helium-4 nucleus has 4 nucleons.
The binding energy per nucleon is thus 28.4/4 = **7.1MeV per nucleon**.

2 Mass of nuclear constituents of U-238 = (92 × 1.00728) + (146 × 1.00867)
= 239.93558 u
Mass defect = 239.93558u − 237.9997 u = 1.93588 u
Binding energy per nucleon = (1.93588 × 934)/238 = **7.5971MeV per nucleon**

Questions

3 The binding energy of a nitrogen-14 nucleus is 101 MeV. Calculate the binding energy per nucleon.

4 Data for the masses of some particles is given in Table 23.1.

nucleus or particle	mass/u
proton	1.0073
neutron	1.0087
zirconium $^{97}_{40}Zr$	97.0980

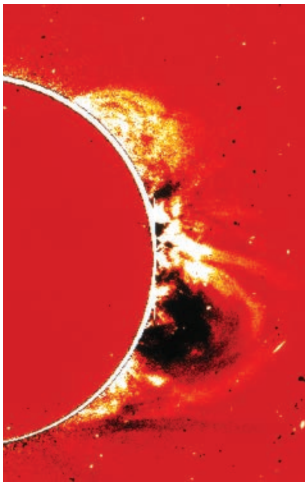
▲ **Table 23.1**

Determine for a zirconium-97 nucleus:

a the mass defect

b the binding energy

c the binding energy per nucleon.



▲ **Figure 23.3** The Sun's surface

Nuclear fusion

Most of the energy on Earth comes from the Sun, where it is produced by nuclear fusion reactions. Light nuclei, such as isotopes of hydrogen, join together to produce heavier, more stable nuclei, and in doing so release energy.

Figure 23.3 shows eruptions from the Sun's surface caused by particularly energetic fusion reactions.

> Nuclear fusion occurs when two light nuclei combine to form a nucleus of greater mass.

One such fusion reaction is

$$^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n + \text{energy}$$

From the binding energy per nucleon curve (Figure 23.1) we see that the binding energy per nucleon for light nuclei, such as hydrogen, is low. But if two light nuclei are made to fuse together, they may form a new heavier nucleus which has a higher binding energy per nucleon. It will be more stable than the two lighter nuclei from which it was formed. Because of this difference in stability, a fusion reaction such as this will release energy.

Although fusion reactions are the source of solar energy, we are, at present, unable to duplicate this reaction in a controlled manner on Earth. This is because the nuclei involved in fusion have to be brought very close together. Conditions of extremely high temperature and pressure, similar to those found at the centre of the Sun, are required. Reactions requiring these conditions are called **thermonuclear reactions**. Some fusion reactions involving hydrogen isotopes have been made to work in the Joint European Torus (JET), although not yet in a controlled, sustainable manner. In 2006, an international consortium agreed to undertake the ITER (International Tokamak Engineering Research) project, which is designed to produce up to 500 MW of fusion power sustained for over 400 seconds by the fusion of a $^{2}_{1}H - ^{3}_{1}H$ mixture. Construction on a site in southern France will take several years.

Nuclear fission

Within the nucleus of an atom, the nucleons experience both attractive and repulsive forces. The attractive force is called the **strong nuclear force**. This acts like a 'nuclear glue' to hold the nucleons together. The repulsive forces are the electric (Coulomb law) forces between the positively charged protons. Gravitational forces of attraction exist, but are negligible in comparison to the other forces. Stable nuclei have much larger attractive forces than repulsive forces. Stable nuclides generally have approximately the same number of neutrons and protons in the nucleus. That is, the neutron-to-proton ratio is close to one. In heavy nuclei such as uranium and plutonium, there are far more neutrons than protons, giving a neutron-to-proton ratio of more than one. For example, uranium-235 has 92 protons and 143 neutrons, giving a neutron-to-proton ratio of 1.55. This leads to a much lower binding energy per nucleon compared with iron, and such nuclides are less stable. Any further increase in the number of neutrons in such nuclei is likely to cause the nucleus to undergo **nuclear fission**.

Nuclear fission is the splitting of a heavy nucleus into two lighter nuclei of approximately the same mass.

When a uranium-235 nucleus absorbs a neutron, it becomes unstable and splits into two lighter, more stable nuclei. There are many possible nuclear reactions, one of which is

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n} + \text{energy}$$

This process is called *induced* nuclear fission, because it is started by the capture of a neutron by the uranium nucleus.

WORKED EXAMPLE 23E

Use the data in Table 23.2 to calculate for the fission reaction:

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n}$$

a the change in mass as a result of the fission

b the energy released per fission.

nucleus or particle	mass/u
neutron	1.009
$^{235}_{92}\text{U}$	235.123
$^{141}_{45}\text{Ba}$	140.912
$^{92}_{36}\text{Kr}$	91.913

▲ **Table 23.2**

Answers

a total mass of reactants = 235.123 + 1.009

= 236.132 u

total mass of products = 140.912 + 91.913 + (3 × 1.009)

= 235.852 u

(the electrons have negligible mass)

mass change = 236.132 − 235.852

= **0.28 u**

b energy released = $c^2\Delta m$

$= (3.00 \times 10^8)^2 \times 0.28 \times 1.66 \times 10^{-27}$

$= \mathbf{4.18 \times 10^{-11}\,J}$

Question

5 One fusion reaction that takes place in the Sun is the joining together of a deuterium nucleus $^{2}_{1}\text{H}$ and a proton $^{1}_{1}\text{p}$ to form a helium-3 nucleus.

The binding energies per nucleon of deuterium and of helium-3 are 0.864 MeV and 2.235 MeV.

a Write down the nuclear equation for this reaction.

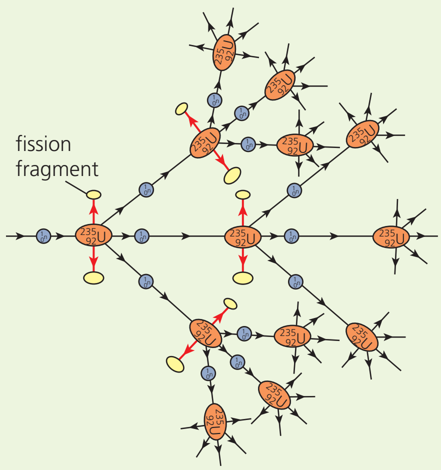
b Explain why the binding energy per nucleon of a proton is zero.

c Calculate the energy released in this reaction.

EXTENSION

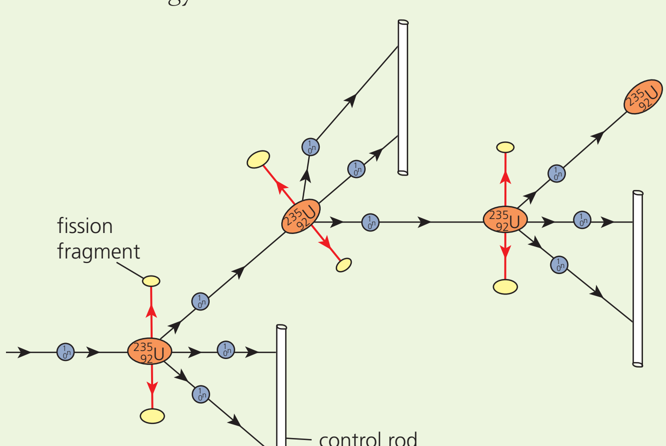
Chain reactions

Each of the fission reactions described by the equation on page 371 results in the release of three neutrons. Other possible fission reactions release two or three neutrons. If these neutrons are absorbed by other uranium-235 nuclei, these too may become unstable and undergo fission, thereby releasing even more neutrons. The reaction is described as being a **chain reaction** which is accelerating. This is illustrated in Figure 23.4. If this type of reaction continues uncontrolled, a great deal of energy is released in a short time, and a nuclear explosion results.



▲ **Figure 23.4** Accelerating chain reaction

If the number of neutrons which take part in the chain reaction is controlled so that the number of fissions per unit time is constant, rather than increasing, the rate of release of energy can be controlled. This situation is illustrated in Figure 23.5.



▲ **Figure 23.5** Controlled chain reaction

These conditions apply in the reactor of a modern nuclear power station, where some of the neutrons released in fission reactions are absorbed by control rods in order to limit the rate of fission reactions.

Two of the neutrons produced by fission are absorbed by control rods. The third neutron induces further nuclear fission.

23.2 Radioactive decay

Some elements have nuclei which are unstable. In order to become more stable, they emit particles and/or electromagnetic radiation (see Topic 11.1). The nuclei are said to be **radioactive**.

Detection of fluctuations in the count rate of radioactive sources provides evidence for the *random* nature of radioactive decay. The decay is random in that it is not possible to predict which nucleus in a sample will decay next.

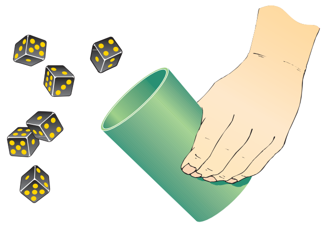
Radioactive decay is a random process in that it cannot be predicted which nucleus will decay next. There is a constant probability that a nucleus will decay in any fixed period of time.

The rate of decay of a sample of a radioactive material decreases with time. However, the rate of decay cannot be changed by changing any environmental factors (e.g. temperature or pressure). This shows that radioactive decay is a spontaneous process.

Radioactive decay is a spontaneous process because it is not affected by any external factors, such as temperature or pressure.

Random decay

We now look at some of the consequences of the random nature of radioactive decay. If six dice are thrown simultaneously (Figure 23.6), it is likely that one of them will show a six.



▲ **Figure 23.6** The dice experiment

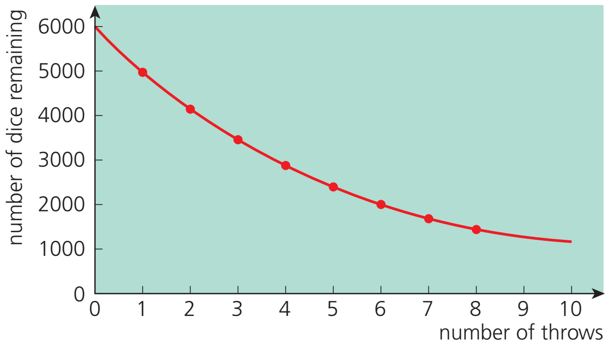
If 12 dice are thrown, it is likely that two of them will show a six, and so on. While it is possible to predict the likely number of sixes that will be thrown, it is impossible to say which of the dice will actually show a six. We describe this situation by saying that the throwing of a six is a **random process**.

In an experiment similar to the one just described, some students throw a large number of dice (say 6000). Each time a six is thrown, that die is removed. The results for the number of dice remaining after each throw are shown in Table 23.3.

number of throws	number of dice remaining	number of dice removed
0	6000	1000
1	5000	827
2	4173	696
3	3477	580
4	2897	483
5	2414	402
6	2012	335
7	1677	280
8	1397	

▲ **Table 23.3** Results of dice-throwing experiment

Figure 23.7 is a graph of the number of dice remaining against the number of throws.

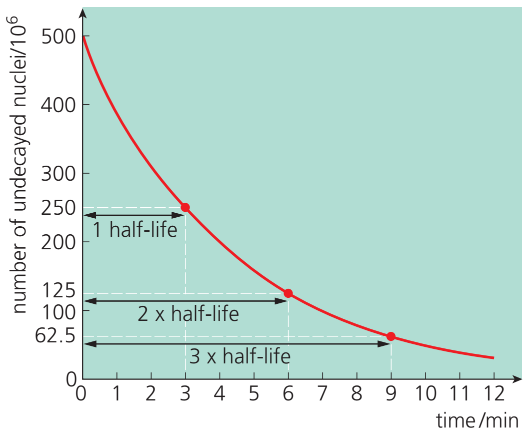


▲ **Figure 23.7**

This kind of graph is called a **decay curve**. The rate at which dice are removed is not linear, but there is a pattern. After between 3 and 4 throws, the number of dice remaining has halved. Reading values from the graph shows that approximately 3.8 throws would be required to halve the number of dice. After another 3.8 throws the number has halved again, and so on.

We can apply the dice experiment to model radioactive decay. The 6000 dice represent radioactive nuclei. To score a six represents radioactive emission. All dice scoring six are removed, because once a nucleus has undergone radioactive decay, it is no longer available for further decay. Thus, we can describe how rapidly a sample of radioactive material will decay.

A graph of the number of undecayed nuclei in a sample against time has the typical decay curve shape shown in Figure 23.8.



▲ **Figure 23.8** Radioactive decay curve

It is not possible to state how long the entire sample will take to decay (its 'life'). However, after 3 minutes the number of undecayed nuclei in the sample has halved. After a further 3 minutes, the number of undecayed atoms has halved again. We describe this situation by saying that this radioactive isotope has a **half-life** $t_{\frac{1}{2}}$ of 3 minutes.

> The half-life of a radioactive nuclide is the time taken for the number of undecayed nuclei to be reduced to half its original number.

The half-lives of different isotopes have a very wide range of values. Examples of some radioactive isotopes and their half-lives are given in Table 23.4.

radioactive isotope	half-life
uranium-238	4.5×10^9 years
radium-226	1.6×10^3 years
radon-222	3.8 days
francium-221	4.8 minutes
astatine-217	0.03 seconds

▲ **Table 23.4** Examples of half-life

We shall see later that half-life may also be expressed in terms of the activity of the material.

If you measure the count rate from an isotope with a very long half-life, you might expect to obtain a constant value. In fact, the count rate fluctuates about an average value. This demonstrates the random nature of radioactive decay.

WORKED EXAMPLE 23F

The half-life of francium-221 is 4.8 minutes. Calculate the fraction of a sample of francium-221 remaining undecayed after a time of 14.4 minutes.

Answer

The half-life of francium-221 is 4.8 min, so after 4.8 min half of the sample will remain undecayed. After two half-lives (9.6 min), 0.5 × 0.5 = 0.25 of the sample will remain undecayed. After three half-lives (14.4 min), 0.5 × 0.25 = 0.125 will remain undecayed. So the fraction remaining undecayed is **0.125** or **1/8**.

Question

6 Using the half-life values given in Table 23.4, calculate:

a the fraction of a sample of uranium-238 remaining undecayed after 9.0×10^9 years

b the fraction of a sample of astatine-217 remaining undecayed after 0.30 s

c the fraction of a sample of radium-226 that has decayed after 3200 years

d the fraction of a sample of radon-222 that has decayed after 15.2 days.

In carrying out experiments with radioactive sources, it is important to take account of background radiation. In order to determine the count-rate due to the radioactive source, the background count-rate must be subtracted from the total measured count-rate. Allowance for background radiation gives the **corrected count rate**.

Note that a detector placed near a radioactive source measures the count rate of the radiation emitted in the direction of the detector. It does not measure the activity of the source. The activity is the total rate of decay of nuclei in the source.

Mathematical descriptions of radioactive decay

Activity and decay constant

As we saw in the dice experiment, increasing the number of dice increases the number of sixes that appear with each throw. Similarly, if we investigate the decay of a sample of radioactive material, we find that the greater the number of radioactive nuclei in the sample the greater the rate of decay.

If there are N nuclei in a sample at time t, then at time $(t + dt)$ some nuclei will have decayed so that the number remaining is $(N - dN)$.

For radioactive decay, the probability of decay per unit time is constant, known as the **decay constant λ**.

For radioactive decay, the decay constant λ is the probability per unit time of the decay of a nucleus.

The decay constant λ has the units s^{-1}, yr^{-1} and so on.

In this case,

probability of decay = $-dN/N$

and

probability of decay per unit time = $\lambda = -(dN/dt)N$

This gives

$$\frac{dN}{dt} = -\lambda N$$

Note that a negative sign has been included. This is because as time t increases, the number of nuclei remaining, N decreases.

dN/dt is the rate at which the number of nuclei in the sample is changing, so dN/dt represents the rate of decay. $-dN/dt$ is known as the **activity** A of the source, and is measured in **becquerels**.

The activity of a radioactive source is the number of nuclear decays occurring per unit time in the source.

Activity is measured in becquerels (Bq), where 1 becquerel is 1 decay per second.

$$1\,\text{Bq} = 1\,\text{s}^{-1}$$

Note that writing the rate of change of the quantity N with time t in the form dN/dt is a shorthand way of expressing the rate, but is not required in Cambridge International AS & A Level Physics (see Maths Note in Topic 20.5).

Combining $A = -\,dN/dt$ and $dN/dt = -\lambda N$, we have

$$A = \lambda N$$

This is an important equation because it relates a quantity we can measure (activity, or the rate at which nuclei decay) to a quantity which cannot, in practice, be determined (N, the number of undecayed nuclei). We shall see later that the decay constant λ is directly related to the half-life $t_{\frac{1}{2}}$ and half-life can be obtained by experiment. This opens the way to calculating the number of undecayed nuclei in a sample. Trying to count nuclei when they are decaying is similar to counting sheep in a field while some are escaping through a gap in the hedge!

WORKED EXAMPLE 23G

Calculate the number of phosphorus-32 nuclei in a sample which has an activity of 5.0×10^6 Bq.

(Decay constant of phosphorus-32 = $5.6 \times 10^{-7}\,\text{s}^{-1}$.)

Answer

From $dN/dt = -\lambda N$, $N = (-dN/dt)/\lambda = -5.0 \times 10^6/5.6 \times 10^{-7} = -8.9 \times 10^{12}$.

The minus sign in this answer arises because dN/dt is the rate of decay. The quantity measured by a ratemeter is the rate of decay, and so should be negative, but it is always displayed as a positive quantity. Similarly, activities in becquerel are always quoted as positive. So don't be worried about discarding the minus sign here! The number of phosphorus-32 nuclei = $\mathbf{8.9 \times 10^{12}}$.

Note: In this type of calculation, because the activity is measured in becquerel (s^{-1}), the decay constant λ must be measured in consistent units, that is s^{-1}. If λ had been quoted as $4.8 \times 10^{-2}\,\text{day}^{-1}$, it would have been necessary to convert to s^{-1}.

Question

7 Calculate the activity, in Bq, of the following samples of radioactive materials:

a 2.4×10^{15} atoms of strontium-90
(Decay constant of strontium-90 = $5.0 \times 10^{-8}\,\text{minute}^{-1}$.)

b 2.5 μg of uranium-238 (0.238 kg of uranium-238 contains 6.0×10^{23} atoms)
(Decay constant of uranium-238 = $5.0 \times 10^{-13}\,\text{s}^{-1}$.)

To solve the equation $dN/dt = -\lambda N$ requires mathematics beyond the scope of Cambridge International AS & A Level Physics. However, it is important to know the solution, in order to find the variation with time of the number of nuclei remaining in the sample. The solution is

$$N = N_0 e^{-\lambda t} \text{ or } N = N_0 \exp(-\lambda t)$$

where N_0 is the initial number of undecayed nuclei in the sample, and N is the number of undecayed nuclei at time t.

The equation represents an **exponential decay**. The decay curve of N against t is as shown in Figure 23.8. Since activity A is proportional to N ($A = \lambda N$), the curve of A against t is the same shape, and we can write

$$A = A_0 e^{-\lambda t}$$

The equation is also true for received count rate. Thus radioactive decay can be represented by the equation

$$x = x_0 e^{-\lambda t}$$

where x represents any of activity, number of undecayed nuclei or received count rate.

WORKED EXAMPLE 23H

A sample of phosphorus-32 contains 8.6×10^{12} nuclei at time $t = 0$. The decay constant of phosphorus-32 is 4.8×10^{-2} day^{-1}. Calculate the number of undecayed phosphorus-32 nuclei in the sample after 10 days.

Answer

From $N = N_0 e^{-\lambda t}$, we have $N = 8.6 \times 10^{12} \times e^{-0.048 \times 10}$, so $N = \mathbf{5.3 \times 10^{12}}$.

(Again, it is important to measure λ and t in consistent units. Here λ is in day^{-1} and t is in days, so there is no problem.)

Question

8 The activity of radon-222 in 1.0 m^3 of air in a room is 350 Bq. The decay constant of radon-222 is 2.1×10^{-6} s^{-1}. Calculate the time, in days, before the activity in 1.0 m^3 of the air is reduced to 210 Bq.

Decay constant and half-life

Using the equation $N = N_0 e^{-\lambda t}$, we can derive an equation which relates the half-life to the decay constant. For any radioactive isotope, the number of undecayed nuclei after one half-life is, by the definition of half-life, equal to $\frac{1}{2} N_0$, where N_0 is the original number of undecayed nuclei. Using the radioactive decay equation

$$N = N_0 e^{-\lambda t}$$

we have, at time $t = t_{\frac{1}{2}}$

$$\tfrac{1}{2} N_0 = N_0 \exp(-\lambda t_{\frac{1}{2}})$$

and, dividing each side of the equation by N_0,

$$0.5 = \exp(-\lambda t_{\frac{1}{2}})$$

or

$$2 = \exp(\lambda t_{\frac{1}{2}})$$

Taking natural logarithms of both sides,

$$\ln 2 = \lambda t_{\frac{1}{2}}$$

So that

$$t_{\frac{1}{2}} = \ln 2/\lambda$$

or

$$t_{\frac{1}{2}} = 0.693/\lambda$$

and $\lambda = 0.693/t_{\frac{1}{2}}$.

WORKED EXAMPLE 23I

Calculate the half-life, in years, of radium-226, which has a decay constant of $1.42 \times 10^{-11}\,s^{-1}$.

Answer

Using $t_{\frac{1}{2}} = 0.693/\lambda$, we have $t_{\frac{1}{2}} = 0.693/1.42 \times 10^{-11} = \mathbf{4.88 \times 10^{10}\,s = 1550\ years}$.

Questions

9 Calculate the half-lives of the following radioactive nuclides:

- **a** bismuth-214, which has a decay constant of $5.83 \times 10^{-4}\,s^{-1}$
- **b** carbon-14, which has a decay constant of $4.1 \times 10^{-12}\,s^{-1}$.

Give your answers in years.

10 Calculate the decay constant, in s^{-1}, of sodium-24, which has a half-life of 15 h.

SUMMARY

- The mass defect of a nucleus is the difference between the total mass of the separate nucleons and the mass of the nucleus.
- Einstein's mass–energy equivalence relation $E = mc^2$.
- The binding energy of a nucleus is the energy needed to separate completely all its constituent nucleons.
- The binding energy per nucleon is a measure of the stability of a nucleus. A higher binding energy per nucleon means the nucleus is more stable.
- Nuclear fusion is the joining together of light nuclei to form a larger, heavier nucleus.
- Nuclear fission is the splitting of a heavy nucleus into two smaller, lighter nuclei of approximately equal mass.
- Decay equations and nuclear reactions can be represented using nuclide notation
- The mass difference can be used to calculate the energy released in nuclear reactions (including radioactive decay equations) using $E = c^2\Delta m$.
- The graph of binding energy per nucleon against nucleon number shows that nuclei of low nucleon number, and also those of high nucleon number, have smaller binding energy per nucleon than those nuclei with a nucleon number around 56.
- For this reason, fusion of low nucleon number nuclei and fission of a high nucleon number nuclei are processes that release energy.
- Radioactive decay is a spontaneous, random process.
- The half-life $t_{\frac{1}{2}}$ of a radioactive nuclide is the time taken for the number of undecayed nuclei to be reduced to half the original number.
- The activity of a radioactive source is the number of nuclei that decay per unit time. The unit of activity is the becquerel (Bq). 1 becquerel = $1\,s^{-1}$.
- The activity A of a source is related to the number N of undecayed nuclei by the equation $A = -\lambda N$ where λ is the decay constant.
- The decay constant is defined as the probability of decay per unit time of a nucleus.
- Radioactive decay is represented by the exponential decay equation $x = x_0 e^{-\lambda t}$ where x is the activity or the number of undecayed nuclei or the received count rate
- The half-life $t_{\frac{1}{2}}$ and the decay constant λ are related by the equation $\ln 2 = \lambda t_{\frac{1}{2}}$ or $t_{\frac{1}{2}} = 0.693/\lambda$.

END OF TOPIC QUESTIONS

1 Fig. 23.9 shows the variation with time t of the activity of a sample of a radioactive nuclide X. The average background count during the experiment was 36 min^{-1}.

t/hour	0	1	2	3	4	5	6	7	8	9	10
activity/min^{-1}	854	752	688	576	544	486	448	396	362	334	284

▲ **Figure 23.9**

a Plot a graph to show the variation with time t of the corrected count rate.
b Use the graph to determine the half-life of the nuclide X.

2 Calculate the mass defect, the binding energy of the nucleus, and the binding energy per nucleon of a nucleus of zirconium-97 ($^{97}_{40}Zr$) having a nuclear mass of 97.09801 u.

(Proton mass = 1.00728 u; neutron mass = 1.00867 u.)

3 One possible reaction taking place in the core of a nuclear reactor is

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{0}_{-1}e$

For this reaction, calculate:
a the change in mass after fission has taken place,
b the energy released per fission of uranium-235,
c the energy available from the complete fission of 1.00 g of uranium-235,
d the mass of uranium-235 used by a 500 MW nuclear power station in one hour, assuming 30% efficiency.

(Masses: U, 235.123 u; Mo, 94.945 u; La, 138.955 u; proton, 1.007 u; neutron, 1.009 u. 0.235 kg of uranium-235 contains 6.0×10^{23} atoms.)

4 Two fusion reactions which take place in the Sun are described below.
a A hydrogen-2 (deuterium) nucleus absorbs a proton to form a helium-3 nucleus.
b Two helium-3 nuclei fuse to form a helium-4 nucleus plus two free protons.
For each reaction, write down the appropriate nuclear equation and calculate the energy released.

(Masses: $^{2}_{1}H$, 2.01410 u; $^{3}_{2}He$, 3.01605 u; $^{4}_{2}He$, 4.00260 u; $^{1}_{1}p$, 1.00728 u; $^{1}_{0}n$, 1.00867 u.)

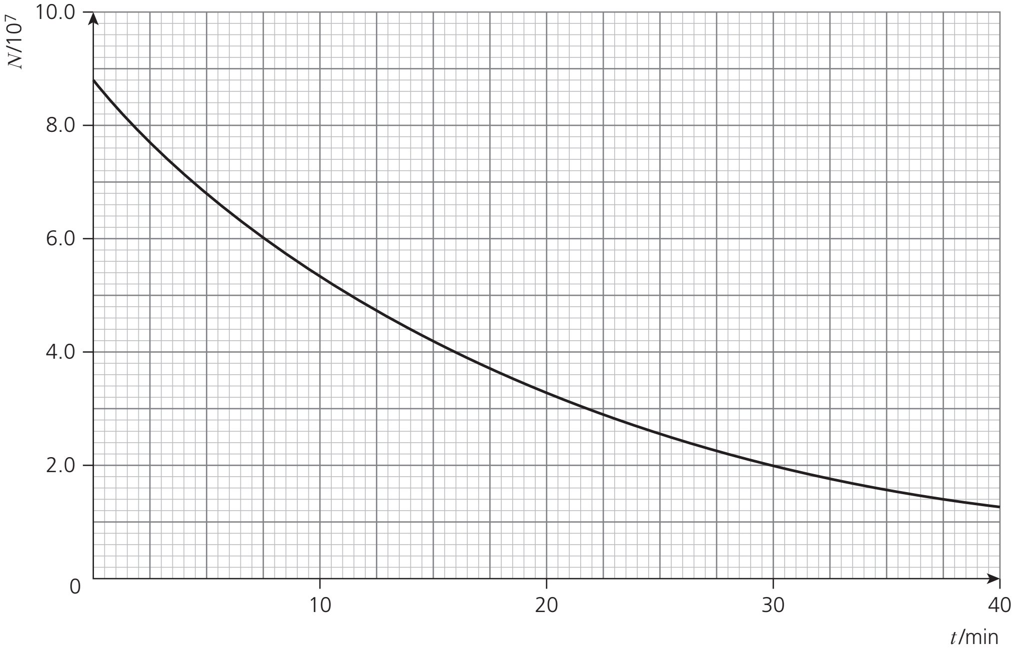
5 Calculate the mass of caesium-137 that has an activity of 2.5×10^{5} Bq.

The number of atoms in 0.137 kg of caesium-137 is 6.0×10^{23}. The half-life of caesium-137 is 30 years.

6 **a** The activity of a radioactive source X falls from 4.5×10^{10} Bq to 1.2×10^{10} Bq in 5.0 hours.
Calculate the half-life.
b The activity of a certain mass of carbon-14 is 3.60×10^{9} Bq. The half-life of carbon-14 is 5570 years.
Calculate the number of carbon-14 nuclei in the sample.

7 **a** State what is meant by *radioactive decay*. **[2]**

b The variation with time t of the number N of technetium-101 nuclei in a sample of radioactive material is shown in Fig. 23.10.



▲ **Figure 23.10**

i Use Fig. 23.10 to determine the activity, in Bq, of the sample of technetium-101 at time t = 14.0 minutes. Show your working. **[4]**

ii Without calculating the half-life of technetium-101, use your answer in **i** to determine the decay constant λ of technetium-101. **[2]**

Cambridge International AS and A Level Physics (9702) Paper 41 Q13 May/June 2018

24 Medical physics

Learning outcomes

By the end of this topic, you will be able to:

24.1 Production and use of ultrasound

1. understand that a piezo-electric crystal changes shape when a p.d. is applied across it and that the crystal generates an e.m.f. when its shape changes
2. understand how ultrasound waves are generated and detected by a piezo-electric transducer
3. understand how the reflection of pulses of ultrasound at boundaries between tissues can be used to obtain diagnostic information about internal structures
4. define the specific acoustic impedance of a medium as $Z = \rho c$, where c is the speed of sound in the medium
5. use $I_R/I_0 = (Z_1 - Z_2)^2/(Z_1 + Z_2)^2$ for the intensity reflection coefficient of a boundary between two media
6. recall and use $I = I_0 e^{-\mu x}$ for the attenuation of ultrasound in matter

24.2 Production and use of X-rays

1. explain that X-rays are produced by electron bombardment of a metal target and calculate the minimum wavelength of X-rays produced from the accelerating p.d.
2. understand the use of X-rays in imaging internal body structures, including an understanding of the term contrast in X-ray imaging
3. recall and use $I = I_0 e^{-\mu x}$ for the attenuation of X-rays in matter
4. understand that computed tomography (CT) scanning produces a 3D image of an internal structure by first combining multiple X-ray images taken in the same section from different angles to obtain a 2D image of the section, then repeating this process along an axis and combining 2D images of multiple sections

24.3 PET scanning

1. understand that a tracer is a substance containing radioactive nuclei that can be introduced into the body and is then absorbed by the tissue being studied
2. recall that a tracer that decays by β^+ decay is used in positron emission tomography (PET scanning)
3. understand that annihilation occurs when a particle interacts with its antiparticle and that mass-energy and momentum are conserved in the process
4. explain that, in PET scanning, positrons emitted by the decay of the tracer annihilate when they react with electrons in the tissue, producing a pair of gamma-ray photons travelling in opposite directions
5. calculate the energy of the gamma-ray photons emitted during the annihilation of an electron–positron pair
6. understand that the gamma-ray photons from an annihilation event travel outside the body and can be detected, and an image of the tracer concentration in the tissue can be created by processing the arrival times of the gamma-ray photons

Starting points

- The wave nature of sound and X-rays.
- X-rays have much shorter wavelengths than visible light.
- All types of waves can be reflected.
- An understanding of the terms wavelength, frequency, speed, amplitude and intensity.
- X-ray radiation is part of the electromagnetic spectrum.
- X-ray radiation is highly penetrating in soft body tissues.
- Isotopes are different forms of the same element.
- Radioactive decay can occur by alpha, beta or gamma decay.
- A positron along with an electron neutrino may be emitted during radioactive decay.
- The positron is the antiparticle of an electron.

24.1 Production and use of ultrasound in diagnosis

The production and use of ultrasound in diagnosis

Ultrasound is sound with frequencies above the range of human hearing, typically above about 20 kHz. The range of frequencies of ultrasound used in medical diagnosis is typically even higher, up to about 10 MHz. As with sound waves, ultrasound waves are longitudinal pressure waves

Ultrasound waves may be generated using a **piezo-electric transducer**. A transducer is the name given to any device that converts energy from one form to another. In this case, electrical energy is converted into ultrasound energy by means of a piezo-electric crystal such as quartz.

The structure of quartz is made up of a large number of tetrahedral silicate units, as shown in Figure 24.1. These units build up to form a crystal of quartz that can be represented, in two dimensions, as shown in Figure 24.2.

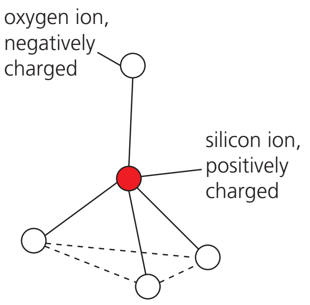


Figure 24.1 Tetrahedral silicate unit

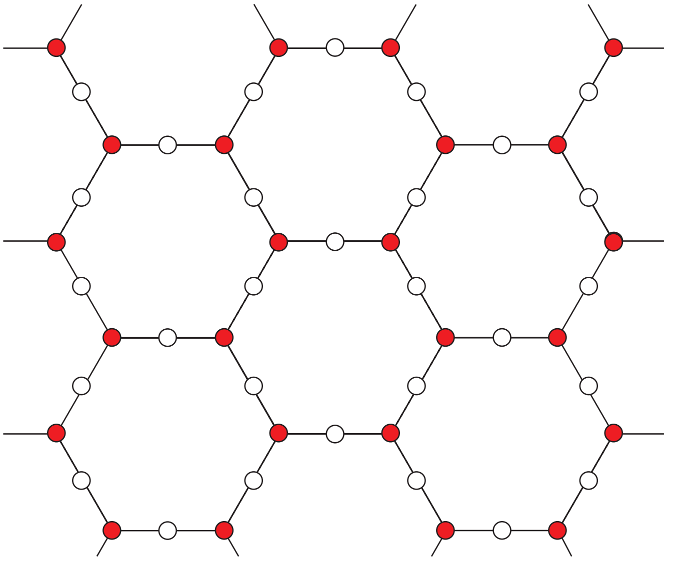
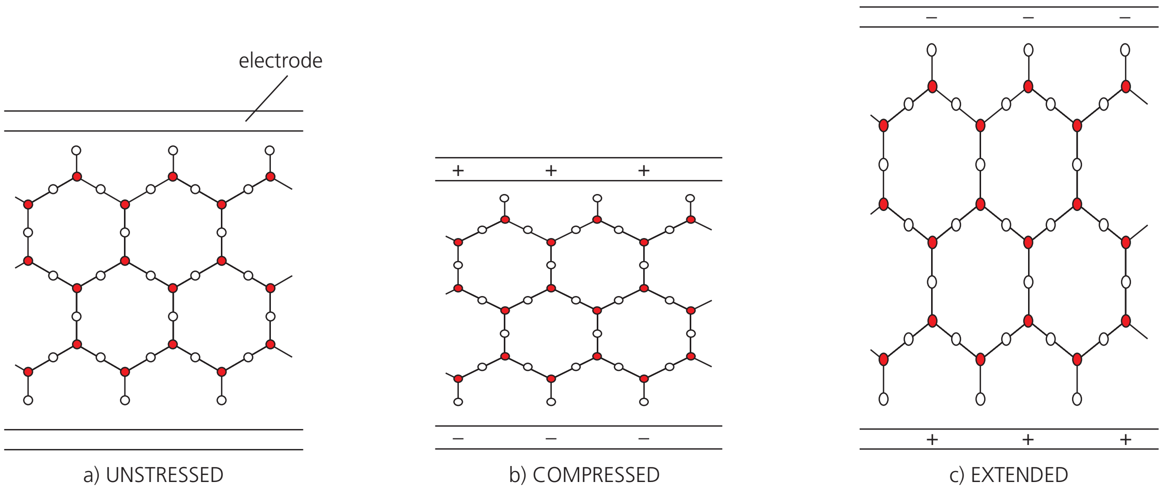


Figure 24.2 Two-dimensional representation of a quartz crystal

When the crystal is unstressed, the centres of charge of the positive and the negative ions in any one unit coincide, as shown in Figure 24.3a. Electrodes may be formed on opposite sides of the crystal by depositing silver on its surfaces. When a potential difference is applied between the electrodes, an electric field is set up in the crystal.

This field causes forces to act on the ions. The oxygen ions are negatively charged and the silicon ions have a positive charge. The ions are not held rigidly in position and, as a result, they will be displaced slightly when the electric field is applied across the crystal. The positive ions will be attracted towards the negative electrode and the negative ions will be attracted to the positive electrode. Dependent on the direction of the electric field, the crystal will become slightly thinner (Figure 24.3b) or slightly thicker (Figure 24.3c).

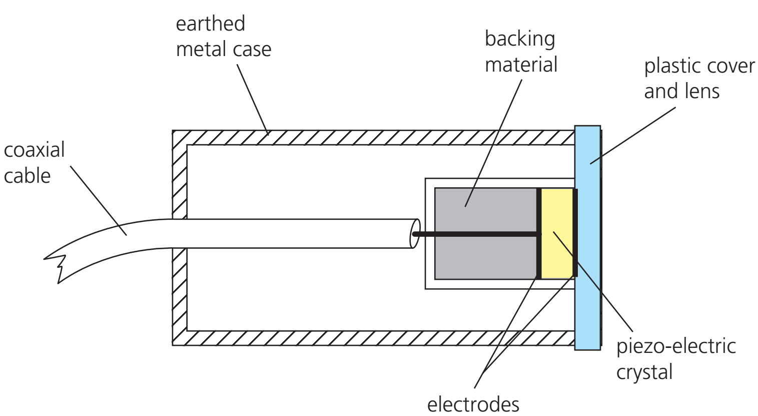


▲ **Figure 24.3** **The effect of an electric field on a quartz crystal**

An alternating voltage applied across the electrodes causes the crystal to vibrate with a frequency equal to that of the applied voltage. These oscillations are likely to have a small amplitude. However, if the frequency of the applied voltage is equal to the natural frequency of vibration of the crystal, resonance will occur (see Topic 17.3) and the amplitude of vibration will be a maximum. The dimensions of the crystal can be such that the oscillations are in the ultrasound range of frequencies (greater than about 20 kHz). These oscillations will give rise to ultrasound waves in any medium surrounding the crystal.

If a stress is applied to an uncharged quartz crystal, the forces involved will alter the positions of the positive and the negative ions, creating a potential difference across the crystal. Therefore, if an ultrasound wave is incident on the crystal, the pressure variations in the wave will give rise to voltage variations across the crystal. An ultrasound transducer may, therefore, also be used as a detector (or receiver).

A simplified diagram of a piezo-electric transducer/receiver is shown in Figure 24.4.

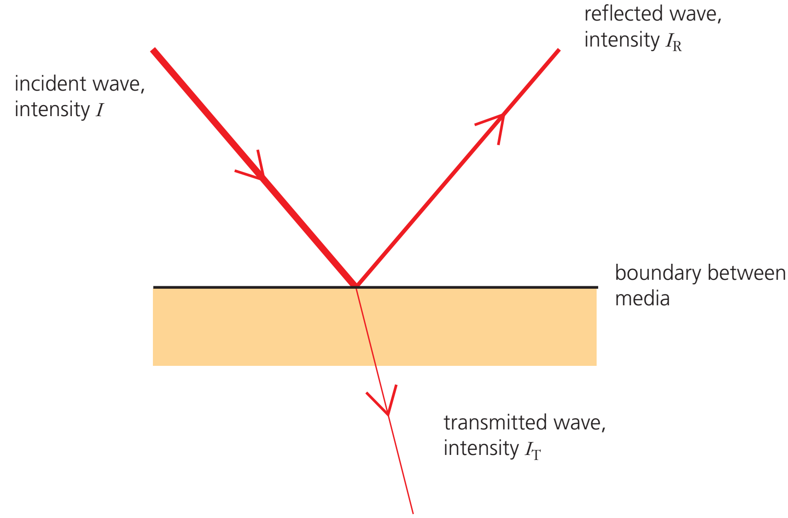


▲ **Figure 24.4** **Piezo-electric transducer/receiver**

A transducer such as this is able to produce and detect ultrasound in the megahertz frequency range, which is typical of the frequency range used in medical diagnosis.

The reflection and absorption of ultrasound

Ultrasound is typical of many types of wave in that, when it is incident on a boundary between two media, some of the wave power is reflected and some is transmitted. This is illustrated in Figure 24.5.



▲ **Figure 24.5** **The reflection and transmission of a wave at a boundary**

For a wave of incident intensity I, reflected intensity I_R and transmitted intensity I_T, by conservation of energy, then

$$I = I_R + I_T$$

Although, for a beam of constant intensity, the sum of the reflected and transmitted intensities is constant, their relative magnitudes depend not only on the angle of incidence of the beam on the boundary but also on the media themselves. The relative magnitudes of I_R and I_T are quantified by reference to the **specific acoustic impedance Z** of each of the media. This is defined as the product of the density ρ of the medium and the speed c of the wave in the medium. That is,

$$Z = \rho c$$

For a wave incident normally on a boundary between two media having specific acoustic impedances of Z_1 and Z_2, the ratio of the reflected intensity I_R to the incident intensity I_0 is given by

$$\frac{I_R}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

The ratio I_R/I_0 is known as the **intensity reflection coefficient** for the boundary and is given the symbol α. As the above equation shows, α depends on the difference between the specific acoustic impedances of the two media. As such, the intensity reflection coefficient can give information about the nature of the two media making up the boundary. Some typical values of specific acoustic impedance are given in Table 24.1, together with the approximate speed of ultrasound in the medium.

medium	speed/m s^{-1}	specific acoustic impedance/kg m^{-2} s^{-1}
air	330	430
blood	1600	1.6×10^6
bone	4100	$5.6 \times 10^6 - 7.8 \times 10^6$
fat	1500	1.4×10^6
muscle	1600	1.7×10^6
soft tissue	1600	1.6×10^6
water	1500	1.5×10^6

▲ **Table 24.1** **Values of speed of ultrasound and specific acoustic impedance for some media**

WORKED EXAMPLE 24A

Using data from Table 24.1, calculate the intensity reflection coefficient for a parallel beam of ultrasound incident normally on the boundary between:

a air and soft tissue

b muscle and bone that has a specific acoustic impedance of $6.5 \times 10^6\,\text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$.

Answers

a $\alpha = (Z_2 - Z_1)^2/(Z_2 + Z_1)^2$

$= (1.6 \times 10^6 - 430)^2/(1.6 \times 10^6 + 430)^2$

$= \mathbf{0.999}$

b $\alpha = (6.5 \times 10^6 - 1.7 \times 10^6)^2/(6.5 \times 10^6 + 1.7 \times 10^6)^2$

$= \mathbf{0.34}$

Question

1 Using data from Table 24.1:

a suggest why, although the speed of ultrasound in blood and muscle is approximately the same, the specific acoustic impedance is different

b calculate the intensity reflection coefficient for a parallel beam of ultrasound incident normally on the boundary between fat and muscle.

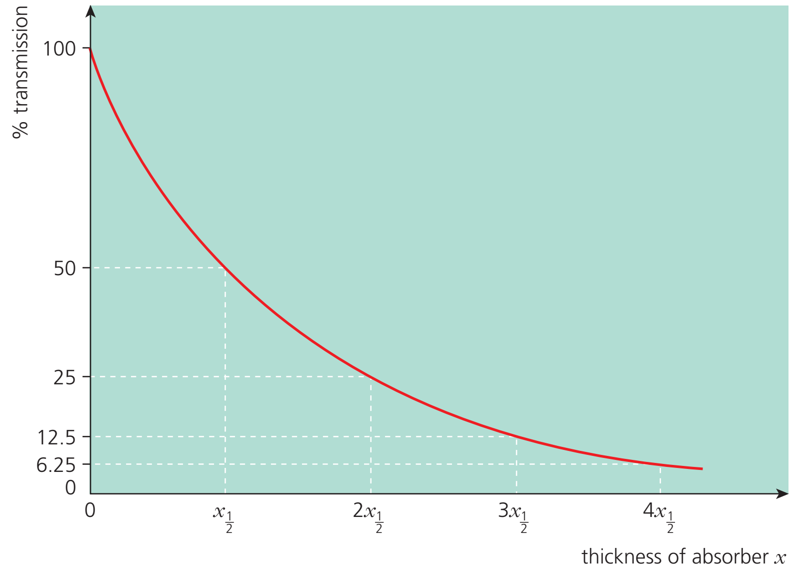
It can be seen that the intensity reflection coefficient for a boundary between air and soft tissue is approximately equal to unity. This means that, when ultrasound is incident on the body, very little is transmitted into the body. In order that ultrasound may be transmitted into the body and also that the ultrasound may return to the transducer, it is important that there is no air between the transducer and the skin (soft tissue). This is achieved by means of a water-based jelly. This jelly has a specific acoustic impedance of approximately $1.5 \times 10^6\,\text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$ which is very similar to the specific acoustic impedance of skin. Thus there is very little reflection of the ultrasound beam as it enters the body.

The attenuation of ultrasound

Once the ultrasound wave is within the medium, the intensity of the wave will be reduced (attenuated) by absorption of energy as it passes through the medium. The medium is heated. In fact, the heating effect produced by ultrasound of appropriate frequencies is used in physiotherapy to assist recovery from sprains and similar injuries.

Mathematical description of reduction in intensity

For a parallel beam of ultrasound, the reduction in intensity is approximately **exponential** and is shown in Figure 24.6 (overleaf). The percentage transmission decreases by the same fraction over equal thicknesses of absorber. A similar exponential attenuation is observed when a parallel beam of X-ray radiation passes through a medium (see Topic 24.2).



▲ **Figure 24.6** The percentage transmission of ultrasound in a medium

For such a beam of ultrasound incident normally on a medium of thickness x, the transmitted intensity I is related to the incident intensity I_0 by the expression

$$I = I_0 e^{-\mu x} \text{ or } I = I_0 \exp(-\mu x)$$

where μ is a constant for the medium known as the **linear absorption (or attenuation) coefficient**. The coefficient μ depends not only on the medium itself but also on the frequency of the ultrasound. Some typical values of the linear absorption (attenuation) coefficient are shown in Table 24.2. The unit of μ is mm^{-1} or cm^{-1}. If the unit of x is given in cm the unit of μ must be given in cm^{-1}.

medium	linear absorption (attenuation) coefficient/cm^{-1}
air	1.2
bone	0.13
muscle	0.23
water	0.0002

▲ **Table 24.2** Some values of linear absorption (attenuation) coefficient for ultrasound

Note that the expression for the change in the transmitted intensity applies only to a *parallel* beam. If the beam is divergent, then the intensity would decrease without any absorption by the medium.

WORKED EXAMPLE 24B

A parallel beam of ultrasound is incident on the surface of a muscle and passes through a thickness of 3.5 cm of the muscle. It is then reflected at the surface of a bone and returns through the muscle to its surface. Using data from Tables 24.1 and 24.2, calculate the fraction of the incident intensity that arrives back at the surface of the muscle.

Answer

The beam passes through a total thickness of 7.0 cm of muscle. For the attenuation in the muscle,

$I = I_0 \exp(-0.23 \times 7.0) = 0.20 I_0$

Fraction reflected at muscle–bone interface $= (6.5 \times 10^6 - 1.7 \times 10^6)^2 / (6.5 \times 10^6 + 1.7 \times 10^6)^2$

$= 0.34$

Fraction received back at surface $= 0.20 \times 0.34 =$ **0.068**

Question

2 A parallel beam of ultrasound passes through a thickness of 4.0 cm of muscle. It is then incident normally on a bone having a specific acoustic impedance of $6.4 \times 10^6\,kg\,m^{-2}\,s^{-1}$. The bone is 1.5 cm thick. Using data from Tables 24.1 and 24.2, calculate the fraction of the incident intensity that is transmitted through the muscle and bone.

Obtaining diagnostic information using ultrasound

The ultrasound transducer is placed on the skin, with the water-based jelly excluding any air between the transducer and the skin (Figure 24.7).

▲ **Figure 24.7** Ultrasound diagnosis

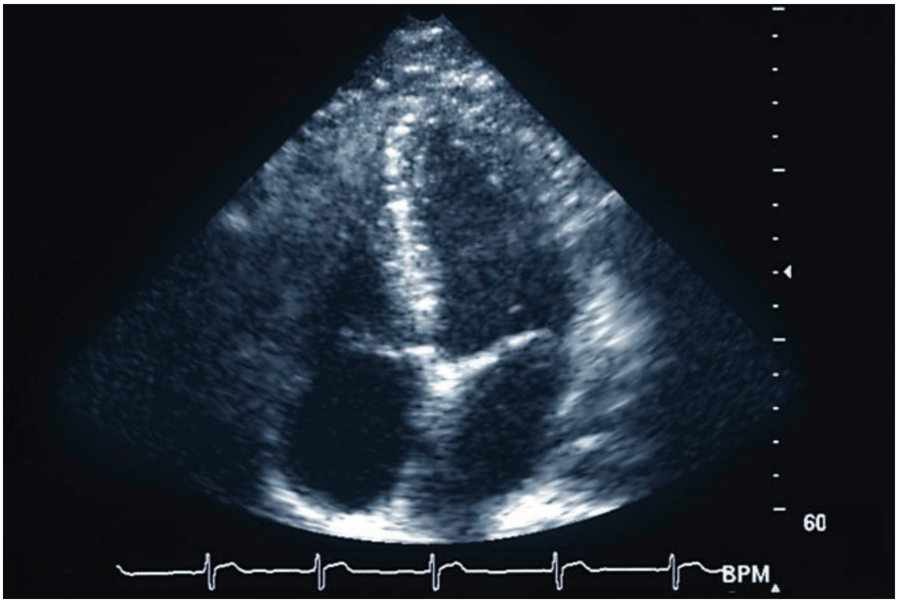
Short **pulses** of ultrasound are transmitted into the body where they are partly reflected and partly transmitted at the boundaries between media in the body such as fat–muscle and muscle–bone. The reflected pulses return to the transducer where they are detected and converted into voltage pulses. These voltage pulses can be amplified and processed by electronic circuits such that the output of the circuits may be displayed on a screen as in, for example, a cathode-ray oscilloscope.

Pulses of ultrasound are necessary so that the reflected ultrasound pulses can be detected in the time intervals between the transmitted pulses. The time between the transmission of a pulse and its receipt back at the transducer gives information as to the distance of the boundary from the transducer. The intensity of the reflected pulse gives information as to the nature of the boundary. Two techniques are in common use for the display of an ultrasound scan.

In an **A-scan**, a short pulse of ultrasound is transmitted into the body through the coupling medium (the water-based jelly). At each boundary between media, some of the energy of the pulse is reflected and some is transmitted. The transducer detects the reflected pulses as it now acts as the receiver. The signal is amplified and displayed on a cathode-ray oscilloscope (CRO). Reflected pulses (echoes) received at the transducer from deeper in the body tend to have lower intensity than those reflected from boundaries near the skin. This is caused not only by absorption of wave energy in the various media but also, on the return of the reflected pulse to the transducer, some of the energy of the pulse will again be reflected at intervening boundaries. To allow for this, echoes received later at the transducer are amplified more than those received earlier. A vertical line is observed on the screen of the CRO corresponding to the detection of each reflected pulse. The time-base of the CRO is calibrated so that, knowing the speed of the ultrasound wave in each medium, the distance between boundaries can be determined. An example of an A-scan is illustrated in Figure 24.8.

▲ **Figure 24.8** An A-scan

A **B-scan** consists of a series of A-scans, all taken from different angles so that, on the screen of the CRO, a two-dimensional image is formed. Such an image is shown in Figure 24.9.



▲ **Figure 24.9** An image of a healthy heart produced from a B-scan

The ultrasound probe for a B-scan does not consist of a single crystal. Rather, it has an array of small crystals, each one at a slightly different angle from its neighbours. The separate signals received from each of the crystals in the probe are processed. Each reflected pulse is shown on the screen of the CRO as a bright spot in the direction of orientation of the particular crystal that gave rise to the signal. The pattern of spots builds up to form a two-dimensional image representing the positions of the boundaries within the body. The image may be either viewed immediately or photographed or stored in a computer memory.

The main advantage of ultrasound scanning compared to X-ray imaging is that the health risk to both the patient and to the operator is very much less. Also, ultrasound equipment is much more portable and is relatively simple to use.

Higher-frequency ultrasound enables greater resolution to be obtained since the wavelength will be shorter and there will be less diffraction around small features. That is, more detail can be seen. Furthermore, as modern techniques allow for the detection of very low-intensity reflected pulses, boundaries between tissues where there is little change in specific acoustic impedance can be detected.

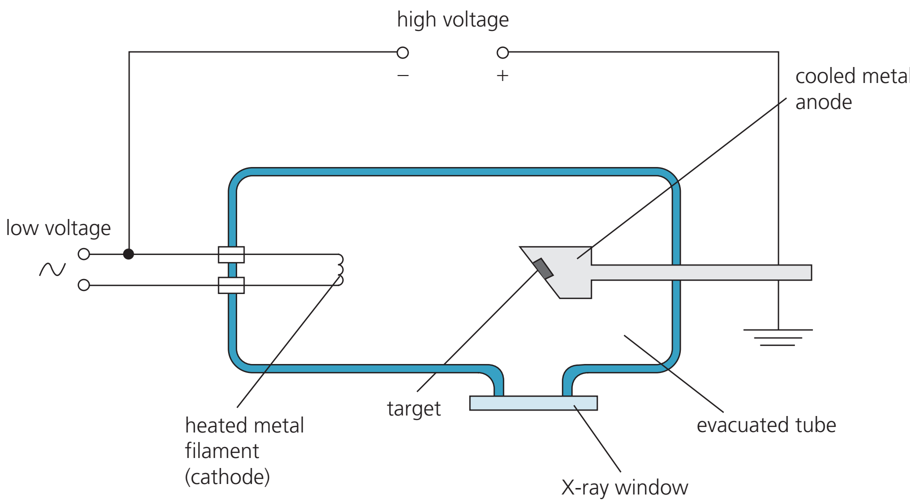
24.2 Production and use of X-rays

▲ **Figure 24.10** Modern X-ray machine

Whenever a charged particle is accelerated, electromagnetic radiation is emitted. This radiation is known as Bremmstrahlung radiation or 'braking' (slowing down) radiation. The frequency of the radiation depends on the magnitude of the acceleration. The larger the acceleration (or deceleration), the greater is the frequency of the emitted photon.

X-ray photons may be produced by the bombardment of metal targets with high-speed electrons. The electrons are first accelerated through a potential difference of many thousands of volts so that they have high energy and high speed. This acceleration is, however, not sufficient for X-ray radiation to be emitted. The high-speed electrons strike a metal target, which causes the electrons to change direction and to lose kinetic energy very rapidly. Large decelerations are involved that give rise to the emission of X-ray photons. It should be remembered that not all of the energy of the electrons is emitted as X-ray photons. The majority is transferred to thermal energy in the target metal.

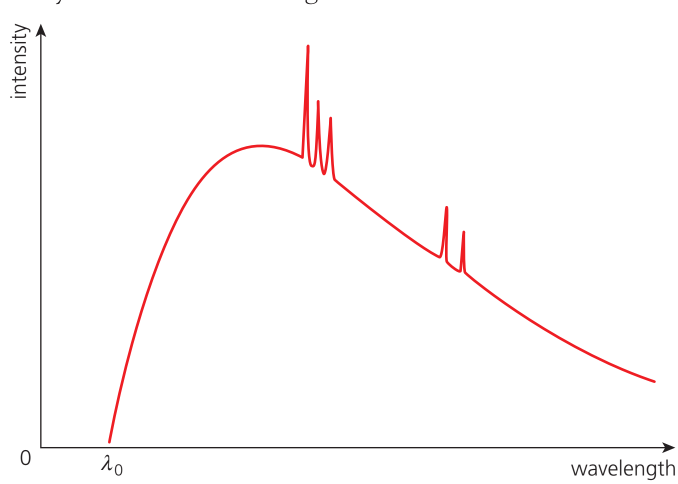
A simplified design of an X-ray tube is shown in Figure 24.11.



▲ **Figure 24.11** Design of an X-ray tube

Varying the X-ray wavelength produced

A typical X-ray spectrum showing the variation with wavelength of the intensity of an X-ray beam is shown in Figure 24.12.



▲ **Figure 24.12** Typical X-ray spectrum

The spectrum has two distinct components. First, there is a continuous distribution of wavelengths with a sharp cut-off at the shortest wavelength λ_0. Second, sharp peaks may be observed. These sharp peaks correspond to the emission line spectrum of the target metal and are, therefore, a characteristic of the target (see Topic 22.4).

The continuous distribution comes about because the electrons, when incident on the metal target, will not all have the same deceleration but will, instead, have a wide range of values. Since the wavelength of the emitted radiation is dependent on the deceleration, there will be a distribution of wavelengths. The cut-off wavelength corresponds to an electron that is stopped in one collision in the target so that all of its kinetic energy is given up as one X-ray photon.

The kinetic energy E_k of an electron is equal to the energy gained by the electron when it is accelerated from the cathode to the anode.

$$E_k = eV$$

where e is the charge on the electron and V is the accelerating potential difference.

The energy E of a photon of wavelength λ is given by (see Topic 22.1)

$$E = hc/\lambda$$

where h is the Planck constant. Thus, at the cut-off (minimum) wavelength λ_0,

$$eV = hc/\lambda_0$$

and so

$$\lambda_0 = \frac{hc}{eV}$$

The accelerating potential V thus determines the cut-off wavelength λ_0. The larger the potential difference, the shorter the wavelength. The hardness (penetrating ability) of the X-ray beam is, therefore, controlled by variation of the accelerating potential difference between the cathode and the anode.

The continuous distribution of wavelengths implies that there will be X-ray photons of long wavelengths that would not penetrate the person being investigated and so would not contribute towards the X-ray image. Such long-wavelength photons would add to the radiation dose received by the person without serving any useful purpose. For this reason, the X-ray beam emerging from the X-ray tube frequently passes through aluminium filters that absorb these long-wavelength photons.

WORKED EXAMPLE 24C

The accelerating potential difference between the cathode and the anode of an X-ray tube is 30 kV. Given that the Planck constant is 6.63×10^{-34} J s, the charge on the electron is 1.60×10^{-19} C and the speed of light in free space is 3.00×10^{8} m s^{-1}, calculate the minimum wavelength of photons in the X-ray beam.

Answer

For the minimum wavelength,

energy gained by electron = energy of photon

$$eV = hc/\lambda_0$$

$$1.60 \times 10^{-19} \times 30 \times 10^{3} = \frac{(6.63 \times 10^{-34} \times 3.00 \times 10^{8})}{\lambda_0}$$

$$\lambda_0 = \mathbf{4.14 \times 10^{-11}\ m}$$

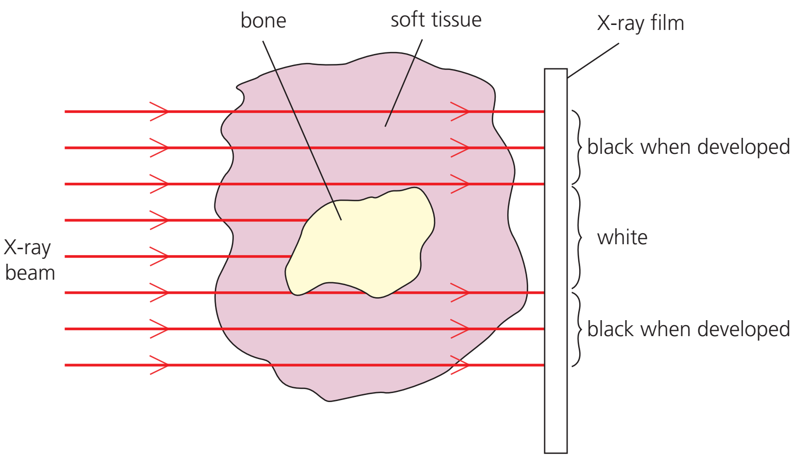
Question

3 Calculate the minimum wavelength of photons produced in an X-ray tube for an accelerating potential difference of 75 kV.

The X-ray image

An X-ray image is shown in Figure 24.13. This is not really an image in the sense of the real image produced by a lens. Rather, the image is like a shadow, as illustrated in Figure 24.14.

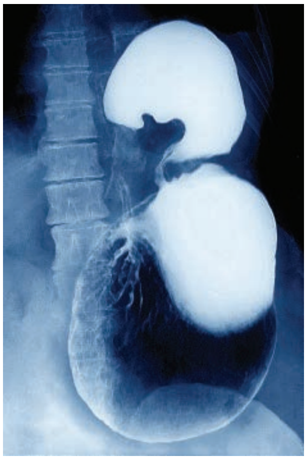
▲ **Figure 24.13** X-ray image showing a fractured collar bone



▲ **Figure 24.14** How an X-ray image is produced

The X-ray beam is incident on the body part of the patient. The X-ray beam can penetrate soft tissues (skin, fat, muscle, etc.) with little loss of intensity and so photographic film, after development, will show a dark area corresponding to these soft tissues. Bone, however, causes a greater attenuation (reduces the intensity by a greater extent) than the soft tissues and, therefore, the photographic film will be lighter in colour in areas corresponding to the positions of bones. What is produced on the film is a two-dimensional shadow of the bone and the surrounding tissues.

The quality of the shadow image produced depends on its contrast. A shadow image where the bones and other organs are clearly outlined is said to be a 'sharp image'. Although an image may be sharp, it may still not be clearly visible because there is little difference in the degree of blackening between, say, the bone and the surrounding tissue. An X-ray image having a wide range of degrees of blackening in different regions is said to have good **contrast**.



▲ **Figure 24.15** X-ray of stomach after a barium meal

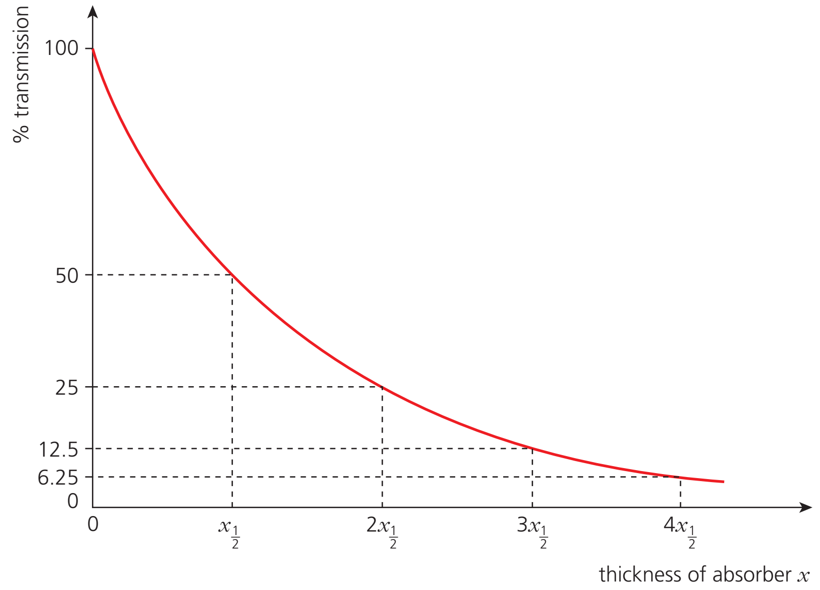
Good contrast is achieved when neighbouring body organs and tissues absorb the X-ray photons to very different extents (see The attenuation of X-rays, below). This is usually the case for bone and muscle. This is, however, not the case where, for example, the stomach is to be investigated. The patient is then asked to swallow a solution of barium sulfate – a 'barium meal' (Figure 24.15).

Barium is a good absorber of X-ray photons. As a result, when the barium sulfate solution coats the inside of the stomach, the outline of the stomach will show up clearly on the image. Similarly, blood vessels can be made visible by injecting a radio-opaque dye into the bloodstream.

Contrast also depends on other factors, such as exposure time. Contrast may be improved by backing the X-ray film with a fluorescent material.

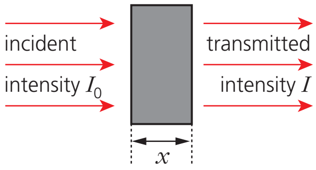
The attenuation of X-rays

When a beam of X-ray photons passes through a medium, absorption processes occur that reduce the intensity of the beam. The intensity of a parallel beam is reduced by the same fraction each time the beam passes through equal thicknesses of the medium. Consequently, the variation of the percentage of the intensity transmitted with thickness of absorber may be shown as in Figure 24.16 (overleaf).



▲ **Figure 24.16** The percentage transmission of X-rays in a medium

It can be seen that the same thickness of medium is always required to reduce the transmitted beam intensity by 50%, no matter what starting point is chosen. This is analogous to the constant half-life of radioactive decay, and is due to the exponential nature of both radioactive decay and the attenuation with distance of a parallel beam of radiation.



▲ **Figure 24.17** Absorption of X-rays in a medium

The decrease in transmitted X-ray intensity is an exponential decrease. Consider a parallel beam having an incident intensity I_0. The medium (the absorber) has thickness x and the transmitted intensity is I, as illustrated in Figure 24.17.

The transmitted intensity is given by the expression

$$I = I_0 e^{-\mu x} \text{ or } I = I_0 \exp(-\mu x)$$

where μ is a constant that is dependent on the medium and on the energy of the X-ray photons, known as the **linear attenuation coefficient** or the **linear absorption coefficient** of the medium. The unit of μ is mm^{-1} or cm^{-1}.

Approximate values of the linear attenuation (absorption) coefficient μ for some substances are given in Table 24.3.

substance	μ/cm^{-1}
copper	7
water	0.3
bone	3
fat	0.9

▲ **Table 24.3** Some approximate values of linear attenuation (absorption) coefficient

Note that the expression $I = I_0\exp(-\mu x)$ applies to a parallel beam. If the beam is not parallel, then there will be further changes in intensity without any absorption. For example, the intensity of a divergent beam decreases with distance from the source.

WORKED EXAMPLE 24D

The linear absorption coefficient of copper is $0.693\,mm^{-1}$. Calculate:

a the thickness of copper required to reduce the incident intensity by 50%

b the fraction of the incident intensity of a parallel beam that is transmitted through a copper plate of thickness 1.2 cm.

Answers

a $I/I_0 = 0.50 = \exp(-0.693 \times x)$

$\ln 0.50 = -0.693x$

$x = \mathbf{1.0\,mm}$

b $I/I_0 = \exp(-0.7 \times 1.2)$

$I/I_0 = \mathbf{2.4 \times 10^{-4}}$

Questions

4 a State what is meant by the *contrast* in an X-ray image.
b Explain how the contrast in an X-ray image may be improved.

5 For one particular energy of X-ray photons, water has a linear attenuation (absorption) coefficient of $0.29\,\text{cm}^{-1}$. Calculate the depth of water required to reduce the intensity of a parallel beam of these X-rays to 3.0×10^{-3} of its incident intensity.

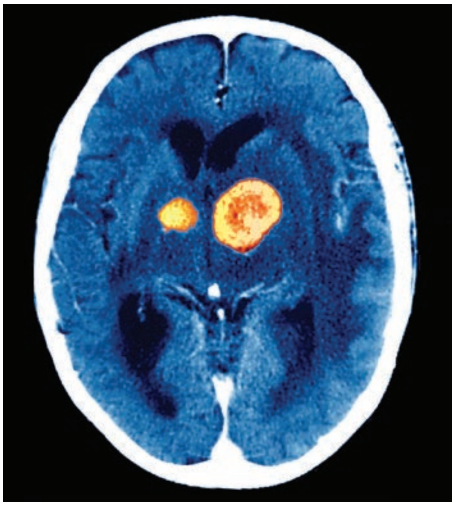
6 The linear attenuation (absorption) coefficients of bone and of the soft tissues surrounding the bone are $2.9\,\text{cm}^{-1}$ and $0.95\,\text{cm}^{-1}$ respectively. A parallel beam of X-rays is incident, separately, on a bone of thickness 3.0 cm and on soft tissue of thickness 5.0 cm. Calculate the ratio:

$$\frac{\text{intensity transmitted through bone}}{\text{intensity transmitted through soft tissue}}$$

Computed tomography (CT scanning)

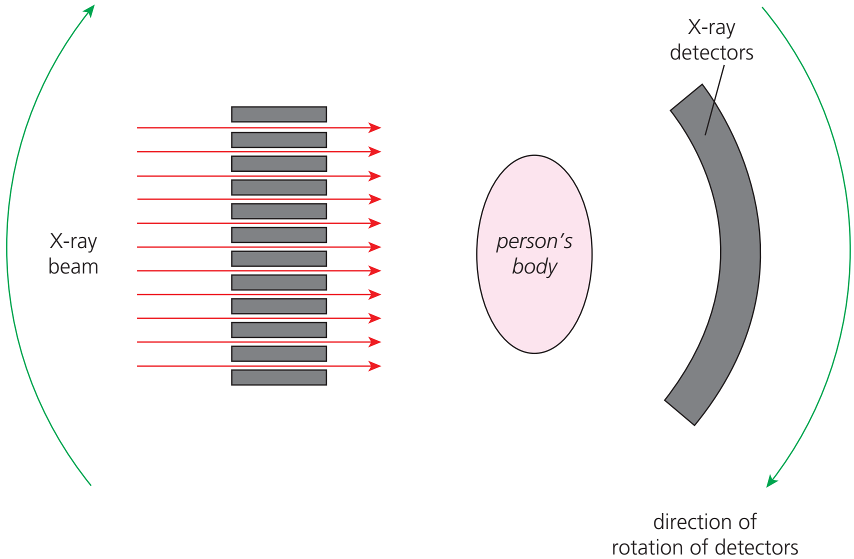
The image produced on X-ray film, as outlined earlier, is a 'shadow' or 'flat' image. There is little, if any, indication of depth. That is, the position of an organ within the body is not apparent. Also, soft tissues lying behind structures that are very dense cannot be detected. Tomography is a technique whereby a three-dimensional image or 'slice' through the body may be obtained. The image is produced by **computed tomography** using what is known as a CT scanner (Figure 24.18). An example of such an image is shown in Figure 24.19.

▲ **Figure 24.18** Radiologist moving a patient into a CT scanner



▲ **Figure 24.19** Scan through the head of a patient with a cerebral lymphoma

In this technique, a series of X-ray images is obtained. Each image is taken through the same section or slice of the body from a different angle, as illustrated in Figure 24.20.

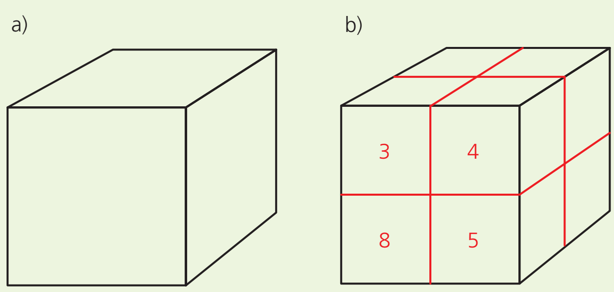


▲ **Figure 24.20** Arrangement for a CT scan

Data for each individual X-ray image and angle of viewing is fed into a high-power computer. A two-dimensional image of the slice is computed. This is then repeated for successive slices. The computer enables the images of each slice to be combined so that a complete three-dimensional image of the whole object is obtained, which can then be viewed from any angle.

EXTENSION

The basic principles of CT scanning may be illustrated using a simple cubic shape as shown in Figure 24.21.

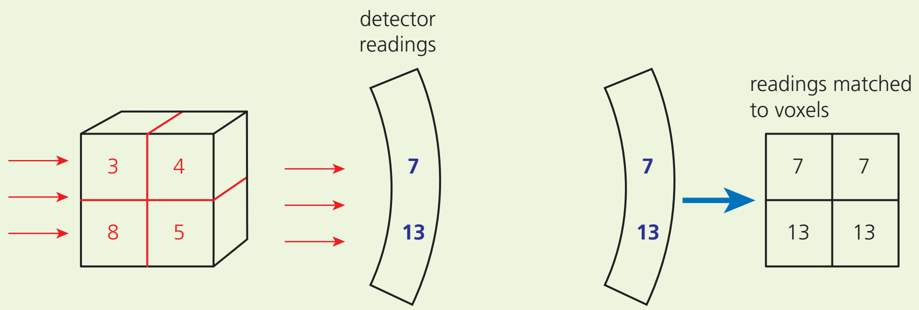


▲ **Figure 24.21** The first section showing the pixels

The aim of CT scanning is to produce an image of a section through the cube from measurements made about its axis. The section, or 'slice' through the cube is divided up into a series of small units, called **voxels**. Each voxel will absorb the X-ray beam to a different extent. The intensity transmitted through each voxel alone can be given a number, referred to as a **pixel**. The various pixels are built up from measurements of the X-ray intensity along different directions through the section or slice.

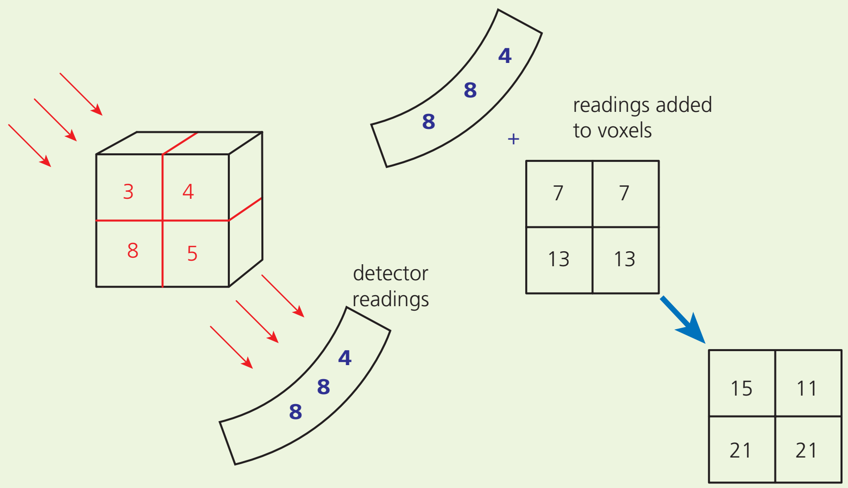
Suppose that the cube in Figure 24.21a is divided into eight voxels. The cube can be thought to consist of two slices or sections. For the first section, let the pixels be as shown in Figure 24.21b. The purpose of the CT scan is to reproduce these pixels in their correct positions.

When the X-ray beam is directed at the section from the left, as shown in Figure 24.22, the detectors will give readings of 7 and 13. The voxels will be partially completed as shown.



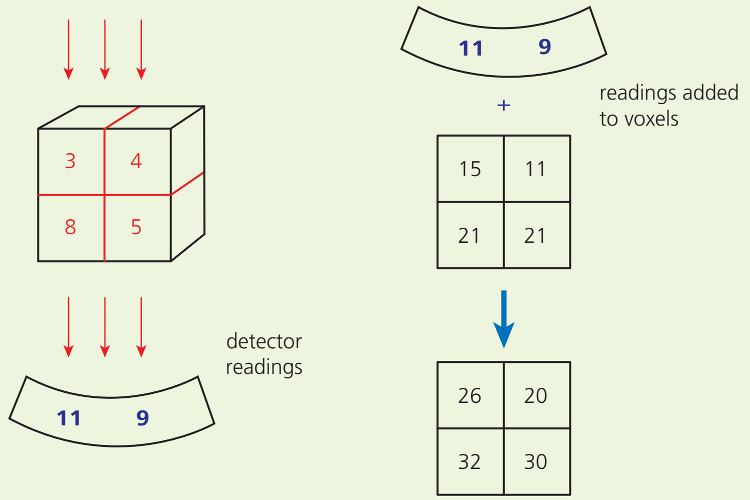
▲ **Figure 24.22** **The first set of detector readings**

The X-ray tube and the detectors are now rotated through 45°. The new detector readings are 4, 8 and 8. These readings are added to the readings already in the voxels, as shown in Figure 24.23.



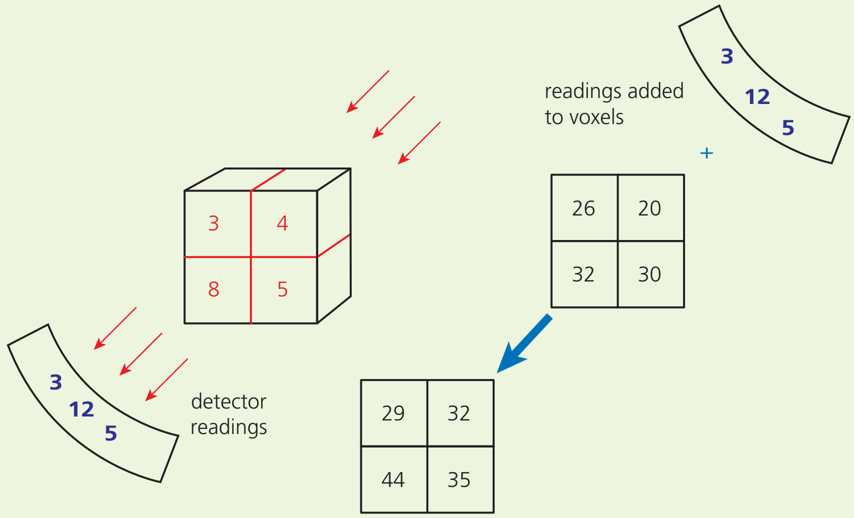
▲ **Figure 24.23** **The second set of detector readings**

After rotation through a further 45°, a third set of detector readings is taken. These readings are added to the voxel readings. The result is shown in Figure 24.24.



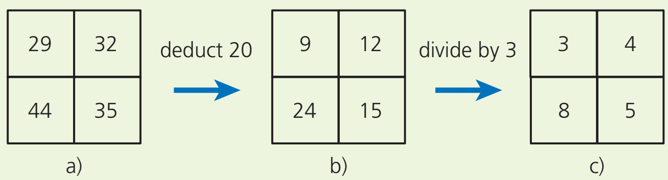
▲ **Figure 24.24** **The third set of detector readings**

After rotation through a further 45° a final set of readings is taken. Once again the readings are added to those already in the voxels, giving the result shown in Figure 24.25 (overleaf).



▲ **Figure 24.25** The fourth set of detector readings

The resulting pattern of the pixels is shown in Figure 24.26a.



▲ **Figure 24.26** The final result

Having summed all the measurements, it remains to reduce these measurements to that of the original. This is achieved in two stages.

1 The background intensity must be removed. This background is equal to the sum of the detector readings for any one position. In this case, the sum is 20. This background is deducted from each pixel, as shown in Figure 24.26b.
2 Allowance must now be made for the fact that more than one view was made of the section. In this example, there were four sets of readings and consequently each pixel reading is divided by 3.

The final result is shown in Figure 24.26c – and, note, this is the same as Figure 24.21b.

Once the pattern of pixels for one section has been obtained, the CT scanner is moved relative to the object so that the next neighbouring section is analysed. This procedure is repeated until the whole of the object to be analysed has been scanned.

The analysis above is a very simple example. In practice, the image of each section is built up from a large number of units or voxels. The larger the number of voxels, the better the definition. This is similar, in principle, to the digital camera. The use of a large number of voxels implies that measurements must be made from a large number of different angles. The storage of the data for each angle and its construction into a final image on a screen requires a powerful computer. The reconstruction of the intensity in each voxel will involve more than one million separate computations. All the data for all the sections is stored in the computer memory so that a three-dimensional image of the whole object is formed. This enables sections of the image to be viewed from many different angles. Finally, the computer enables the brightness and contrast of the image to be varied so that the optimum image may be obtained.

WORKED EXAMPLE 24E

Compare the image produced during an X-ray investigation and that produced in CT scanning.

Answer

An X-ray image is a two dimensional projection onto a flat screen of a three-dimensional object. A CT scan is a three-dimensional image. The computer in which the data for the image is stored enables different sections to be viewed at different angles.

Questions

7 The principles of CT scanning have been understood for some time. However, scanners could not be developed until large powerful computers were available. By reference to the image produced in a CT scan, suggest why such a computer is necessary.

8 a Outline how X-ray images are used to build up the image produced in a CT scan.
b Explain why the radiation dose received during a CT scan is greater than that for an X-ray 'photograph'.

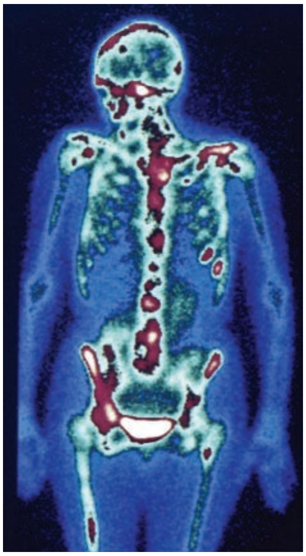
24.3 PET scanning

Radioactive tracers

Isotopes are different forms of the same element that have different number of neutrons in their nuclei (Topic 11.1) but have the same chemical properties. Some isotopes of an element have unstable nuclei and so are radioactive which means that these isotopes give off radiation (α-particles, β-particles and/or γ-radiation). This radiation can be detected and thus the presence of the radioactive material can be established.

If a chemical compound has one or more of its atoms replaced by radioactive atoms of the same element then, as a result of the radioactive decay of these atoms, the location of the compound can be determined or its progress in living tissues can be followed. Such compounds are known as **tracers**.

A tracer is a chemical compound in which one or more of its atoms have been replaced by radioactive nuclei of the same element that can then be used to locate or follow the progress of the compound in living tissues.



▲ **Figure 24.27** This gamma scan used the tracer technetium-99 to detect abnormal activity in bone metabolism.

Tracers used in medical imaging are usually introduced into the body by injection or are swallowed. They then travel in the blood to tissues where the tracer compound is absorbed. Different tracer compounds are absorbed in different amounts by different tissues depending on the biological process being carried out, so the choice of tracer depends on the tissue being studied.

Since the radiation emitted by the tracer must be detected from outside the body, generally, only gamma emitters can be used as tracers. Alpha and beta radiation are much less penetrating and would be absorbed by the body.

The advantage of tracers is that the radioactive compound behaves chemically in the same way as the compound without the tracer and so any biological process is not affected.

In many uses of tracers in medical imaging, the tracer has a half-life of just a few hours so that the risk to the patient or medical staff of exposure to radiation is reduced. Radiation exposure is a disadvantage of the use of tracers but this is offset by a technique for diagnosis that is non-invasive and does not require surgery or the taking of biopsies (samples of tissue).

A commonly used tracer is technetium-99^m. This radioactive isotope has a half-life of 6 hours and can be generated where it is to be used. Technetium-99^m is used to image the skeleton and organs including heart muscle, the brain and the thyroid gland. Accumulation of technetium-99^m in the skeleton is shown in Figure 24.27.

Other frequently used tracers include iodine-131 (for thyroid conditions), iron-59 (for spleen metabolism) and potassium-42 (for blood composition).

Positron emission and annihilation

Positron emission tomography (PET) uses radioactive tracers. The tracers used in a PET scan decay by **β^+ emission**. The emission of a **positron** during β^+ decay occurs when a proton inside a nucleus is converted into a neutron (see Topic 11.1). The emission also includes an electron neutrino. An example of such a decay is that of fluorine-18:

$$^{18}_{9}F \rightarrow\ ^{18}_{8}O \rightarrow +\ ^{0}_{1}e +\ ^{0}_{0}\nu$$

Fluorine-18 is very frequently used as the tracer in PET scanning (see below). Other positron emitters include magnesium-23 and sodium-23.

Within a very short time of a positron being emitted, the positron slows and it collides with an electron. An electron is the antiparticle of a positron. **Annihilation** occurs when a particle interacts with its antiparticle. The mass of the two particles is converted into energy through Einstein's mass–energy relation $E = \Delta mc^2$, where c is the speed of light (see Topic 23.1).

When particles and antiparticles meet, they annihilate each other, releasing their combined mass as energy in the form of photons.

During the process of annihilation, conservation laws are followed. Namely

- charge: the positive charge and the negative charge on the particle and antiparticle are equal in magnitude. The overall charge remains as zero.
- mass-energy: when the particle and the antiparticle collide, since mass and energy are equivalent, the total mass-energy before annihilation must equal the total mass-energy after the annihilation.
- momentum: momentum is conserved so the total momentum before annihilation must equal the total momentum after annihilation.

Annihilation produces energy in the form of photons. Since we can assume that the positron and electron collide with negligible initial momentum, for the momentum to remain zero after annihilation, then the resulting energy is seen as *two* photons of equal energy moving in *opposite* directions. Each photon has momentum and the total momentum of the photons is zero.

Also, since we can assume that the positron and electron collide with negligible initial kinetic energies, the total energy of the two photons produced must be equal to the energy equivalent of the loss in mass.

Consider the annihilation of an electron–positron pair:

$$^{0}_{-1}e +\ ^{0}_{1}e \rightarrow \gamma + \gamma$$

Each particle has mass m of 9.11×10^{-31} kg. The total photon energy is given by $E = \Delta mc^2$, where c is the speed of light.

$$E = 2 \times (9.11 \times 10^{-31}) \times (3.00 \times 10^{8})^2$$
$$= 1.64 \times 10^{-13}\,\text{J}$$

Since 1MeV = 1.60×10^{-13} J, then

$$E = (1.64 \times 10^{-13}/(1.60 \times 10^{-13}) = 1.02\,\text{MeV}$$

The energy of each photon emitted as a result of the annihilation is thus 0.512 MeV.

The frequency of each photon is found from $E = hf$:

$$f = E/h = \frac{1}{2} \times 1.64 \times 10^{-13}/6.63 \times 10^{-34} = 1.24 \times 10^{20}\,\text{Hz}$$

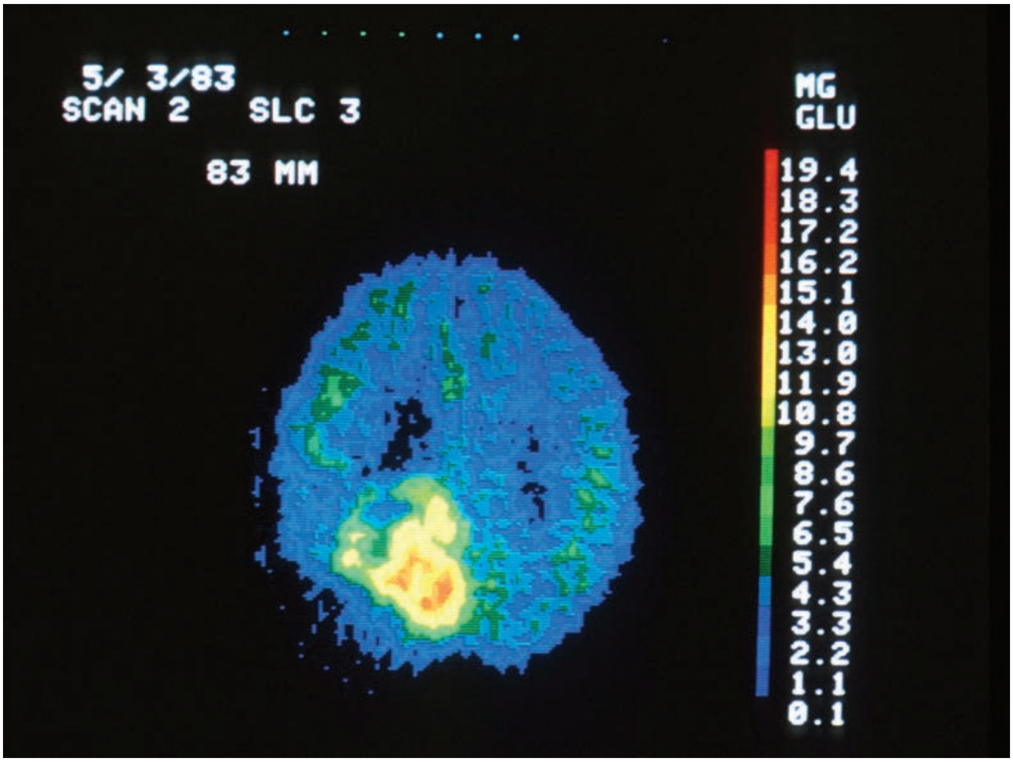
This frequency is in the gamma ray range of the electromagnetic spectrum.

The PET scanner

A PET scanner is used in medical imaging and diagnosis to determine how well certain body functions are operating and to identify abnormalities.

A commonly used tracer in PET scanning is fluorodeoxyglucose (FDG) which is labelled with radioactive fluorine. This fluorine is a positron emitter (see above) and has a half-life of 118 minutes. FDG is a naturally occurring sugar that does not interfere with any body functions. Using the radioactive tracer it is possible to analyse where the FDG builds up, or does not build up. For example, cancer tissues tend to accumulate sugars to a greater extent than healthy tissue.

For a PET scan, the patient is surrounded by a ring of γ-ray detectors. When a positron annihilates, two γ-ray photons are emitted, having the same energy but in opposite directions. The energy of the photons is sufficient for them to leave the body and be detected. If two detectors, on opposite sides of the patient, detect a γ-ray photon simultaneously, then it is known that the positron annihilation took place along the straight line joining the two detectors. The direction of this line is stored in a computer. A set of data for a PET scan consists of millions of these lines which can be processed by computer to give an image of the organ or tissue where the tracer accumulated. A PET scan image is shown in Figure 24.28.



▲ **Figure 24.28** PET scan of patient with a brain tumour (bright white area in the centre of the scan)

Question

9 During the radioactive decay of a nucleus, a mass of 1.5×10^{-3}u is converted into the energy of a γ-ray photon. Calculate this energy, in MeV.

SUMMARY

- Ultrasound may be generated and detected by piezo-electric crystals.
- Ultrasound images are formed as a result of the detection and processing of ultrasound pulses that have been reflected from tissue boundaries.
- The acoustic impedance Z of a medium is ρc.
- The intensity reflection coefficient at a boundary between two media is $I_R/I_0 = (Z_2-Z_1)^2/(Z_2+Z_1)^2$.
- The attenuation of a parallel beam of ultrasound is given by $I = I_0 e^{-\mu x}$.
- Two-dimensional US scans may be obtained using a generator/detector consisting of many separate crystals all at different angles of orientation.
- X-rays are produced when high-speed electrons are stopped by a metal target.
- The minimum wavelength of X-rays produced depends on the accelerating p.d.
- An X-ray image is a 'shadow' of structures in which the X-ray beam is attenuated.
- Contrast in an X-ray image depends on neighbouring tissues having very different absorption coefficients.
- The attenuation in the intensity of a parallel X-ray beam is given by $I = I_0 e^{-\mu x}$.
- Computed tomography (CT scanning) enables an image of a section through the body to be obtained by combining many X-ray images, each one taken from a different angle.
- A tracer is a compound in which one of its atoms has been replaced by a radioactive isotope of the same element.
- The location or progress of tracers in living tissues can be followed without affecting the tissue function.
- The tracers used in PET scanning are positron emitters.
- A positron, mass m_e and charge $+q$, is the antiparticle of an electron.
- Annihilation occurs when a particle and an antiparticle interact.
- Annihilation of an electron–positron pair results in two γ-ray photons having equal energies and moving off in opposite directions.
- A PET scan image is created by processing the directions of movement of large numbers of pairs of simultaneously detected γ-ray photons resulting from electron–positron annihilation.

END OF TOPIC QUESTIONS

1 Explain why, when obtaining an ultrasound scan:

a the ultrasound is pulsed and is not continuous,

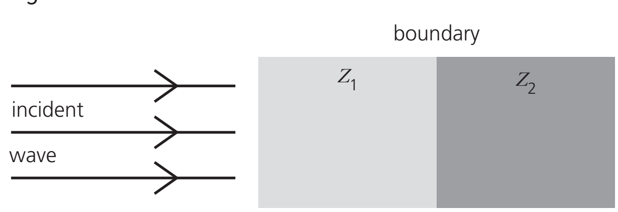
b the reflected signal received from deeper in the body is amplified more than that received from near the skin.

2 The specific acoustic impedance of fat, muscle and bone are $1.4 \times 10^6\,\text{kg m}^{-2}\text{s}^{-1}$, $1.6 \times 10^6\,\text{kg m}^{-2}\text{s}^{-1}$ and $6.5 \times 10^6\,\text{kg m}^{-2}\text{s}^{-1}$ respectively. The linear absorption coefficients in fat and in muscle are $0.24\,\text{cm}^{-1}$ and $0.23\,\text{cm}^{-1}$ respectively.

A parallel beam of ultrasound of intensity I is incident on the layer of fat. Discuss quantitatively, in terms of I, the reflection and the transmission of the beam of ultrasound as it passes through the layer of fat of thickness 4.0 mm, into the muscle of thickness 43.5 mm and finally into the bone.

3 **a** By reference to ultrasound waves, state what is meant by *acoustic impedance*.

b An ultrasound wave is incident on the boundary between two media. The acoustic impedances of the two media are Z_1 and Z_2, as illustrated in Fig. 24.29.



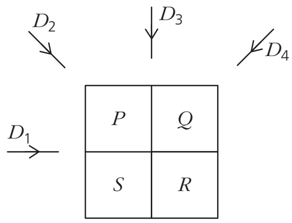
▲ **Figure 24.29**

Explain the importance of the difference between Z_1 and Z_2 for the transmission of ultrasound across the boundary.

4 During PET scanning, annihilation of a positron and an electron occurs. Suggest why the annihilation:

a occurs close to the point where the positron is created,

b results in two gamma-ray photons of equal energy moving in opposite directions.



▲ **Figure 24.30**

5 a Outline briefly the principles of CT scanning. **[5]**

b In a model for CT scanning, a section is divided into four voxels. The pixel numbers P, Q, R and S of the voxels are shown in Fig. 24.30.

The section is viewed from the four directions D_1, D_2, D_3 and D_4.

The detector readings for each direction are noted.

The detector readings are summed as shown below.

49	61
73	55

The background reading is 34.

Determine the pixel numbers P, Q, R, and S as shown below.

P	Q
S	R

[4]

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6 a State what is meant by a *radioactive tracer*.

b Explain why the annihilation of a positron in matter gives rise to two gamma-ray photons moving in opposite directions.

c Calculate the energy of one of the photons emitted as a result of positron annihilation.

25 Astronomy and cosmology

Learning outcomes

By the end of this topic, you will be able to:

25.1 Standard candles

1. understand the term luminosity as the total power of radiation emitted by a star
2. recall and use the inverse square law for radiant flux intensity F in terms of the luminosity L of the source $F = L/(4\pi d^2)$
3. understand that an object of known luminosity is called a standard candle
4. understand the use of standard candles to determine distances to galaxies

25.2 Stellar radii

1. recall and use Wien's displacement law $\lambda_{max} \propto 1/T$ to estimate the peak surface temperature of a star
2. use the Stefan–Boltzmann law $L = 4\pi\sigma r^2 T^4$
3. use Wien's displacement law and the Stefan–Boltzmann law to estimate the radius of a star

25.3 Hubble's law and the Big Bang theory

1. understand that the lines from the emission spectra from distant objects show an increase in wavelength from their known values
2. use $\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$ for the redshift of electromagnetic radiation from a source moving relative to an observer
3. explain why redshift leads to the idea that the Universe is expanding
4. recall and use Hubble's law $v \approx H_0 d$ and explain how this leads to the Big Bang theory (candidates will only be required to use SI units)

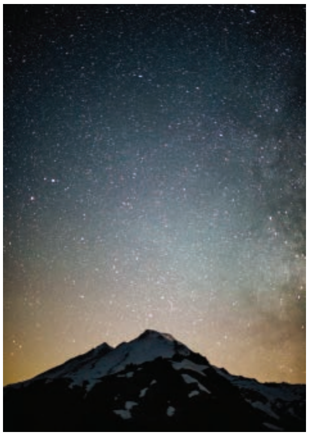
Starting points

- ★ When a source of waves moves relative to a stationary observer there is a change in the observed frequency (Doppler effect).
- ★ When a spectrum of continuous light passes through a cool gas the continuous spectrum is crossed by a series of darker lines (absorption spectra).

25.1 Standard candles

When we look at the stars at night, it is obvious that some of the stars are brighter than others. This may be because the star is giving out more light or because it is closer to us than other stars.

This effect can be illustrated in the laboratory using two lamps, one emitting 3 W of light and the other, 5 W of light. When they are the same distance from the observer, the 5 W lamp is the brighter (or more luminous) of the two. However, if the 3 W lamp is just 1 metre away from the observer and the 5 W lamp is 10 m away, the 3 W lamp would appear to be the brighter of the two. This is because the light spreads out as it moves away from the source.



▲ **Figure 25.1** The night sky

Luminosity

The **luminosity** of an object is the total power (the total energy emitted per unit time) of the object.

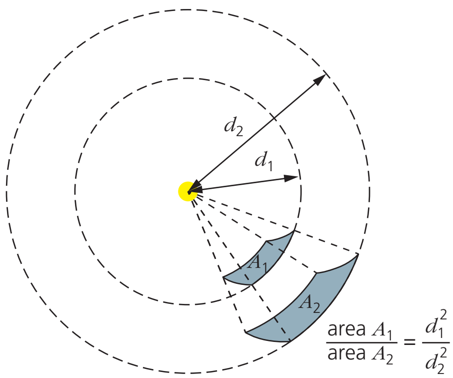
The object may be a star, a lamp, etc. This energy is emitted as electromagnetic radiation.

The unit of luminosity is the unit of power (W) which is also equivalent to the total energy emitted per second ($J\,s^{-1}$). The symbol for luminosity is L.

For example, the luminosity L of the Sun is 3.9×10^{26} W.

Radiant flux intensity (apparent brightness)

The energy emitted by a star moves out in all directions. This light illuminates an ever-increasing area as it moves out from the star (see Figure 25.2). If no energy is absorbed and the energy is emitted uniformly in all directions, then at distance d from the star, the energy will be spread over the surface area of a sphere. This area is given by the expression $4\pi d^2$ and since the total energy per second is the luminosity L, then the energy passing per second through unit area is given by $L/(4\pi d^2)$.



▲ **Figure 25.2** The inverse square law

The quantity $L/(4\pi d^2)$ is known as the **radiant flux intensity** F (or apparent brightness) and is given by the expression

$$F = L/(4\pi d^2)$$

The unit of F is $W\,m^{-2}$.

Radiant flux intensity is the radiant power per unit area passing normally through unit area.

WORKED EXAMPLE 25A

The luminosity of the Sun is 3.9×10^{26} W. The Earth orbits the Sun at a mean distance of 1.5×10^8 km. Calculate the radiant flux intensity of the Sun, near to the Earth.

Answer

$F = L/(4\pi d^2)$

$= (3.9 \times 10^{26})/(4\pi \times \{1.5 \times 10^{11}\}^2)$ [N.B. convert km to m]

$=$ **1400 W m^{-2}**

Question

1 The radiant flux intensity at Earth due to the Sun's radiation is 1400 $W\,m^{-2}$. The mean orbital radii about the Sun of Earth and of Mars are 1.5×10^8 km and 2.3×10^8 km respectively.

Determine the radiant flux intensity of the Sun at Mars.

Standard candles

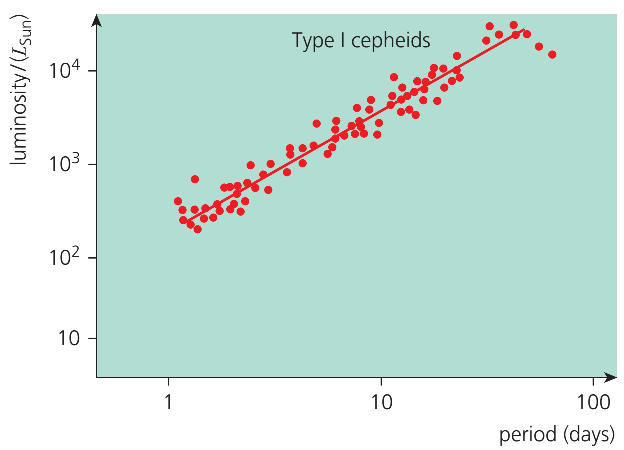
An important aspect of the work of astronomers is to determine the distance of stars and galaxies from Earth. A galaxy is a group of hundreds of millions of stars, stellar remnants, gas and dark matter, held together by gravity. Galaxies are so far from Earth that, to the naked eye, they appear either as a small speck of light or cannot be seen at all.

A standard candle is a class of stellar object which has a known luminosity and whose distance can be determined by calculation using its radiant flux intensity (apparent brightness) and luminosity.

To measure distance, the radiant flux intensity F on Earth (observed brightness) is measured. If the actual luminosity L of the star or galaxy can be found, then the distance can be calculated using the expression $F = L/(4\pi d^2)$. The difficulty is in the determination of the luminosity. One way in which this problem is overcome is the use of stars known as **Cepheid variables**.

A Cepheid variable star is a star whose radius varies periodically. The varying radius of the star causes the temperature of the star to change and consequently, the luminosity varies periodically. The period of this variation of luminosity (and also the star's brightness observed from Earth) ranges from 1 day to 100 days. Cepheid variable stars were first identified by Henrietta Swan Leavitt in 1908. Leavitt also discovered that more luminous Cepheids had longer periods, and other astronomers extended her work to show that there is a relationship between the period of the star's variation and its luminosity (see Figure 25.3).

Since all Cepheids of a given period have the same luminosity, a Cepheid's luminosity L can be estimated from the period of the variation of brightness. Hence the distance to the star is found after measuring F.



▲ **Figure 25.3 Relationship between luminosity and period for Cepheid variable stars. The luminosity is plotted as a multiple of the Sun's luminosity, L_{Sun}. Note both axes have a logarithmic scale.**

Using this technique, distances to Cepheid variables of up to 13 million light-years can be determined with Earth-bound telescopes. With space-bound telescopes, distances to Cepheid variables even further away can be determined. A Cepheid variable in a galaxy in the Virgo cluster was found to be 56 million light-years distant. Note that a light-year (ly) is the distance that a photon of light travels through space in one year i.e. 9.46×10^{15} m:

$$1.0\,\text{ly} = 3.0 \times 10^8 \times 365 \times 24 \times 60 \times 60$$
$$= 9.46 \times 10^{15}\,\text{m}$$

WORKED EXAMPLE 25B

The luminosity L of a Cepheid variable is estimated from its period to be 4.6×10^{15} W. Its radiant flux intensity (observed brightness) F measured on Earth is $1.3 \times 10^{-23}\ \mathrm{W\,m^{-2}}$.

Determine the distance of the Cepheid variable from Earth.

Answer

$$F = \frac{L}{(4\pi d^2)}$$

$$1.3 \times 10^{-23} = \frac{(4.6 \times 10^{15})}{(4\pi \times d^2)}$$

$$d^2 = 2.8 \times 10^{37}$$

$$d = \mathbf{5.3 \times 10^{18}\ m\ (560\ ly)}$$

Question

2 A Cepheid variable has a period of 10 days. Its radiant flux intensity (apparent brightness) as measured in Earth orbit is $1.4 \times 10^{-10}\ \mathrm{Wm^{-2}}$.

Use data from Fig. 25.3 to estimate, for the Cepheid variable:

a its luminosity

b its distance from Earth.

Other standard candles

Cepheid variables method does not work for galaxies that are a very long distance from Earth. Early observations were limited to Cepheid variables and to galaxies relatively close to our own galaxy. Other standard candles, such as some **supernovae** may be used for more distant galaxies.

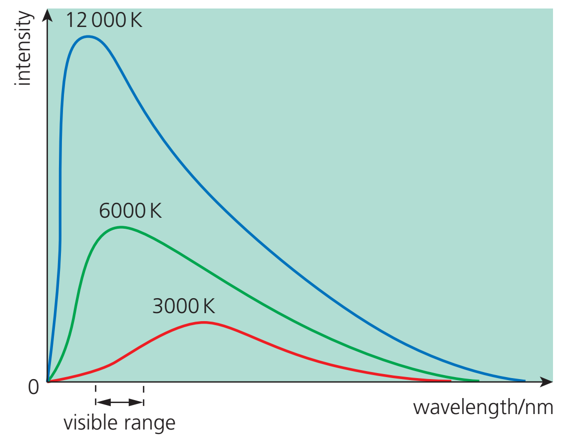
25.2 Stellar radii

When a steel rod is heated very strongly, at first it glows dull red. As heating continues, the brightness of the glow increases and the colour changes from dull red to orange. The rod is radiating energy to its surroundings as electromagnetic radiation. The brightness of the glow depends on the rate at which radiation is emitted (the intensity) and the colour seen depends on the relative intensities of the wavelengths of the emitted radiation.

Experiments show that at any particular temperature an object emits radiation with a *continuous range* of wavelengths, and that the intensity and the spread of wavelengths emitted depend on the temperature of the object.

The theoretical idea of a 'black body' was developed to explain the intensity of radiation of different wavelengths. The radiation emitted from an ideal black body is known as 'black-body radiation'. Stars behave approximately as black bodies. Studying the nature of black-body radiation in the laboratory improves our understanding of the nature of radiation from stars.

The variation with wavelength of the intensity of the electromagnetic radiation emitted from the surface of a black body (e.g. a star) at different temperatures is illustrated in Figure 25.4 (overleaf).



▲ **Figure 25.4** Variation of intensity of radiation with wavelength for a black body at temperatures of 3000 K, 6000 K and 12000 K

It can be seen that:

- at all temperatures, radiation is emitted over a continuous range of wavelengths
- the peak of the graph moves towards shorter wavelengths as the temperature increases
- the higher the temperature, the greater the power radiated.

The wavelength corresponding to the maximum intensity of emission at any temperature is given the symbol $\lambda_{max.}$ Clearly, λ_{max} depends on temperature.

Wien's displacement law

Wilhelm Wien (1864–1928) discovered a simple relationship between λ_{max} and the thermodynamic temperature T which is known as Wien's displacement law. Namely

$$\lambda_{max} \propto 1/T$$

or

$$\lambda_{max} = b/T$$

where b is known as Wien's displacement constant and is equal to 2.898×10^{-3} m K.

Thus, by measuring the wavelength of the peak intensity of radiation emitted from a star, its surface temperature may be determined.

WORKED EXAMPLE 25C

The wavelength of the peak intensity of radiation emitted by the Sun is 510 nm. Use Wien's displacement law to calculate a value for the surface temperature of the Sun.

Answer

$$\lambda_{max} = (2.898 \times 10^{-3})/T$$

$$510 \times 10^{-9} = (2.898 \times 10^{-3})/T$$

$$T = \mathbf{5700\,K}$$

Question

3 Rigel and Betelgeuse are two stars in the constellation of Orion. The wavelengths for the peak intensities of emission of radiation from Rigel and from Betelgeuse are 240nm and 878nm respectively.

Calculate the surface temperature of each of the stars.

(Wien's displacement constant = 2.898×10^{-3} m K.)

Stefan–Boltzmann law

The luminosity L of a star is the total energy emitted per second. On Figure 25.4, the luminosity increases as the area under the curve increases. As a result, it can be seen that the luminosity increases as the temperature rises. The observation that $L \propto T^4$ was formulated by Josef Stefan (1835–93) and is known as the Stefan–Boltzmann law.

For a spherical object of radius r emitting black-body radiation at thermodynamic temperature T, its luminosity L is given by the expression $L = 4\pi\sigma r^2 T^4$.

where σ is the **Stefan–Boltzmann constant**, equal to $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

Finding stellar radii

In Topic 25.1, we saw how, using standard candles, it is possible to determine the luminosity L of stars. Furthermore, the variation with wavelength of the intensity of the radiation emitted by a star can be measured and hence the wavelength λ_{max} giving rise to the maximum intensity can be found. Substituting λ_{max} into the expression for Wien's displacement law enables the surface temperature T of the star to be determined.

So, knowing the luminosity L of a star and its surface temperature T then substituting these values into the expression for the Stefan–Boltzmann law, a value for the radius of the star can be found. It should be remembered that the value is only an estimate, since an accurate value for the luminosity of the star cannot be obtained – only an estimate.

WORKED EXAMPLE 25D

Measurements taken of the star Sirius give its luminosity L as $1.6 \times 10^{28}\,\mathrm{W}$ with its intensity maximum at 290 nm. Determine a value for the radius of Sirius.

(Wien's displacement constant = $2.898 \times 10^{-3}\,\mathrm{mK}$; Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$.)

Answer

Use the expression $L = 4\pi\sigma r^2 T^4$ where the temperature T can be found using Wien's displacement law.

$$\lambda_{max} = b/T$$

$$290 \times 10^{-9} = 2.898 \times 10^{-3}/T$$

$$T = 1.0 \times 10^4\,\mathrm{K}$$

$$L = 4\pi\sigma r^2 T^4$$

$$1.6 \times 10^{28} = 4\pi \times 5.67 \times 10^{-8} \times r^2 \times (1.0 \times 10^4)^4$$

$$r = \mathbf{1.5 \times 10^9\,m}$$

(approximately 2.2 times radius of Sun)

Question

4 The red giant star Aldebaran has a luminosity of $1.95 \times 10^{29}\,\mathrm{W}$. The wavelength corresponding to its peak intensity of emission of radiation is 725 nm.

Calculate, to 2 significant figures, the radius of Aldebaran.

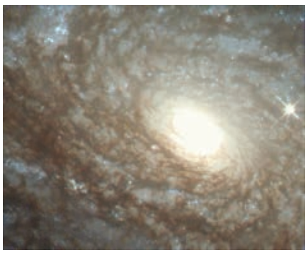
(Wien's displacement constant = $2.898 \times 10^{-3}\,\mathrm{mK}$; Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$.)

25.3 Hubble's law and the Big Bang theory

In Topic 22.4 we saw that when white light passes through a low-pressure gas and its spectrum is analysed, the continuous spectrum is crossed by a series of darker lines. This is known as an **absorption spectrum**. The darker lines correspond to the wavelengths of the **emission line spectrum** of the gas.

The spectrum of the light from a star is an absorption spectrum. The hot interior of the star emits white light and then this white light passes through the cooler outer layers of the star. On Earth, the absorption spectrum produced can be analysed to identify the elements in the star's outer layers.

Galaxies



▲ **Figure 25.5** A spiral galaxy

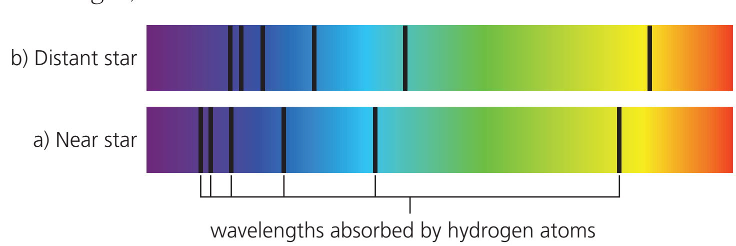
The Milky Way is a faint continuous band of light crossing the sky that can be seen on clear moonless nights. In 1750, Thomas Wright suggested that this band is, in fact, a very large collection of stars shaped like a convex lens. Such a collection of stars, held together by gravitational forces, is referred to as a **galaxy**.

The Milky Way galaxy, in which our Sun is just one star, contains approximately 10^{11} stars in the shape of a spiral. Such a galaxy is shown in Figure 25.5. The spiral of the Milky Way galaxy has a diameter of approximately 9.5×10^{20} m and a thickness of 1.9×10^{19} m. (Light travels 9.46×10^{15} m in one year.)

It is now known that many of the dots of light seen in the night sky are not stars in the Milky Way but are, in fact, very distant galaxies. With the advent of space telescopes, many more galaxies have been discovered that are so distant from Earth that they are only visible from space. Approximately 77% of galaxies are spiral in shape (see Figure 25.5). The majority of the remainder are elliptical. It is known that there are at least 10^{11} galaxies in the Universe with each galaxy containing approximately 10^{11} stars!

Redshift

It may be thought that the dark lines in the absorption spectrum from a star (or any other very hot astronomical object) produced by the light passing through cooler gaseous elements would correspond exactly to the lines in the emission spectrum of those elements in the laboratory. However, this is not the case. For all stars except the Sun, the dark lines all have a small but significant difference from the wavelengths of the emission spectrum of those elements in the laboratory, as illustrated in Figure 25.6. In the visible spectrum, wavelength increases as the colour changes from blue to red. When the spectral lines in the absorption spectrum are seen to have an increase in wavelength from their known values measured in a laboratory, the effect is known as **redshift**. If the spectral lines in the absorption spectrum are seen to have a decrease in wavelength, the effect is known as **blueshift**.



▲ **Figure 25.6** Simplified absorption spectrum for hydrogen for near and distant stars

Measurement of speed using redshift or blueshift

In Topic 7.2 it was seen that where a sound source moves relative to an observer, the observed frequency is different from the source frequency. This is known as the **Doppler effect**. Sound is a wave motion and so is light, the difference being that light has a much greater speed and much shorter wavelengths. However, the Doppler effect is observed with light waves.

Consider light of speed c emitted at frequency f_0 from a star or galaxy moving at speed v away from an observer. Then, in one second, f_0 waves will be emitted in a distance of $(c + v)$ towards the observer.

The apparent wavelength λ that is observed is given by

$\lambda = (c + v)/f_0$

and since $f_0 = c/\lambda_0$ where λ_0 is the wavelength of the light emitted from a stationary source, then

$\lambda = (c + v)/f_0 = \lambda_0 \times (c + v)/c$

$\lambda = \lambda_0 \times (1 + v/c)$

so, $\lambda = \lambda_0 + \lambda_0 \times v/c$

and $\lambda - \lambda_0 = \lambda_0 \times v/c$,

$(\lambda - \lambda_0)/\lambda_0 = v/c$

$(\lambda - \lambda_0)$ is the redshift $\Delta\lambda$ (or the blueshift) that is observed and so,

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta f}{f} = \frac{v}{c}$$

Comparison of wavelengths in the spectral lines of a star with the wavelengths determined in a laboratory enables the speed and direction of the star relative to Earth to be calculated. It should be remembered that only the velocity along the direction of sight can be found.

Where $\Delta\lambda$ is positive (the wavelength observed is greater than the wavelength measured in the laboratory) then redshift is seen to occur and the object is moving away from the observer. A negative value of $\Delta\lambda$ implies blueshift with the object moving towards the observer.

For objects that are a very long distance from Earth (such as quasars, discussed further below) their speeds are very large, approaching the speed of light.

For objects moving close to the speed of light the formula has to be corrected and the formula above is, therefore, approximate.

WORKED EXAMPLE 25E

One wavelength in the hydrogen spectrum of light from Ursa Majoris is 486.112 nm.

In the laboratory, this spectral line is found to have a wavelength of 486.133 nm.

Determine the velocity of Ursa Majoris relative to Earth.

The speed c of light in free space is $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.

Answer

Use the expression $\Delta\lambda/\lambda \approx v/c$

where $\Delta\lambda = (486.133 - 486.112) = 0.021\,\mathrm{nm}$

and $\lambda = 486.133\,\mathrm{nm}$

So, $0.021/486.133 = v/(3.0 \times 10^8)$

$v \approx 1.3 \times 10^4\,\mathrm{m\,s^{-1}}$

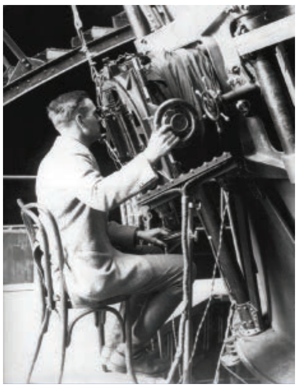
$\approx$ **$13\,\mathrm{km\,s^{-1}}$** towards Earth as the light is blue-shifted

Question

5 Measurements of the spectrum of the light from the galaxy NGC 4889 indicate a wavelength of 401.8 nm for the wavelength of the K-line of singly ionised calcium. In the laboratory, the wavelength is measured to be 393.3 nm.

What is the speed and direction of this galaxy relative to Earth?

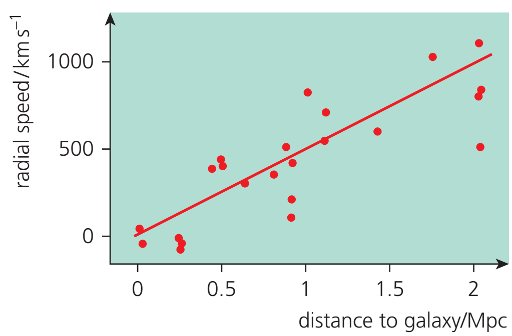
(The speed c of light in free space is $3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$.)



▲ **Figure 25.7** Edwin Hubble using a telescope at Mount Wilson Observatory, Pasadena, California

Hubble's law

Edwin Hubble (1889–1953) studied the absorption spectra of galaxies and calculated that most are red-shifted and, therefore, moving away from Earth. Using standard candle methods (see Topic 25.1) he determined the distances from Earth of some nearby galaxies for which the radial speeds from Earth had been calculated in the 1920s using the Doppler formula. Plotting a graph of radial speed v against distance d gave a straight line of best fit, as illustrated in Figure 25.8. This showed that the radial speed is proportional to distance.



◀ **Figure 25.8** Hubble's original data for radial speed against distance

That is

$$v \approx H_0 \times d$$

His conclusion is known as **Hubble's law** and the constant H_0, determined from the gradient of the graph, is known as the **Hubble constant**. Hubble's original data was for galaxies in the range 0 to 2 Mpc where 1 Mpc (1 megaparsec) is a distance unit used in astronomy that is equal to 3.09×10^{19} km. Recent observational data has extended Hubble's plot to galaxies at much greater distances.

The unit of the Hubble constant H_0 is normally expressed as $\text{km}\,\text{s}^{-1}$ per Mpc but in SI units H_0 has units ms^{-1} per m, which simplifies to s^{-1}.

There is much debate as to the value of the constant H_0 since it depends on the accuracy to which distances to galaxies can be measured and, hence, the uncertainty in the gradient of the graph of radial speed against distance. It is thought that the value of H_0 lies between $50\,\text{km}\,\text{s}^{-1}$ per Mpc and $100\,\text{km}\,\text{s}^{-1}$ per Mpc with the present accepted value being $75\,\text{km}\,\text{s}^{-1}$ per Mpc. This, in SI units, is approximately $2.4 \times 10^{-18}\,\text{m}\,\text{s}^{-1}$ per m.

WORKED EXAMPLE 25F

The light from a nearby galaxy is observed to be red-shifted, indicating a radial speed from Earth to be $1.2 \times 10^6\,\text{m}\,\text{s}^{-1}$. Assuming a value for the Hubble constant of $2.4 \times 10^{-18}\,\text{m}\,\text{s}^{-1}$ per m, estimate the distance from Earth of the galaxy.

Answer

Using Hubble's law, $v \approx H_0 \times d$,

$1.2 \times 10^6 = 2.4 \times 10^{-18} \times d$

$\boldsymbol{d \approx 5.0 \times 10^{23}\,\text{m}}$

(*Note*: Light takes approximately 1.7×10^{15} s to travel this distance which is 54 million years.)

Questions

6 Using redshift, the radial speed away from Earth of galaxy NGC 4889 was found to be $6500\,km\,s^{-1}$.

Using Hubble's law and assuming a value of $2.4 \times 10^{-18}\,m\,s^{-1}$ per m for the Hubble constant, determine the time taken, in years, for light to travel from NGC 4889 to Earth.

7 The Hubble constant has been determined from one recent set of measurements as $75\,km\,s^{-1}$ per Mpc. If $1\,Mpc = 3.09 \times 10^{19}\,km$, show that in SI units this is $2.4 \times 10^{-18}\,s^{-1}$.

Quasars

A mysterious object in the Universe is the **quasar**. Quasars are very distant objects that have very large redshifts and a huge luminosity of up to $10^{40}\,W$. This great luminosity enables the study of objects that are at great distances from Earth.

The expanding Universe

Einstein proposed that, on a large enough scale, the Universe is both homogeneous and isotropic. This is known as the **Cosmological Principle**. It means simply the Universe would have the same general appearance from anywhere else in the Universe as it appears from Earth.

The conclusion from redshift data is that nearly all galaxies are moving away from Earth, and that galaxies on the edge of the observable Universe have very large recession speeds, some close to the speed of light. Using the Cosmological Principle, Hubble's formula also leads to the important conclusion that every galaxy 'sees' every other galaxy receding from it, and the greater the separation of different objects (e.g. galaxies), the greater their speed of separation.

This means that the Universe is expanding – the space between each galaxy is expanding – and that this expansion has been taking place for billions of years.

Age of the Universe

Consider two galaxies that are moving apart with speed v. The galaxies are separated by a distance x. If this speed were to be reversed in direction, then the galaxies would approach one another and the time T taken before the galaxies collide is given by

$$T = x/v$$

Using Hubble's law,

$$v = H_0 x$$

and combining these two expressions,

$$T = x/H_0 x = 1/H_0.$$

Note that the distance of separation x has cancelled out in the final expression for T and so T is the same for all galaxies. This leads to the conclusion that T is the time when all galaxies were at the same point. That is, the time of the Big Bang.

Assuming that $H_0 = 2.4 \times 10^{-18}\,m\,s^{-1}$ per m, then

$$T \approx 4.2 \times 10^{17}\,s \approx 13 \text{ billion years.}$$

As already mentioned, there is doubt as to the actual value of H_0 and so the age of the Universe is thought to be between 11 billion and 20 billion years.

WORKED EXAMPLE 25G

The oldest stars that have been detected have an age of approximately 15 billion years.

Deduce a probable value for the Hubble constant based on this value.

Answer

Assume 15 billion years is the age of the Universe.

Then, 15×10^9 years $= 1/H_0$.

Using SI units

$15 \times 10^9 \times 365 \times 24 \times 3600 = 1/H_0$

$H_0 = \mathbf{2.1 \times 10^{-18}\,m\,s^{-1}}$ **per m**

Question

8 The distance between the Moon and the Earth is approximately 3.8×10^5 km. Calculate a value for the change per year in distance between the Moon and the Earth as a result of expansion of the Universe.
Assume the Hubble constant H_0 is $2.4 \times 10^{-18}\,m\,s^{-1}$ per m.

SUMMARY

- Luminosity is a measure of the total power of radiation emitted by an object.
- Radiant flux intensity (apparent brightness) varies inversely with the square of the distance from a source that may be considered as a point source, $F = L/(4\pi d^2)$.
- Objects of known luminosity are referred to as standard candles.
- Standard candles may be used to determine distances to galaxies.
- Wien's displacement law $\lambda_{max} = b/T$ enables the temperature of a black body to be determined
- The Stefan–Boltzmann law relates luminosity to temperature $L = 4\pi\sigma r^2 T^4$.
- Combining Wien's displacement law with the Stefan–Boltzmann law enables the radii of stars to be determined.
- The absorption spectra from distant objects shows redshift and, using the Doppler formula $\Delta\lambda/\lambda \approx v/c$, speeds of recession can be estimated.
- Redshift leads to the concept of an expanding Universe.
- Hubble's law $v \approx H_0 \times d$ leads to the Big Bang theory of the creation of the Universe.

END OF TOPIC QUESTIONS

Data

Speed of light in free space $c = 3.00 \times 10^8\,m\,s^{-1}$

Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,W\,m^{-2}\,K^{-4}$

Formulae

Stefan–Boltzmann law: $L = 4\pi\sigma r^2 T^4$

Doppler redshift: $\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$

1 **a** Distinguish between the luminosity L of a star and radiant flux intensity F.

b Explain how luminous intensity and radiant flux intensity may be used to determine distances to galaxies.

c A space probe is 1.4×10^9 km from the Sun. Power for the space probe is provided by solar panels that collect radiant energy from the Sun. The solar panels have a total area of $31\,m^2$ and are directly facing the Sun.
The luminosity of the Sun is 3.9×10^{26} W.
Calculate the radiant power incident on the panels.

2 a Explain how the intensity distribution curve of the radiant energy from a star may be used to estimate its peak surface temperature.

b The star Betelgeuse in the constellation of Orion is known as a red giant. It has a luminosity L of 3.9×10^{30} W. The wavelength λ_{max} of the peak intensity of radiation emitted from Betelgeuse is 970 nm.

Wien's displacement constant = 2.9×10^{-3} m K.

Use:

i Wien's displacement law to determine the surface temperature of Betelgeuse,

ii the Stefan–Boltzmann law to calculate the radius of Betelgeuse.

3 a By reference to the Doppler effect, explain what is meant by *redshift.*

b One wavelength in the hydrogen spectrum of light from the star Vega is observed to be 656.255 nm.

In the laboratory, the wavelength of this spectral line is measured to 656.285 nm.

What is the speed and direction of travel of Vega relative to Earth?

4 a Explain why redshift leads to the idea that the Universe is expanding.

b i State Hubble's law.

ii The redshift in the light from Cygnus A indicates a speed of recession from Earth of 1.7×10^4 km s^{-1}.

Assuming a value of the Hubble constant H_0 of 2.4×10^{-18} m s^{-1} per m, calculate the time taken, in years, for light to travel to Earth from the position where Cygnus A emitted the radiation.

5 a Explain what is meant by *redshift.*

b Light from a distant galaxy is shifted towards the red end of the spectrum. The amount of redshift is found to differ in different parts of the galaxy.

i State what can be deduced from the fact that the light is redshifted.

ii Explain why the amount of redshift differs.

6 The Stefan–Boltzmann law may be represented by the expression

$$L = \sigma A T^4$$

where σ is the Stefan–Boltzmann constant.

a Name the quantities represented by the symbols L, A and T.

b Show that the SI units of the constant σ are W m^{-2} K^{-4}.

7 The Earth, situated 1.5×10^{11} m from the Sun receives 1370 W m^{-2} of solar power.

The Sun has radius 7.0×10^8 m.

The Stefan–Boltzmann constant is 5.7×10^{-8} W m^{-2} K^{-4}.

Determine the surface temperature of the Sun.

26 Practical work

An important part of any Physics course is practical work. Practical work should help your understanding of the theoretical aspects of the course, as well as be exciting and interesting. The practical skills you gain should improve your understanding of the principles of scientific enquiry and scientific theory. The development of good practical skills will also assist you in future scientific courses or careers.

Throughout your Cambridge International AS & A Level Physics course, you will develop the following experimental skills:

1 manipulation, measurement and observation
2 presentation of data and observations
3 analysis, conclusions and evaluation
4 planning

During the course, you will have opportunities to develop these skills. As you become more experienced, you will learn how to use logarithms to test possible relationships and be able to analyse uncertainties in greater detail.

Safety

In all your practical work, you must consider appropriate safety precautions. You should always follow any general laboratory rules and procedures. For a specific practical activity, you should also consider any hazards and special precautions which are relevant to that activity. It is your responsibility to be constantly vigilant about potential hazards, not only for yourself but also for others in the laboratory.

Further guidance

The *Cambridge International AS & A Level Physics Practical Skills Workbook* is a write-in resource designed to be used throughout the course and provides you with extra opportunities to test your understanding of the practical skills required by the syllabus.

Expectations of experimental skills

1 Manipulation, measurement and observation

Throughout the course, there are a number of measuring instruments that you need to learn to use skilfully in order to take accurate measurements. Topic 1 gives information on a number of these instruments. You should be able to take measurements using apparatus such as a millimetre scale, protractor, top-pan balance, newton meter, analogue and digital electrical meters, measuring cylinder, calipers, micrometer screw gauge and thermometer. You should also be able to use a stop-watch to measure intervals of time, including the period of an oscillating system by timing an appropriate number of consecutive oscillations.

It is important that you are able to follow both written instructions and diagrams for setting out basic apparatus, including electrical circuits. The skills you develop will enable you to identify the best choice of instrument, as well as the type of measurements to take. For example, to minimise the uncertainty in the measurement of a time interval for the period of an oscillation, it is good practice to time a number n of oscillations so that the measured time t is at least 10 seconds. You can then use the following to determine the period T

$$T = \frac{t}{n}$$

You should understand that it is helpful to repeat measurements to reduce random errors. For example, to determine the diameter of a metallic wire, you may repeat the measurements of the diameter along the wire and then find the average diameter.

For any experiment, you will need to consider the number of measurements needed to give you reliable data. For example, if you expect a straight-line trend, you should take a minimum of six data measurements. If your graph is likely to be a curved trend, you should take at least nine sets of measurements.

You will also need to consider the *range* of measurements that will give you reliable data. For example, if you need to measure the resistance of a wire of length 85.0 cm, you could take six measurements of the resistance for lengths of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm 65.0 cm and 80.0 cm. These readings use the full length of the wire and are spread out. After plotting the graph of resistance against length, the resistance for 85.0 cm can be found. If the plotted points are scattered somewhat, you may decide to take further readings.

2 Presentation of data and observations

During your practical work, you will take many measurements and make many observations. Your data should always be presented in a concise form and be easy to understand. It is good practice to record your measurements and observations in a table. Using your data, you will usually then plot a graph to investigate the relationship between two quantities.

Table of results

You will be expected to record numerical data in a table of results. It is recommdended that you plan your table before you begin your practical work. When you start the experiment, you will then already have a plan for what you are going to do and it will be easier to record the data.

Your table should include columns for all the raw data, as well as columns for calculated data. If you are taking repeat measurements, there should be a column for each repeat and a column for the average mesaurement.

Column headings

Each column heading should include the quantity and an appropriate SI unit. The accepted scientific convention is to have the quantity and unit separated by '/' (see Figure 1.8 in Topic 1). For example, if a diameter d is measured in millimetres, the column heading would be written as d/mm. It is sometimes helpful to write the unit without the prefix, so diameter d measured in millimetres could be written as $d/10^{-3}$ m. Units should not be included in the main part of the table.

Recording measurements

You should record raw readings of a given quantity to the same degree of precision. For example, if you measure a length using a metre rule with a millimetre scale, all measurements should be recorded to the nearest millimetre, e.g. 20.0 cm and 23.4 cm are recorded to the nearest millimetre.

As a general rule, all raw measurements should be recorded to the same number of decimal places. This is because the number of decimal places in data gives information about the accuracy with which the measurements were taken. For example, if you measure a length using a metre rule with a millimetre scale, then 20 cm would not be an acceptable measurement because this implies you measured to the nearest centimetre,

▲ Figure 26.1 Ammeter reading

i.e. the measurement could be between 19.5 cm and 20.5 cm. Similarly, 23.40 cm would not be acceptable either, as this implies you have measured to the nearest 0.1 mm, which is not possible with a millimetre scale. A reading of 20.0 cm indicates that the measurement is to the nearest millimetre, i.e. the measurement could be between 19.95 cm and 20.05 cm.

Figure 26.1 shows an ammeter reading. The reading of 1.36 A shows that the current was measured to the nearest 0.01 A.

Calculating values

In a table of results, you may need to calculate other quantities. When a value is calculated from measured quantities, the 'appropriate number of significant figures' depends on the measured quantity with the *least* number of significant figures. The calculated value should have the same number of significant figures (or one more) than the number of significant figures in the quantity with the least significant figures. (See the Maths Note on significant figures in Topic 1, page 3.)

For example, if the current I = 1.36 A and the current squared (I^2) needs to be calculated,

$$I^2 = 1.36^2 = 1.8496\,\text{A}^2$$

The column heading for this would be I^2/A^2.

Since I is given to 3 significant figures, then I^2 should also be given to 3 significant figures (or one more), i.e. 1.85 A^2 or 1.850 A^2 would be appropriate.

Similarly, to calculate resistance R from current I and potential difference V, for the values I = 0.24 A and V = 4.52 V,

$$R = \frac{V}{I} = \frac{4.52}{0.24} = 18.8333333\,\Omega$$

Since I is given to 2 significant figures and V is given to 3 significant figures, R should be given to 2 significant figures (or one more), i.e. 19 Ω or 18.8 Ω are acceptable.

Example

In an experiment, a student investigated how the extension of a rubber cord varied with the diameter of the cord. The length of each cord was 80.0 cm. A mass of 500 g was attached to each cord. The student measured and recorded the diameter d of each cord using a caliper and determined the extension e of each cord using a metre rule.

The student then calculated values of $\frac{1}{d^2}$ and completed a table of results, as shown in Table 26.1.

Note the unit for $\frac{1}{d^2}$.

The caliper measured d to the nearest 0.1 mm.

Since e was measured using a metre rule, all values are given to the nearest millimetre. 1 cm or 1.00 cm would be incorrect.

d/mm	$\frac{1}{d^2}$/mm^{-2}	e/cm
1.0	1.0	4.2
1.2	0.69	2.9
1.4	0.51	2.1
1.6	0.39	1.6
1.8	0.31	1.3
2.0	0.25	1.0

▲ Table 26.1 Table of results for how extension e varied with diameter d of a rubber cord

Values of $\frac{1}{d^2}$ were calculated. Since d was recorded to 2 significant figures, $\frac{1}{d^2}$ should also be calculated to 2 (or 3) significant figures. For the first row, 1.00 would also be acceptable, but not 1.

Graph

A graph enables experimental results to be displayed so that further information may be obtained. Similar to a table of results, a graph should always be clear. It is recommended that you use a sharp pencil and a transparent 30 cm ruler.

Axes

Both the *x*-axis and *y*-axis should be labelled with the quantity and unit following accepted scientific convention (see Figure 1.8 in Topic 1). It is easy to do this by copying the relevant column heading from your table of results.

It is important that you are able to easily plot the data on your graph and read the data on your graph. Simple scales should be used on each axis. Each 2 cm square should have a value and each 2 cm square should increase in 1, 2 or 5 units. Examples of simple scales include 0.2, 0.4, 0.6, etc. (increasing each time by 0.2) and 50, 100, 150, etc. (increasing by 50).

The scale on each axis should allow the plotted points to occupy at least half the graph grid in both the *x*-direction and the *y*-direction.

Plotting of points

You should plot the points using a sharp pencil. Indicate each point using a cross or an encircled dot. The diameter of each point should be less than 1 mm. The data should be plotted to better than 1 mm.

Before you draw a trend line, it is a good idea to check that the points do follow a trend. Any points that do not obviously follow the trend should be re-checked and re-plotted if necessary. This could indicate an incorrect measurement. It is good practice to repeat such practical measurements.

Trend line

A trend line effectively averages your results and makes an allowance for random errors. A trend line should show an even distribution of points on either side of the line along its whole length. The line may not necessarily pass through all the points. Drawing the line from the top point to the bottom point may not result in a balanced line.

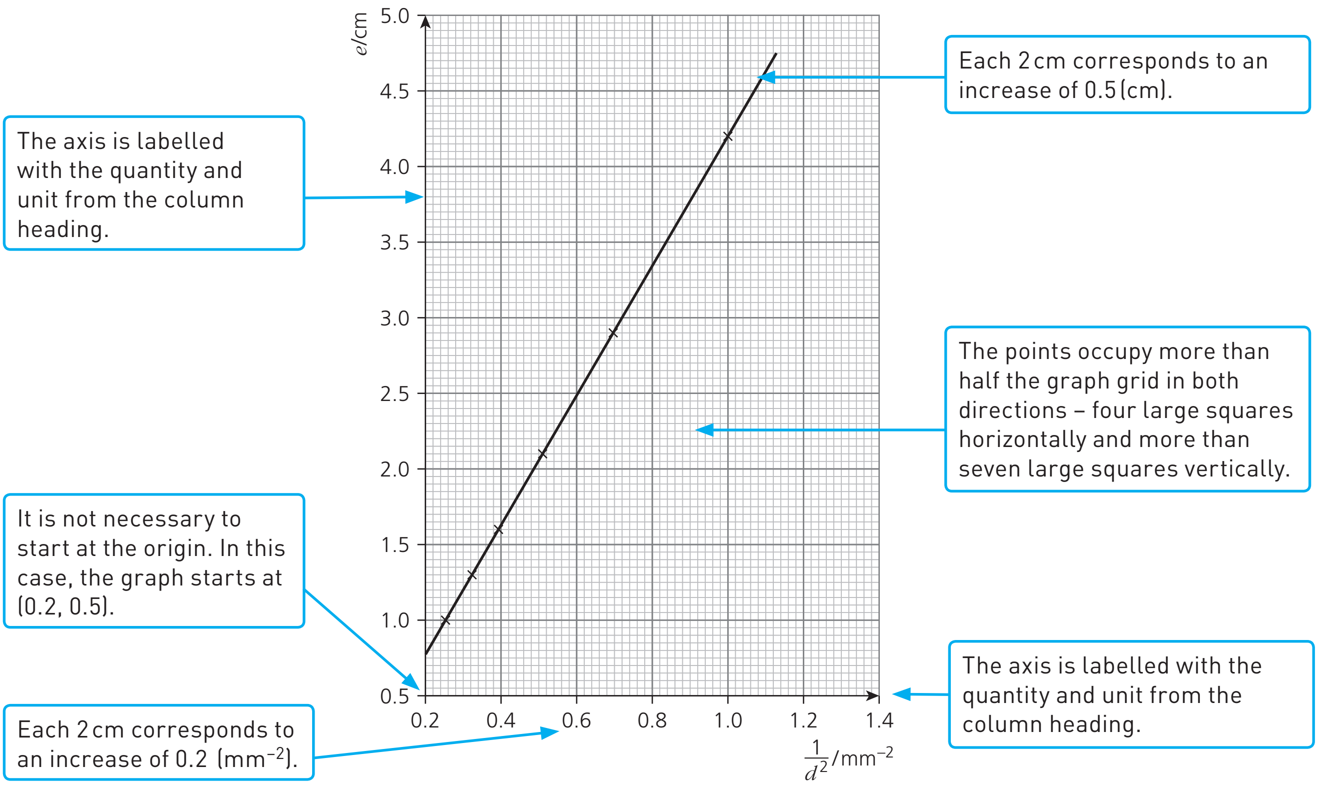
Again, use a sharp pencil so that the thickness of the line is less than 1 mm. To draw a straight line, use a transparent 30 cm ruler so that you can see all the plots and to ensure you do not have any bumps in the line.

When you plot your data, if one point does not follow the trend, circle this point and label it 'anomalous'. When you draw the trend line, you should ignore this anomolous point.

For curved trends, you will need to draw a smooth curve. It is best to rotate the graph paper so that the arc follows naturally with the movement of the pencil. Again, the curved line may not pass through every point. After you have drawn your curve, you may need to draw a tangent to the curve at a given point.

Example

The graph in Figure 26.2 shows the data plotted from Table 26.1.



▲ **Figure 26.2 Graph shows how extension *e* varied with diameter *d* of a rubber cord**

3 Analysis, conclusions and evaluation

It can be useful to interpret a graph in order to determine the conclusion for an experiment. Straight-line graphs are helpful for determining relationships. The equation of a straight-line, $y = mx + c$, where m is the gradient and c is the y-intercept, may be used to compare physical quantities. (See the Maths Note on straight-line graphs in Topic 2, page 38.)

Interpretation of a graph

You should be able to read data from a graph. You should read graph data to better than 1 mm, in the same way that you plotted the data. It is helpful to read data from clear intersections on the graph paper – this may not be the same as data points plotted from the table.

Straight-line graphs are useful because quantities plotted on the x-axis and y-axis can be related to the equation of a straight line

$$y = mx + c$$

where m is the gradient and c is the y-intercept.

For example, consider the relationship between the length L of a metallic wire and the current I in an electrical circuit

$$\frac{1}{I} = \frac{K}{E}L + \frac{r}{E}$$

where E is the electromotive force (e.m.f.), and K and r are constants.

When a graph of $1/I$ (y-axis) is plotted against L (x-axis), then from the equation for this relationship,

$$\text{gradient} = \frac{K}{E}$$

$$y\text{-intercept} = \frac{r}{E}$$

If the e.m.f. is measured, the constants may be determined using the gradient and y-intercept by

$$K = E \times \text{gradient}$$

$$r = E \times y\text{-intercept}$$

Units

In the example above, if I is measured in A, L is measured in cm and E is measured in V, then

the unit of $\frac{1}{I}$ is A^{-1}

the unit of K is $V \times A^{-1}\,cm^{-1}$ or $V\,A^{-1}\,cm^{-1}$ or $\Omega\,cm^{-1}$

the unit of r is $V \times A^{-1}$ or $V\,A^{-1}$ or Ω

Determining the gradient and y-intercept

To determine the gradient from a straight line or tangent to a curve, you should look for two data points (x_1, y_1) and (x_2, y_2) that are on the trend line. The points should be easy to read from the graph grid with either or both the x- and y-values coinciding with grid lines. The two chosen points on the line should be separated by more than half the length of the line drawn.

The gradient is calculated by the following expression

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

You should include the powers of ten included in labels for each axis.

To determine the y-intercept from a straight line where the x-axis begins at the origin, the y-intercept may be read directly from the y-axis. The reading of the data point should be better than 1 mm.

To determine the y-intercept c from a straight line where the x-axis does *not* begin at the origin, the coordinates of a point on the trend line and gradient should be substituted into the equation of a straight line

$$y = mx + c$$

$$c = y - mx$$

Example calculations of gradient and y-intercept

For the graph in Figure 26.2, the gradient could be determined as follows.

Choose two points: (0.40, 1.65) and (1.10, 4.65). These two data points are more than half the length of the line apart. The line crosses grid lines so it is easy to read the values.

$$\text{gradient} = \frac{4.65 - 1.65}{1.10 - 0.40} = \frac{3.00}{0.7} = 4.29$$

Since the x-axis does not start at the origin (it starts at 0.2), the y-intercept should be determined by substituting the coordinates of a point on the trend line and the gradient into the equation of a straight line, $y = mx + c$. A simple way to do this is to use one of the data points from the gradient calculation.

For Figure 26.2,

$$c = y - mx = 1.65 - 4.29 \times 0.40 = -0.066$$

Alternatively,

$$c = 4.65 - 4.29 \times 1.10 = -0.069$$

You should not worry about the difference between the two values. This is likely to be due to differences in rounding.

For the data given in Table 26.1 and the graph in Figure 26.2, the suggested relationship is

$$e = PML\frac{1}{d^2} + Q$$

where M is the mass attached to the cord, L is the length of the cord, and P and Q are constants.

Comparing this equation to the equation of a straight line,

$$\text{gradient} = PML$$

$$y\text{-intercept} = Q$$

Using the data given, it is possible to determine values with appropriate units for the constants P and Q.

$$Q = y\text{-intercept} = -0.066\,\text{cm (or } -0.069\,\text{cm)}$$

$$Q = -6.6 \times 10^{-4}\,\text{m}$$

Note that the unit (cm) is the same as the y-axis label.

To determine P, the expression for the gradient needs to be rearranged to

$$P = \frac{\text{gradient}}{ML} = \frac{4.29}{0.500 \times 0.800} = 10.7$$

However, we also need to consider the units. The extension e was measured in centimetres and the diameter was measured in millimetres. The gradient needs to be changed to a consistent unit.

Using metres for both the diameter and extension gives

$$\text{gradient} = \frac{3.00 \times 10^{-2}}{0.7 \times 10^{6}} = 4.29 \times 10^{-8}\,(\text{m}^3)$$

This means that $P = 10.7 \times 10^{-8}\,\text{m}^2\,\text{kg}^{-1}$.

To determine the unit of a constant, you need to know the commonly used prefixes (see Table 1.5 on page 4) and be confident in using derived units (see Topic 1).

Estimating uncertainties

All measurements have an uncertainty. The uncertainty of a measurement gives an indication of the range of values within which the measurement is likely to lie. The uncertainty depends on the measuring instrument used and how the measurement is taken.

To estimate the uncertainty in a measurement, the precision of the instrument is often used. For example, in Table 26.1 the diameter d of the cord is measured using a caliper which measures to the nearest 0.1 mm. This means that the first measurement of d could be recorded as 1.0 ± 0.1 mm. The '± 0.1' is the absolute uncertainty.

Table 1.7 in Topic 1 indicates some examples of the uncertainty in different measuring instruments.

Fractional uncertainty and percentage uncertainty

It is often useful to determine the fractional uncertainty or percentage uncertainty.

$$\text{fractional uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}}$$

$$\text{percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100$$

When estimating the absolute uncertainty in a measurement, the method of measurement needs to be considered. In order to determine the circumference of a cylinder, a string may be wrapped tightly around the cylinder. Then, if there are N turns of string and the length of string for these N turns is L, the circumference is L/N. The uncertainty in the result depends on the thickness of the string as well the divisions on the rule. Clearly, the string should be thin, i.e. use thread.

Determining the uncertainty in repeated readings

There will be many occasions when it is sensible to take repeat readings and average the results. In experiments to determine the period of an oscillating system, it is common for the timing to be repeated to reduce random errors. Other instances could be repeating measurements of the diameter of a wire spirally along the length of wire. In this case, the absolute uncertainty in a repeated measurement is half the range of the repeated readings. For example, if the diameter of a wire is measured five times along its length as 0.93 mm, 0.87 mm, 0.95 mm, 0.91 mm and 0.89 mm, then

$$\text{mean diameter} = \frac{0.93 + 0.87 + 0.95 + 0.91 + 0.89}{5} = 0.91\,\text{mm}$$

$$\text{half the range} = \frac{\text{maximum value} - \text{minimum value}}{2} = \frac{0.95 - 0.87}{2} = 0.04\,\text{mm}$$

Thus, the mean diameter of the wire is 0.91 ± 0.04 mm.

Drawing conclusions

Sometimes there is a limited number of results and it is not possible to plot a graph. In these cases, it often still possible to determine the value of a constant.

For example, a student investigated the period T of a simple pendulum for a length L of the pendulum to determine the acceleration of free fall g. The student then repeated the experiment for a different length. The results were

$L_1 = 80.2\,\text{cm}$ and $T_1 = 1.79\,\text{s}$

$L_2 = 44.9\,\text{cm}$ and $T_2 = 1.35\,\text{s}$

It is suggested that the relationship between T and L is

$$T^2 = \frac{4\pi^2 L}{g}$$

The acceleration of free fall can be determined for each of the two sets of data by

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 \times 80.2}{1.79^2} = 990\,\text{cm}\,\text{s}^{-2} \quad \text{and} \quad g = \frac{4\pi^2 \times 44.9}{1.35^2} = 970\,\text{cm}\,\text{s}^{-2}$$

To determine whether this relationship containing a constant is supported by experimental data, the percentage difference between the two values of the constant should be calculated and compared with a calculated percentage uncertainty.

To calculate percentage difference,

$$\text{percentage difference} = \frac{\text{difference between two values}}{\text{one of the values}} \times 100$$

or

$$\frac{\text{difference between two values}}{\text{average value}} \times 100$$

Thus,

$$\text{percentage difference} = \frac{20}{990} \times 100 = 2\%$$

or

$$\text{percentage difference} = \frac{20}{970} \times 100 = 2\%$$

or

$$\text{percentage difference} = \frac{20}{980} \times 100 = 2\%$$

If the percentage uncertainty in this experiment was 5%, the conclusion could be that the experimental data supports the relationship because the percentage difference is less than the percentage uncertainty.

Identifying limitations and suggesting improvements

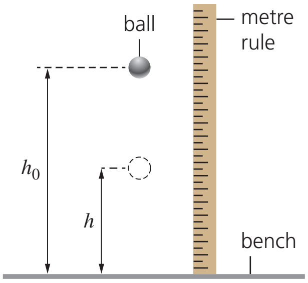
As you carry out practical work, you should be continually evaluating the procedures and measurements so that your data and conclusions are reliable. You need to think about how the accuracy of a particular experiment could be improved.

This means that you need to consider whether you are using appropriate measuring instruments. For example, you might use a rule with a millimetre scale to measure the internal diameter of a beaker. One possible limitation of this procedure is that you are only measuring to the nearest millimetre. Another possible limitation is that it is difficult to judge the inside of the beaker by eye. A possible improvement to both of these limitations would be to use a caliper measuring to the nearest 0.1 mm or a travelling microscope.

You should consider the uncertainty in your measurements. For example, when measuring length to the centre of thick string, the uncertainty in the measurement may be large and the percentage uncertainty will also be large. To improve the procedure, it may be possible to use some thinner string.

A simple method to identify limitations is to consider each quantity that is being measured or determined. For each measurement, consider the uncertainty in the measurement and how easy or difficult it was to take that particular measurement. When you have identified the limitations in the measurement, you should then use your practical experience to suggest improvements.

The limitations and improvements will depend on each experiment and will require you to consider the practical skills that you have gained throughout the course. The example below is an illustration only, to help you understand how you may develop your skills in identifying limitations and suggesting improvements to experiments.



▲ Figure 26.3

Example

Consider an experiment to investigate the height that a ball bounces from a surface, as shown in Figure 26.3.

1 The ball is dropped from a height h_0 of 40.0 cm. The height of the bounce is h.
2 A metre rule with a millimetre scale is held by hand to measure both h_0 and h.
3 The experiment is repeated for the height of 40.0 cm.
4 Steps 1, 2 and 3 are then repeated for a height of 80.0 cm.

It is suggested that $h = kh_0$, where k is a constant.

There are a number of limitations in this experiment and some possible improvements.

Both the measurement of h_0 and h depend on the metre rule. It is difficult to keep a metre rule straight by hand, so a possible solution would be to use a clamp stand. Also, the metre rule may not be vertical so another improvement could be to place a set square between the bench and metre rule. It would be helpful to sketch these improvements.

Consider the measurement of h_0. One limitation is that it is difficult to judge the exact starting position of the ball. A possible improvement could be to have a horizontal bar attached to the rule. You would also need to measure the diameter of the ball so that you could identify the starting position.

It is difficult to judge the maximum height h of the bounce height. An improvement to this would be to set up a video camera level with the approximate bounce height, with the metre rule in view, so that the height can be recorded. The recording can then be played back in slow-motion and then frame-by-frame as the ball reaches its maximum bounce height. Alternatively, if a camera is not available, a set-square moved along the edge of the rule so that the set-square is level with the maximum position would be an improvement.

Since the bounce height h is small and the uncertainty is large, the percentage uncertainty will be high. The experiment may be improved by reducing the percentage uncertainty by increasing the initial height h_0, so that the bounce height is greater.

In this experiment, only two readings are taken for each height, which is not sufficient to determine a valid value of k. An improvement would be to take many more pairs of measurements of h_0 and h, and to plot a graph of h on the y-axis and h_0 on the x-axis so that k is equal to the gradient.

During the experiment, the ball may bounce from the bench to the floor which could cause a trip hazard. An improvement would be to use a box so that the ball does not fall onto the floor.

4 Planning

Planning is a higher-order skill that you will develop throughout the course. As you gain more practical experience, you will learn when and how to use laboratory equipment to take measurements and make observations. Practical work also requires you to understand how you can present your measurements and observations so that you can analyse your experimental data. In order to be able to plan experiments and investigations successfully, you will need to practise writing a plan and then carrying out your plan. You will then need to review your plan to see what additional details could have been included.

There are four key stages to planning an experiment or investigation.

Stage 1 – Defining the problem

Initially, you will need to determine the purpose of your experiment or investigation. You may have a possible relationship to test and/or quantities to determine.

You should identify the quantity you will vary – this is called the **independent variable**. You will then need to identify the quantity that will be measured as a result of changing the independent variable. This quantity is called the **dependent variable**. Next, to ensure that your experiment is fair, you will need to identify the quantities that need to remain constant.

For example, in the earlier example, a student investigated how the extension e of a rubber cord varied with the diameter d of the cord. A mass of 500 g was attached to each cord. The length of each cord was 80.0 cm. In this experiment, the independent variable was the diameter of the cord – this quantity was varied. The dependent variable was the extension – this quantity was measured due the variation in the diameter.

The quantities that were kept constant, to ensure a fair test, were the mass attached to the cord, the length of the cord and the material of the cord.

Stage 2 – Method

Once you understand the experiment, you should describe it in detail so that another person could follow the plan and carry out the experiment safely. The plan must include a description of the method for varying the independent variable. It must also explain how the independent and dependent variables will be measured.

The description of the procedure should be clear and easy to follow. You should include suitable measuring instruments with an appropriate precision. To improve the description, include a clear, labelled diagram of how the apparatus should be assembled, e.g. a circuit diagram to indicate how an oscilloscope is connected. The diagram can include measurements that need to taken. For example, in an experiment to measure the extension of an object, the diagram may include a label L_0 on a rule to indicate the original measurement and a label L_1 to indicate the final measurement of length. The statement that extension = $L_1 - L_0$ should also be included.

In the description, you should include the method for how to measure other quantities that are needed for the final analysis. For example, an investigation may need to know the density of a liquid. In this case, the description should include how the mass of the liquid is measured and how the volume of the liquid is measured, as well as how the density is determined.

The description should also include details of any preliminary experiments that need to be carried out, or perhaps any calibration curves that need to be obtained.

It is often helpful to use technical equipment to measure quantities. For example, Topic 21 described how oscilloscopes are often used to measure voltages and times. From the measurements of voltage and period on an oscilloscope, you should also be able to describe how an oscilloscope can be used to determine current and frequency. You may suggest the use of datalogging equipment, such as light gates and motion sensors. Again, the plan should include a detailed description of the position of the equipment and any measurements that need to be made to determine the required quantity. To measure quantities that change very quickly, it may be appropriate to use a video camera with the ability to playback in slow motion or frame by frame. In such cases, the plan should include the position of the video camera.

Stage 3 – Analysis

In this stage, you should identify how the measurements and observations you are planning to take can be analysed. Usually this will be by plotting a graph. Ideally, the graph should be a straight-line graph. This can be achieved by rearranging the proposed relationship into the equation of straight line, $y = mx + c$. (See the Maths Note on straight-line graphs in Topic 2, page 38.)

In your plan, you should state explicitly the quantities that need to be plotted on the x-axis and the y-axis. Your plan should include an explanation of how any constants may be calculated using the gradient and/or y-intercept of the proposed graph. This could indicate that further measurements may be needed.

For example, a student investigated how the period T of a mass-spring oscillator varied with mass m of the oscillator. It is suggested that the relationship between T and M is

$$T = A\sqrt{\frac{m}{k}}$$

where k is the spring constant and A is a constant.

The student decided to carry out an experiment to determine the value of constant A. The student identified that the mass m was the independent variable and the period

T was the dependent variable. The student also identified that, for the experiment to be fair, k must be kept constant – the same spring must be used, so that the spring constant remains the same. The student planned an experiment to vary m and measure T, and described an experiment to determine the spring constant k.

To determine A, the student rearranged the equation by squaring both sides to give

$$T^2 = \frac{A^2}{k} m$$

Comparing this with the equation of a straight line, $y = mx + c$, the student plotted a graph of T^2 on the y-axis and m on the x-axis.

Comparing the above expression with the equation of a straight line means that

$$\text{gradient} = \frac{A^2}{k}$$

$$y\text{-intercept} = 0$$

Rearranging the gradient with A as the subject gives

$$A = \sqrt{k \times \text{gradient}}$$

Note, the student could have plotted a graph of T on the y-axis and $\sqrt{m}$ on the x-axis, but it is better not to calculate square roots.

When you plan an experiment, you should be able to test relationships of the following forms:

- $y = mx + c$, where m is the gradient and c is the y-intercept
- $y = ax^n$, where a and n are constants
- $y = ae^{kx}$, where a and k are constants (an exponential relationship)

Please see the section below on *Testing relationships and the use of logarithms*.

Stage 4 – Safety considerations

When you plan an experiment, you should consider any specific hazards that could become a danger to you or another person in the laboratory. It is good practice to carry out a safety risk assessment for all activities. This risk assessment should include an explanation of why each safety precaution is needed and how the precaution helps to minimise the risk. Examples include the use of goggles to prevent a metallic wire damaging a person's eyes if the wire should snap and switching off a 'high voltage' power supply when not in use to prevent danger of electrocution.

Testing relationships and the use of logarithms

Ideally, to test a proposed relationship, the plotted graph should produce a straight line. However, throughout the course, you will encounter relationships that do not follow the format of the straight line equation, $y = mx + c$. One solution to this is to use logarithms.

Relationships of the form $P = aQ^n$

One type of experiment where using logarithms is useful is when you need to find the relationship between two variables, P and Q, and the relationship between P and Q is of the form

$$P = aQ^n$$

where a and n are constants.

Taking logarithms to base 10 of both sides of the equation

$$\lg P = \lg a + n \lg Q \qquad \text{or} \qquad \lg P = n \lg Q + \lg a$$

Comparing this with the equation of a straight line, a graph of $\lg P$ on the y-axis and $\lg Q$ on the x-axis will give

gradient = n

y-intercept = $\lg a$

Thus, to determine a,

$a = 10^{y\text{-intercept}}$

Relationships of the form $Y = Y_0e^{-bx}$

Another type of experiment where you might be asked to find the relationship between two variables, Y and x, is when the relationship between Y and x is of the form

$Y = Y_0e^{-bx}$

where Y_0 and b are constants.

This type of relationship occurs, for example, in Topic 19.3 *Discharging a capacitor*, Topic 23.2 *Radioactive decay* and Topic 24.2 *Production and use of X-rays*.

Taking natural logarithms (to base e) of both sides of the equation

$\ln Y = \ln Y_0 - bx$ or $\ln Y = -bx + \ln Y_0$

Comparing this with the equation of a straight line, a graph of $\ln Y$ on the y-axis and x on the x-axis will give

gradient = $-b$

y-intercept = $\ln Y_0$

Thus, to determine Y_0,

$Y_0 = e^{y\text{-intercept}}$

Significant figures in logarithms

Consider a length L measured with a rule with a millimetre scale.

If L = 44.6 cm then $\lg(L/\text{cm})$ = 1.649334 …

Note that $\lg(L/\text{cm})$ is used, rather than $\lg L/\text{cm}$, because the logarithm of a quantity does not have a unit. This is how the quantity should be written in a table of results and on a graph axis.

It is also possible to determine $\lg(L/\text{mm})$.

If L = 446 mm then $\lg(L/\text{mm})$ = 2.649334.

In this case, the digit before the decimal place has changed. In both cases, L has been recorded to 3 significant figures. For logarithmic quantities, the number of decimal places should correspond to the number of significant figures. The number of decimal places in a logarithmic quantity should be the same as (or one more than) the number of significant figures in the raw data.

In this example, if L = 44.6 cm then $\lg(L/\text{cm})$ should be recorded in a table of results as 1.649 (or 1.6493).

Advanced treatment of uncertainties

Throughout your course, you will have estimated the (absolute) uncertainty in a measurement and calculated the fractional uncertainty or percentage uncertainty from an absolute uncertainty. To convert a fractional uncertainty or percentage uncertainty to an absolute uncertainty, the reverse method is used.

$$\text{absolute uncertainty} = \frac{\text{percentage uncertainty}}{100} \times \text{quantity}$$

$$= \text{fractional uncertainty} \times \text{quantity}$$

For example, if a potential difference V measurement is 9.2 V ± 2%, the absolute uncertainty is given by

$$\text{absolute uncertainty} = \frac{2}{100} \times 9.2 = 0.1844$$

Thus, V = 9.2 ± 0.2 V.

Combining uncertainties

There are two simple rules for combining uncertainties (see Topic 1):

1. For quantities which are added or subtracted to give a final result, add the absolute uncertainties.
2. For quantities which are multiplied together or divided to give a final result, add the fractional or percentage uncertainties.

Note that rule 2 applies to quantities that are raised to a power. For example, consider the length L of the side of a cube. The volume $V = L^3 = L \times L \times L$. If L = 25 ± 1 mm, then

$$\text{percentage uncertainty in } L = \frac{1}{25} \times 100 = 4\%$$

$$\text{percentage uncertainty in } L^3 = 4\% + 4\% + 4\% = 3 \times 4\% = 12\%$$

Thus, the volume is 25^3 = 15 625 mm^3 ± 12%, which is written as 16 000 ± 2000 mm^3.

In general, if the percentage uncertainty in X is P, then the percentage uncertainty in X^n is nP.

To combine the uncertainties where several quantities are raised to a power, for example $x = Ay^a z^b$, where A is a constant, the rule is to multiply the fractional uncertainties by the power, so

$$\frac{\Delta x}{x} = a \times \frac{\Delta y}{y} + b \times \frac{\Delta z}{z}$$

Similarly, the percentage uncertainty in x is given by

$$\frac{\Delta x}{x} \times 100 = a \times \frac{\Delta y}{y} \times 100 + b \times \frac{\Delta z}{z} \times 100$$

Maximum and minimum method to determine uncertainties

It is possible to work out absolute uncertainties by using maximum and minimum methods. In the above example, the maximum volume V_{max} and the minimum volume V_{min} are given by

$$V_{max} = (25 + 1)^3 = 17\,576$$

$$V_{min} = (25 - 1)^3 = 13\,824$$

The absolute uncertainty in V could be given by

$$\text{absolute uncertainty} = V_{max} - V = 17\,576 - 15\,625 = 1951 \approx 2000$$

or

$$\text{absolute uncertainty} = V - V_{min} = 15\,625 - 13\,824 = 1801 \approx 2000$$

or

$$\text{absolute uncertainty} = \frac{(V_{max} - V_{min})}{2} = \frac{(17\,576 - 13\,824)}{2} = 1876 \approx 2000$$

This method is often used when completing the uncertainties in calculated quantities in a table of results. Uncertainties in final answers are usually given to 1 significant figure. In a table of results, more than 1 significant figure is acceptable.

Note that care needs to be taken when working out uncertainties where one quantity is divided by another. For example, when calculating a spring constant k from the force F and extension x,

$$\max k = \frac{\max F}{\min x} \qquad \text{and} \qquad \min k = \frac{\min F}{\max x}$$

Uncertainties on a graph

In a table of results, you may have actual uncertainties for one or both quantities that are to be plotted (see Table 26.2 opposite). These uncertainties are plotted on a graph as error bars for each data point (see Figure 26.4 overleaf). A fine pencil should be used to draw these. The extremity of an error bar is shown by a short line at right angles to the direction of the error bar.

The straight line of best fit should be drawn, as already described. To estimate the uncertainty from a graph, a worst acceptable straight line should also be drawn. The worst acceptable line is the steepest or shallowest line that passes through all the error bars, allowing for any anomalous points. The worst acceptable line should be labelled as such or be drawn as a dashed line. It is good practice to ensure that all the dashes pass clearly through the error bars.

After you have drawn the worst acceptable line, the gradient and y-intercept of this line may be determined. It is important that you take care to read the correct information from your graph.

To estimate the absolute uncertainty in the gradient

absolute uncertainty = gradient of line of best fit – gradient of worst acceptable line

or

$$\text{absolute uncertainty} = \frac{\begin{matrix}\text{gradient of steepest} \\ \text{worst acceptable line}\end{matrix} - \begin{matrix}\text{gradient of shallowest} \\ \text{worst acceptable line}\end{matrix}}{2}$$

To estimate the absolute uncertainty in the y-intercept

absolute uncertainty = y-intercept of line of best fit – y-intercept of worst acceptable line

or

$$\text{absolute uncertainty} = \frac{\begin{matrix}y\text{-intercept of steepest} \\ \text{worst acceptable line}\end{matrix} - \begin{matrix}y\text{-intercept of shallowest} \\ \text{worst acceptable line}\end{matrix}}{2}$$

The final answer in any experiment should be expressed as a value, an uncertainty estimate and a unit. For example, the volume of the cube described above should be written as $(16\,000 \pm 2000)\,\text{mm}^3$.

Example

In an experiment, the period T of a mass-spring oscillator is determined by measuring the time t for 10 oscillations. Measurements of t are repeated twice. The experiment is then repeated for different values of mass m.

It is suggested that the period T and mass m are related by the following equation

$$T = 2\pi A m^q$$

where A and q are constants.

The equation can be changed so that a straight-line graph can be plotted by taking logarithms to base 10 of both sides

$$\lg T = q \lg m + \lg 2\pi A$$

A graph of $\lg T$ against $\lg m$ will test this relationship.

The mass is measured on a balance with a percentage uncertainty of 5%.

The table of results for this experiment should include columns for the measurements of mass m and time t for 10 oscillations. The table should also include columns for calculated values of mean t, T, $\lg m$ and $\lg T$. The columns for m and $\lg(m/\text{g})$ will also have absolute uncertainties.

Determining the absolute uncertainties in m and $\lg(m/\text{g})$

Since the percentage uncertainty in m is 5%, for the first row in the table of results, when m = 100 g, the absolute uncertainty is 5% × 100 g = 5 g. This means that the first quantity should be written as 100 ± 5. (Remember that units are *not* included in the body of a table; column headings indicate the units.)

For the second row, the absolute uncertainty in m is 5% × 150 = 7.5, i.e. 150 ± 8. A similar method is used for subsequent rows. In a table of results, it is acceptable to have more than 1 significant figure in the absolute uncertainties, e.g. in the fourth row of the data 250 ± 13 (see Table 26.2).

Now that the uncertainties in m are known, the uncertainties in $\lg(m/\text{g})$ can be determined.

In the first row, lg (100) = 2. Since the mass is given to 3 significant figures, lg (100) should be given to 3 (or 4) decimal places, i.e. 2.000 (or 2.0000).

To calculate the uncertainty in lg (100 ± 5),

$$\lg(105) = 2.021$$

$$\lg(95) = 1.978$$

The uncertainty in lg (100 ± 5) is therefore either

$$2.021 - 2.000 = 0.021$$

or

$$2.000 - 1.978 = 0.022$$

or

$$\frac{1}{2} \times (2.021 - 1.978) = 0.0215$$

As demonstrated, any of these values would be acceptable.

Table 26.2 shows the completed table of results.

Column headings for logarithmic quantities give a quantity and unit within brackets as the logarithm does not have a unit.

This is the usual notation for column headings: a quantity and unit.

The actual uncertainties in m are calculated by taking 5% of the mass values.

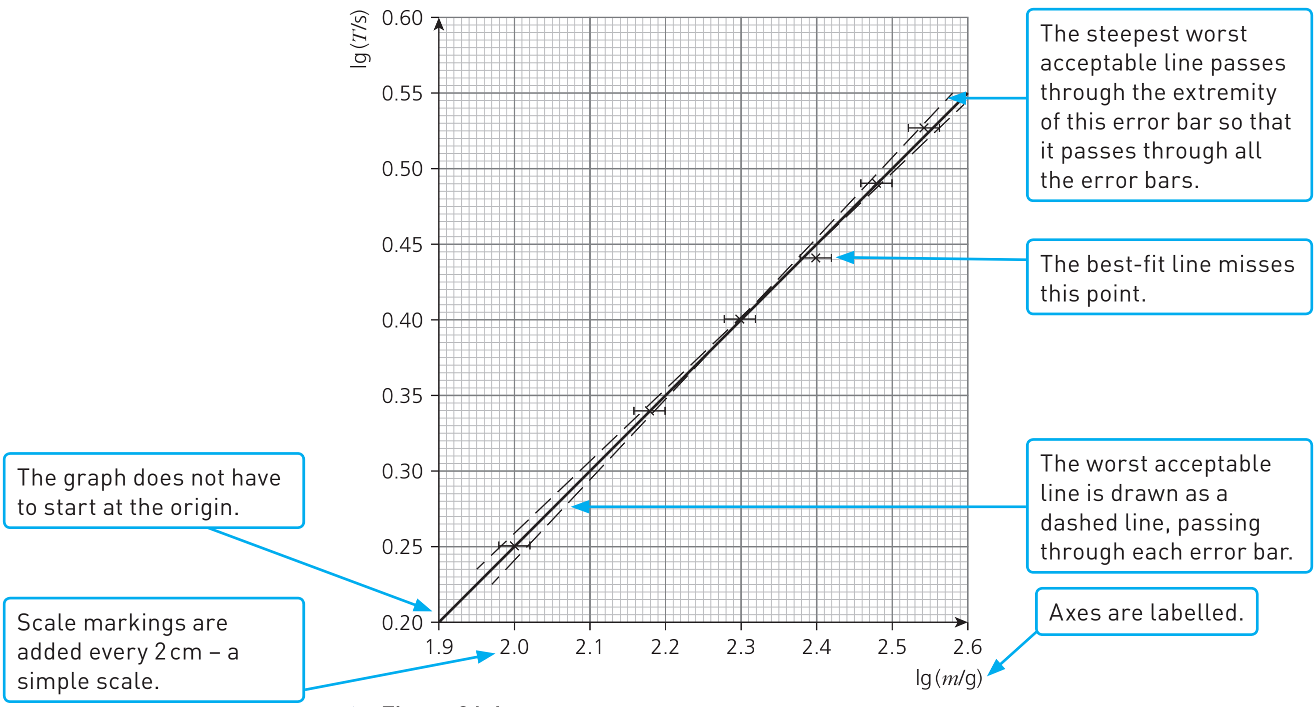
m/g	t_1/s	t_1/s	mean t/s	T/s	lg (m/g)	lg (T/s)
100 ± 5	17.6	18.0	17.8	1.78	2.000 ± 0.021	0.250
150 ± 8	22.0	21.6	21.8	2.18	2.176 ± 0.023	0.338
200 ± 10	24.9	25.3	25.1	2.51	2.301 ± 0.022	0.400
250 ± 13	27.4	27.8	27.6	2.76	2.398 ± 0.023	0.441
300 ± 15	31.0	30.6	30.8	3.08	2.477 ± 0.022	0.489
350 ± 18	33.3	33.7	33.5	3.35	2.544 ± 0.022	0.525

▲ Table 26.2

Since m is recorded to 3 significant figures, $\lg(m/\text{g})$ should be recorded to 3 (or 4) decimal places.

Since t is recoded to 3 significant figures, $\lg(T/\text{s})$ should be recorded to 3 (or 4) decimal places.

A graph can then be plotted of lg(*T*/s) on the *y*-axis against lg(*m*/g) on the *x*-axis, as shown in Figure 26.4.



▲ **Figure 26.4**

Determining the gradient, including the uncertainty

The gradient of the best-fit line is determined using the method already discussed. (See the Maths Note on straight-line graphs in Topic 2, page 38.)

Two data points are chosen from the line of best fit in Figure 26.4 and substituted into the expression to calculate the gradient

$$\text{gradient} = \frac{0.520 - 0.225}{2.54 - 1.95} = \frac{0.295}{0.59} = 0.50$$

The gradient of either the steepest or shallowest worst line from Figure 26.4 is then determined using the same principles of determining the gradient

$$\text{steepest gradient} = \frac{0.535 - 0.235}{2.55 - 1.99} = \frac{0.300}{0.56} = 0.536$$

$$\text{shallowest gradient} = \frac{0.520 - 0.235}{2.55 - 1.95} = \frac{0.285}{0.6} = 0.475$$

The uncertainty in the gradient is either

$0.536 - 0.50 = 0.036$ or

$0.50 - 0.475 = 0.025$ or

$(0.536 - 0.475)/2 = 0.031$

Determining the y-intercept, including the uncertainty

Since the *x*-axis in Figure 26.4 does not start at the origin, the *y*-intercept should be determined by substituting the coordinates of a point on the trend line and the gradient into the equation of a straight line, $y = mx + c$. A simple way to do this is to use one of the data points from the gradient calculation.

For the graph in Figure 26.4,

y-intercept of best-fit line = $y - mx = 0.520 - 0.50 \times 2.54 = -0.75$

y-intercept of steepest line = $y - mx = 0.535 - 0.536 \times 2.55 = -0.83$

y-intercept of shallowest line = $y - mx = 0.520 - 0.475 \times 2.55 = -0.69$

The uncertainty in the y-intercept is either

$-0.83 - -0.75 = 0.08$ or

$-0.75 - -0.69 = 0.06$ or

$(-0.83 - -0.69)/2 = 0.07$

Determining the constants A and q, including the uncertainties

Since $\lg T = q \lg m + \lg 2\pi A$,

gradient = $q = 0.50 \pm 0.05$

y-intercept = $\lg 2\pi A$

$10^{y\text{-intercept}} = 2\pi A$

Remember 'lg' represents the logarithm to base 10.

$$A = \frac{10^{y-\text{intercept}}}{2\pi} = \frac{10^{-0.75}}{2\pi} = 0.028$$

Using the steepest line,

$$A = \frac{10^{y-\text{intercept}}}{2\pi} = \frac{10^{-0.83}}{2\pi} = 0.024$$

Using the shallowest line,

$$A = \frac{10^{y-\text{intercept}}}{2\pi} = \frac{10^{-0.69}}{2\pi} = 0.032$$

Thus,

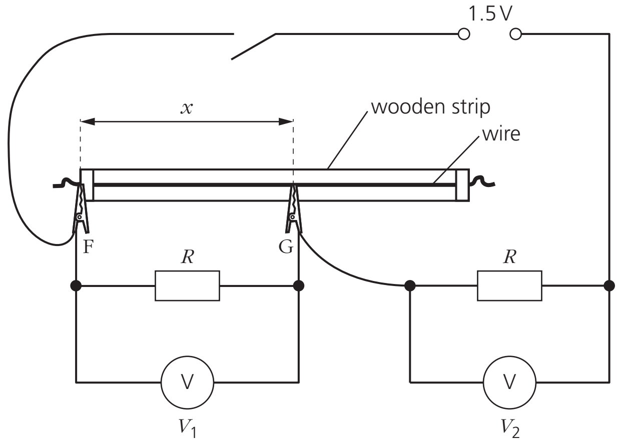
A = 0.028 ± 0.004

END OF TOPIC QUESTIONS

1 You may not need to use all of the materials provided.

In this experiment, you will investigate an electrical circuit.

a Set up the circuit shown in Fig. 26.5.



▲ Figure 26.5

F and G are crocodile clips.

- Place G on the wire so that the distance x between the ends of F and G is approximately 40 cm.
- Measure and record x.
- Close the switch.
- Record the voltages V_1 and V_2.
- Open the switch. **[2]**

b Vary x until you have six sets of readings of x, V_1 and V_2.
Record your results in a table. Include values of $(V_2 - V_1)$ and $\frac{V_1}{x}$ in your table. **[10]**

c **i** Plot a graph of $(V_2 - V_1)$ on the y-axis against $\frac{V_1}{x}$ on the x-axis. **[3]**

ii Draw the straight line of best fit. **[1]**

iii Determine the gradient and y-intercept of this line. **[2]**

d It is suggested that the quantities V_2, V_1 and x are related by the equation:

$$(V_2 - V_1) = \frac{PV_1}{x} + Q$$

where P and Q are constants.
Using your answers in **c iii**, determine values for P and Q. Give appropriate units. **[2]**

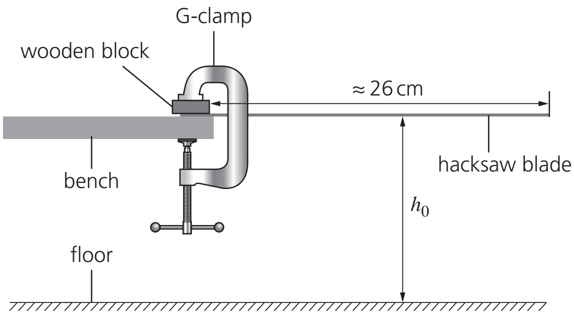
[Total: 20 marks]

Cambridge International AS and A Level Physics (9702) Paper 33 Q1 May/June 2018

2 You may not need to use all of the materials provided.

In this experiment, you will investigate the motion of a hacksaw blade.

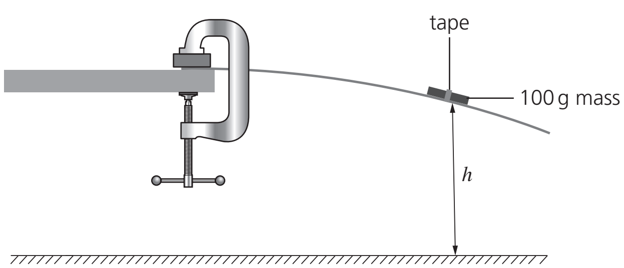
a Assemble the apparatus as shown in Fig. 26.6.



▲ **Figure 26.6** (not to scale)

The vertical distance from the floor to the top surface of the hacksaw blade is h_0, as shown in Fig. 26.6.
Measure and record h_0. **[1]**

b **i** Place a 100 g mass on the blade with its centre approximately 19 cm from the bench and tape it in position.
When released, the hacksaw blade will bend down, as shown in Fig. 26.7.



▲ **Figure 26.7** (not to scale)

The vertical distance from the floor to the top surface of the hacksaw blade at the centre of the mass is h.

Measure and record h. **[1]**

ii Calculate y, where $y = h_0 - h$. **[1]**

c Estimate the percentage uncertainty in your value of y. **[1]**

d Push the end of the hacksaw blade down a small distance and then release it. The blade will oscillate.

Determine the period T of the oscillations. **[2]**

e Move the slotted mass approximately 3 cm further from the bench and fix it with tape.

Measure and record h.

Repeat **b ii** and **d**. **[3]**

f It is suggested that the relationship between T and y is:

$$T = c\sqrt{y}$$

where c is a constant.

i Using your data, calculate two values of c. **[1]**

ii Explain whether your results support the suggested relationship. **[1]**

g Theory suggests that an approximate value of the acceleration of free fall g is given by:

$$g = \frac{4\pi^2}{c^2}$$

Using your second value of c, calculate g. Give an appropriate unit. **[1]**

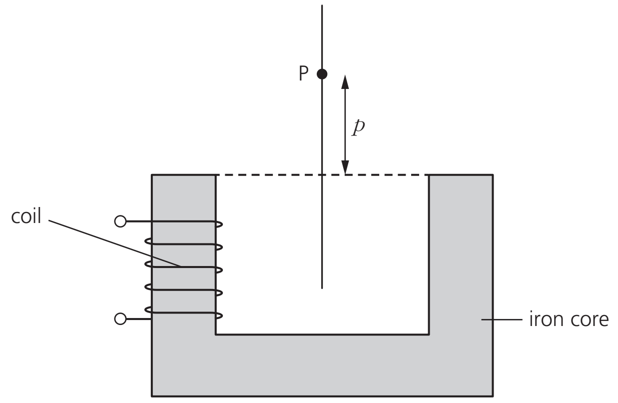
h **i** Describe four sources of uncertainty or limitations of the procedure for this experiment. **[4]**

ii Describe four improvements that could be made to this experiment. You may suggest the use of other apparatus or different procedures. **[4]**

[Total: 20 marks]

Cambridge International AS and A Level Physics (9702) Paper 36 Q2 Oct/Nov 2018

3 A student uses a Hall probe to investigate the magnetic flux density due to a U-shaped electromagnet, as shown in Fig. 26.8.



▲ **Figure 26.8**

Point P is equidistant from the poles of the electromagnet and distance p is the vertical distance between P and the top of the electromagnet. The magnetic flux density is B at point P. It is suggested that the relationship between B and p is:

$$B = kNIe^{-\alpha p}$$

where N is the number of turns on the coil, I is the current in the coil and α and k are constants.

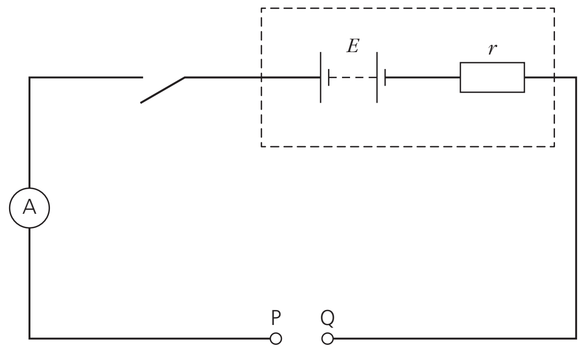
Design a laboratory experiment using a Hall probe to test the relationship between B and p. Explain how your results could be used to determine values for α and k. You should draw a diagram to show the arrangement of your equipment. In your account, you should pay particular attention to:

- the procedure to be followed,
- the measurements to be taken,
- the control of variables,
- the analysis of the data,
- any safety precautions to be taken.

[Total: 15 marks]

Cambridge International AS and A Level Physics (9702) Paper 52 Q1 Oct/Nov 2016

4 A student is investigating the current in a circuit. The circuit is set up as shown in Fig. 26.9.



▲ **Figure 26.9**

Resistors, each of resistance R, are connected in parallel between P and Q. The current I is measured. The experiment is repeated for different numbers n of resistors between P and Q. It is suggested that I and n are related by the equation:

$$E = I\left(\frac{R}{n} + r\right)$$

where E is the electromotive force (e.m.f.) and r is the internal resistance of the power supply.

a A graph is plotted of $\frac{1}{I}$ on the y-axis against $\frac{1}{n}$ on the x-axis.
Determine expressions for the gradient and the y-intercept. **[1]**

b Values of n, I and $\frac{1}{n}$ are given in Fig. 26.10.

n	I/mA	$\frac{1}{n}$	$\frac{1}{I}$/A^{-1}
2	34 ± 2	0.50	
3	46 ± 2	0.33	
4	56 ± 2	0.25	
5	66 ± 2	0.20	
6	76 ± 2	0.17	
7	84 ± 2	0.14	

▲ **Figure 26.10**

Calculate and record values of $\frac{1}{I}$/A^{-1} in Fig. 26.10.

Include the absolute uncertainties in $\frac{1}{I}$. **[2]**

c **i** Plot a graph of $\frac{1}{I}$/A^{-1} against $\frac{1}{n}$.

Include error bars for $\frac{1}{I}$. **[2]**

ii Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. **[2]**

iii Determine the gradient of the line of best fit. Include the absolute uncertainty in your answer. **[2]**

iv Determine the y-intercept of the line of best fit. Include the absolute uncertainty in your answer. **[2]**

d **i** Using your answers to **a**, **c iii** and **c iv**, determine the values of E and r. Include appropriate units.

Data: $R = 470 \pm 5\,\Omega$. **[3]**

ii Determine the percentage uncertainty in r. **[1]**

[Total: 15 marks]

Cambridge International AS and A Level Physics (9702) Paper 52 Q2 Oct/Nov 2018

List of formulae and data

AS Level

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$	$v^2 = u^2 + 2as$
hydrostatic pressure	$\Delta p = \rho g \Delta h$	
upthrust	$F = \rho g v$	
Doppler effect for sound waves	$f_o = f_s v/(v \pm v_s)$	
electric current	$I = Anvq$	
resistors in series	$R = R_1 + R_2 + \ldots$	
resistors in parallel	$1/R = 1/R_1 + 1/R_2 + \ldots$	

Data

acceleration of free fall	$g = 9.81\,\mathrm{m\,s^{-2}}$
speed of light in free space	$c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
elementary charge	$e = 1.60 \times 10^{-19}\,\mathrm{C}$
unified atomic mass unit	$1\mathrm{u} = 1.66 \times 10^{-27}\,\mathrm{kg}$
rest mass of electron	$m_\mathrm{e} = 9.11 \times 10^{-31}\,\mathrm{kg}$
rest mass of proton	$m_\mathrm{p} = 1.67 \times 10^{-27}\,\mathrm{kg}$

A Level

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$	$v^2 = u^2 + 2as$
hydrostatic pressure	$\Delta p = \rho g \Delta h$	
upthrust	$F = \rho g v$	
Doppler effect for sound waves	$f_o = f_s v/(v \pm v_s)$	
electric current	$I = Anvq$	
resistors in series	$R = R_1 + R_2 + \ldots$	
resistors in parallel	$1/R = 1/R_1 + 1/R_2 + \ldots$	
gravitational potential	$\varphi = -GM/r$	
gravitational potential energy	$E_\mathrm{p} = -GMm/r$	
pressure of an ideal gas	$p = \frac{1}{3}Nm\langle c^2\rangle$	
simple harmonic motion	$a = -\omega^2 x$	
velocity of particle in s.h.m.	$v = v_0 \cos \omega t$	$v = \pm\omega\sqrt{(x_0^2 - x^2)}$
electric potential	$V = Q/4\pi\varepsilon_0 r$	
electric potential energy	$E_\mathrm{p} = Qq/4\pi\varepsilon_0 r$	
capacitors in series	$1/C = 1/C_1 + 1/C_2 + \ldots$	
capacitors in parallel	$C = C_1 + C_2 + \ldots$	
discharge of a capacitor	$x = x_0 \exp(-t/RC)$	
Hall voltage	$V_\mathrm{H} = BI/ntq$	
alternating current/voltage	$x = x_0 \sin \omega t$	
radioactive decay	$x = x_0 \exp(-\lambda t)$	
decay constant	$\lambda = 0.693/t_{\frac{1}{2}}$	
intensity reflection coefficient	$\dfrac{I_R}{I_0} = \dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$	
Stefan–Boltzmann law	$L = 4\pi\sigma r^2 T^4$	
Doppler redshift	$\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$	

Data

acceleration of free fall	$g = 9.81\,\mathrm{m\,s^{-2}}$
speed of light in free space	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
elementary charge	$e = 1.60 \times 10^{-19}\,\mathrm{C}$
unified atomic mass unit	$1\,\mathrm{u} = 1.66 \times 10^{-27}\,\mathrm{kg}$
rest mass of electron	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
rest mass of proton	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
molar gas constant	$R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ $(1/4\pi\varepsilon_0 = 8.99 \times 10^{9}\,\mathrm{m\,F^{-1}})$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Stefan–Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

Answers

AS Level

1 Physical quantities and units

Questions

1 10 800 cm^2 or 1.08×10^4 cm^2
2 8.5×10^{-8} m^3
3 1.0×10^9
4 a 6.8×10^{-12} F
b 3.2×10^{-5} C
c 6.0×10^{10} W
5 800
6 4.6×10^4
7 a 100–150 g
b 50–120 kg
c 2–3 m
d 0.5–1.0 cm
e 0.05 mm
f 2×10^{-8} m^3
g 4×10^{-3} m^3
h 10–12 $m\,s^{-1}$
i 200–300 $m\,s^{-1}$
j 50–100 GJ
k 1–2 kJ
8 $kg\,m^{-3}$
9 $kg\,m^{-1}\,s^{-2}$
10 a yes
b yes
11 $kg\,s^{-2}$
13 $kg\,m^{-1}\,s^{-2}$
14 12.52 mm
16 a ±0.06 A
b 2.01 ± 0.09 A
17 a i ± 2 mm
b 6%
18 a end of rule damaged; measure length from 10 cm mark
b parallax error; place pencil in contact with scale, over the graduations and define the ends with set-squares
19 a micrometer screw gauge
b zero error on drum
c averaging reduces random errors; spiral readings allow for a non-circular cross-section, and moving along the length of the wire allows for any taper
20 a 1.2
b ±0.1 cm
21 (870 ± 40) cm^3
22 $(8.9 \pm 0.6) \times 10^3$ $kg\,m^{-3}$
24 a scalar
b vector
c scalar
25 a vector
b scalar
c vector
26 Velocity has direction, speed does not. Velocity is defined in terms of displacement which is a vector, speed is defined in terms of distance which is a scalar.
27 Student is correct. Weight is a force which acts vertically downwards.
28 Direction of arrow gives direction of vector. Length of arrow drawn to scale represents magnitude of vector quantity.
29 a 690 N
b 210 N
c 510 N at an angle of 28° to the 450 N force
30 upstream at 78° to the bank
31 120 N at an angle of 25° to the 50 N force in an anticlockwise direction
32 a 7 $km\,h^{-1}$
b 1 $km\,h^{-1}$
33 11 N at an angle to the 6.0 N force of 56° in an anticlockwise direction
34 a 250 $km\,h^{-1}$
b 180 $km\,h^{-1}$
35 a 1.0 $m\,s^{-1}$
b 9.1 $m\,s^{-1}$

END OF TOPIC QUESTIONS

1 A
2 B
3 B
4 C
5 C
6 C
8 $kg\,m^2$
9 a i $kg\,m^2\,s^{-2}$
ii $kg\,m^2\,s^{-2}$
11 $(9.7 \pm 0.9) \times 10^3$ $kg\,m^{-3}$)
12 b i 92 N
ii 77 N
c i 59 N
ii 59 N
13 a i 18 $m\,s^{-1}$
ii 29° above horizontal
b i 10 $m\,s^{-1}$
ii 33°

14 a 380 pm
b 0.086 Ms
c 8.3 min
e ii 230 m s^{-1}
15 a i m^3
b kg $m^{-1} s^{-1}$
16 a i $(50–150) \times 10^{-3}$ kg
ii 50–300 cm^3
b $(7.9 \pm 0.5) \times 10^3$ kg m^{-3}
17 b m^2
18 b i 0.20 W
ii 7%
iii 0.20 ± 0.01 W

2 Kinematics

Questions

1 5.3×10^{-11} m
2 3200 s
3 6 m s^{-1}
4 6 m s^{-2}
5 3.3 s
6 30 km
7 180 m s^{-1}
9 3.6 h; 610 km h^{-1}
10 –5.0 m s^{-2}
12 9.8 m s^{-2}
13 2.5 m s^{-2}
14 7.5 s; 15 m s^{-1}
15 8.2 m s^{-1} upwards
16 a 1.8 m s^{-2}
b 770 m
17 6.1 m s^{-1}
18 20 m
19 75 m
20 140 m s^{-1}
21 47 m s^{-1}
22 a B
b A
23 3.5°

END OF TOPIC QUESTIONS

2 8
3 a 61°; 2.8 m
b 3.9 m s^{-1}
c 1.4 s
4 a 9.4 m s^{-1}
b 10%
6 B
7 C
8 C
9 b 0.31–0.35 m s^{-2}
c 84 m
d 26 s
10 b ii 9.8 m s^{-2}
iii 1 9.9 m
2 2.0 m
11 b i 10.6
ii 30 m s^{-1}
iv 10 800 N
12 b i 0.39 s
ii 0.75 m
13 a i 7.7 m s^{-1}
ii 9.2 m s^{-1}
c 8.4 m
14 b ii 0.61 s
iii 2.8 m

3 Dynamics

Questions

1 1.7 m s^{-2}
2 5.3 N
3 a 61 N
b 94 N
4 6.8×10^{-24} N s (kg m s^{-1})
6 1.2 m s^{-1}
7 600 N
8 a $\frac{1}{2}u_A$
b 0.5

END OF TOPIC QUESTIONS

1 50 kg
2 a 7.7 m s^{-1}
b 45.3 m s^{-2}
c 2040 N
4 *T*/2
5 3.3×10^4 N s (kg m s^{-1})
6 3.6×10^7 N
7 0.27 N
9 1.03×10^5 m s^{-1}
10 Heavy particle's speed is practically unchanged; light particle moves with a speed 2u in the same direction as the incident heavy particle.
11 the lighter object
12 C
13 D
14 B
15 a 0.54 kg m s^{-1}
b 0.54 N s
c 36 N
16 a i 2.16 kg m s^{-1}
ii 1.07 m s^{-1}
iii 1.16 J
b 0.059 m
17 3080 m s^{-1}
19 a 3u to the left
b i 3u to the right
ii $3t_1$
c 3u/2
20 a 3 m
b 0.25
21 a 75 g

22 a i 1 6.3 s
2 510 J
b ii $7.6\,m\,s^{-2}$
23 b i $3.9\,kg\,m\,s^{-1}$
ii 0.97 N
c i -3.4 and $+3.4\,kg\,m\,s^{-1}$
24 a $0.11\,m\,s^{-1}$
b i 1 $47\,m\,s^{-1}$
2 $0\,m\,s^{-1}$
25 b $3.2\,m\,s^{-1}$
c 97 J

4 Forces, density and pressure

Questions

1 5.6 N m
2 4.5 N
3 14 (13.5) N
4 b 29 N
5 $11\,000\,kg\,m^{-3}$
6 0.18 kg
7 1.4×10^4 Pa
9 3.0 N

END OF TOPIC QUESTIONS

1 B
2 D
3 B
4 a 4.2 N m
b 9.1 N
5 a 2.9 N m
b 8.0 N
6 a 88 N
b 112 N
7 67 N
8 b iii 1.3 N
c $7800\,kg\,m^{-3}$
9 a 1.4 N m
b 1.2 N m
10 a $8800\,kg\,m^{-3}$
b i 5.1 N
ii 44 N
c 4.1kPa
11 1.5×10^5 Pa
13 b i 23 N
iv 1.9 kg
14 b i 12 N m
c i 0.90 N
ii 1.5 N
15 b i $2500\,kg\,m^{-3}$
ii 2.2×10^4 Pa
16 b i 26 N
ii 15 N
17 a ii 190 N
18 0.49 N
19 b i $7.9 \times 10^3\,kg\,m^{-3}$
iv $8.6\,m\,s^{-2}$
v $7.3\,m\,s^{-1}$

5 Work, energy and power

Questions

1 a 180 J
b 30 J
2 a 15.5 J
b 7.6 J
3 a kinetic energy at its lowest point → potential energy at its highest point → kinetic energy at its lowest point, etc. (kinetic and potential energy at points in between)
b potential energy of compressed gas → kinetic energy of spray droplets → heat when droplets have stopped moving(energy transferred to surrounding atmosphere)
c kinetic energy when thrown → potential and kinetic energy at highest point of motion → internal and heat and sound energy when clay hits the ground
4 All the electrical energy is changed to internal energy of the heater and the heater gets hot. The heater releases all this energy to the surroundings so the process is 100% efficient.
5 58%
6 4.3×10^5 J
7 1.2×10^5 N
8 1.5 kW
9 a 1.3×10^4 J gain
b 12 J gain
c 2.4×10^9 J loss
10 3.1×10^5 J
11 a 1000 J
b 3000 J

END OF TOPIC QUESTIONS

1 90 J; 0; 78 J; 0.15 m; 36°; 2.6×10^3 N
2 a 7.9 N
b 0.19 J
3 a chemical
b gravitational potential
c kinetic of the wind
d kinetic of gases, sound, light, gravitational potential energy
e internal potential
4 a 0.51 m
b $80\,J\,s^{-1}$
5 a 11 J
b $14\,m\,s^{-1}$
6 a 16 MJ
b 110 MJ
c 210 MJ

7 15%
9 C
10 A
11 C
12 a i 16.5 kW
ii 3200 N
iii 96 kW
b i 1.32 MJ
ii 3.84 MJ
13 b i 1 8.3 J
2 13 m
ii 3
14 b 1090 N
c ii 76 m
iii 1 2.7×10^5 J
2 3.5×10^5 J
15 a ii 17.6 m
b i 140 N up
ii 17.6 m
16 b 2.9 m
c i 1.6 J
ii 0.60 J
e 0.55 N s
f 5.9 N
17 b i 9.0×10^4 W
ii 1 1.6×10^7 J
2 7.0×10^6 J
3 9.0×10^6 J
iii 1.1×10^3 N

6 Deformation of solids

Questions

2 400 N m^{-1}
3 a 3.0×10^4 N m^{-1}
b 54 N
4 a 1.29×10^7 Pa
b 1.17×10^{-4}
c 0.16 mm
5 5.3×10^6 Pa
7 5.4×10^{-2} J
8 4.2×10^{-2} m

END OF TOPIC QUESTIONS

1 C
2 D
3 B
4 a 500 N m^{-1}
b 13.1 cm
5 7.9 J
6 0.36 mm
8 c ii 1 0.057 J
3 0.020 J
9 b i 4.6×10^{-3} m
ii 4×
10 b ii 78 N m^{-1}
iii 0.26 J
c i 0.12 m
ii 53 N m^{-1}
11 b ii 1.3×10^{11} N m^{-2}
iii 1.2×10^{-3} J
12 b i 4.5×10^7 N m^{-1}
ii 90 J

7 Waves

Questions

1 60° allow $\pi/3$ radians
2 0.40 m s^{-1}
3 0.68 m
4 1.4
5 a 1.2×10^{-3} s
b 830 Hz
6 amplitude = 0.99 cm, time period no change
7 270 Hz
8 460 THz
9 6.3
10 0.038 m
11 The graph of intensity with θ shows a $\cos^2$ graph with a maximum intensity when $\theta = 0$, 180° and 360° and zero intensity when $\theta = 90°$ and 270°.
12 60°

END OF TOPIC QUESTIONS

1 B
2 C
3 C
4 a 200 Hz
b 5.0×10^{-3} s
5 a 7.5×10^{14} Hz to 4.3×10^{14} Hz
b 1.2 m
6 ¼(0.25)
7 a 5.7×10^{-8} J
b 4.9×10^{-5} W m^{-2}
8 2.0×10^9 W m^{-2}
9 b i 0.6 s
ii 4.0 cm
iii 6.7 cm s^{-1}
c ii 3.0
10 b ii 6.0×10^{14} Hz
11 a i 1 $N\lambda$
2 N/f
b 1300 Hz
c ii 180°
iii 9.1 m
12 a 250 Hz
b amplitude 2.8 cm
13 b i 1 1.5 mm
2 4.2×10^{-2} m
ii 0.56 s
14 a i 5.5 mV
ii 530 Hz
iii 14 Hz
b 530 ± 10 Hz

15 b i P and T
ii P and S or Q and T
c $0.18\,m\,s^{-1}$
d 0.44
16 a 71°
b intensity is reduced by 1/9 of its original value
c 55°

8 Superposition

Questions

1 6.25×10^{-7} m (625 nm)
2 63 m
3 880 Hz; 1320 Hz
4 0.38 m
5 340 Hz
6 128 Hz
7 56°
8 22 cm
9 652 nm
10 28.2°; 70.7°
11 2 (3 if zero order included)

END OF TOPIC QUESTIONS

1 A
2 D
3 B
4 C
6 c 3.5 mm
9 b 8.5 cm
10 a $108\,m\,s^{-1}$
b 45 Hz
11 a 2.5×10^{-6} m
b 10.2°
c 5; 3
13 0.73° (1.26×10^{-2} rad)
14 b 180° (π rad)
c 5:1
d $224\,m\,s^{-1}$
15 26.6 cm
17 b 212 mm
c 449 mm
18 c 180 Hz
19 b i 8.0×10^5
ii 2.5×10^6 m
20 a i 0.8 m
b ii 1.2 m
21 b 1.1 cm
c i 3.4×10^{-4} m
22 b ii 1 360° or 0°
2 920 nm
iii 3×10^{-4} m
23 b i 1800 Hz
ii $1.0 \times 10^{-4}\,s\,cm^{-1}$
c i 0.090 m
iii 14 s

9 Electricity

Questions

1 2.0 A
2 4.8×10^5 s
3 a 60 C
b 3.8×10^{20}
4 $3.9 \times 10^{28}\,m^{-3}$
5 1.6×10^{-13} J
6 a 0.25 C
b 2.2 J
7 100 Ω
8 current in both is 0.42 A
9 a 9.2 A
b 26 Ω
10 18 m
11 0.97 mm

END OF TOPIC QUESTIONS

1 A
2 D
3 C
4 a 760 C
b 1000 W
c 57 Ω
5 a i 0.20 A
ii 0.60 W
b 5400 J
6 3.5×10^6 J
7 2.0 kW
8 6.7 m
9 b

I/A	0.20	0.40	0.60	0.80	1.00	1.20	1.40
R/Ω	0.95	1.20	2.45	3.65	4.56	5.47	6.21

10 a 0.62 Ω
b 4.3×10^{-7} Ω m
11 a i 19 Ω
ii 3.5 m
b i 630 W
ii ¼ length or 4 times greater area
12 b i 3200 W
ii 13 A
iii 3.5×10^7 C
iv 8.4×10^{19}
13 b iii 2.4×10^{-2} Ω
14 b i 1 2.8 A
2 8.0 A
ii 1 4.3 Ω
2 1.5 Ω
iii 0.35

10 D.C. circuits

Questions

1 a EI
b I^2R or EI
2 0.56 Ω

3 a 3.0 A, when the cell is short circuited
b 1.1 W, when load resistance equals the internal resistance

4 0.25 A

5 2.0 A towards the junction

6 2.0 V

7 4.3 Ω

8 25 Ω

9 connections are across variable resistor
R = 750 Ω

10 a 12 V
b 0.57 V
c 5.0 kΩ

11 a 0.82 V
b 7.7 V

END OF TOPIC QUESTIONS

1 C

2 C

3 D

4 a 0.05 Ω
b 0.3 Ω

5 a 0.25 A
b 1.6 Ω
c 12 J

7 a 169 Ω
b 13 Ω

8 a 5 Ω
b 3.0 A

9 a 25 Ω

10 a 4.5 V
b i 50 Ω
ii 0.09 A
iii 0.90 V

12 10 resistors each of resistance 12 kΩ, power rating 0.5 W connected in parallel

13 a 4 Ω
b 8 Ω
c 3 Ω
d 1.0 A

14 a i 1.6×10^{-2} Ω
ii 1.1×10^{-3} Ω
iii 27 W
b i 4.4 s
ii 4.4×10^{21}
c i 11.7 V
ii 307 W

15 a 1.02 V; 1.22 W
b ii 7.53 m
iii 1.41 W

16 b i 0.29 A
ii 1.03 V
iii 1.03 V

17 c i 5.7 V
ii 1.7 Ω
d i 4.64 W
ii 51 %

18 b i 1 1.5 A
2 2.5 Ω
ii 0.25

19 b i 1 0.15 A
2 0.90 W
3 2.3×10^{19}
4 8.0 Ω
5 12 Ω

11 Particle physics

Questions

1 19, 40, 21

2 ${}^{232}_{90}\text{Th} \rightarrow {}^{228}_{88}\text{Ra} + {}^{4}_{2}\text{He}$ + energy

3 ${}^{228}_{88}\text{Ra} \rightarrow {}^{228}_{89}\text{Ac} + {}^{0}_{-1}\text{e} + \bar{\nu}$ + energy

4 $2.7 \times 10^{7}\ \text{m s}^{-1}$

5 $4.0 \times 10^{5}\ \text{m s}^{-1}$

END OF TOPIC QUESTIONS

1 B

2 A

3 C

4 a 26, 28
b 47, 62
c 79, 117
d 94, 138

6 ${}^{231}_{90}\text{Y} + {}^{4}_{2}\text{He}$, ${}^{231}_{91}\text{Z}$, ${}^{0}_{0}\gamma$

7 a $2.3 \times 10^{7}\ \text{m s}^{-1}$
b $2.7 \times 10^{5}\ \text{m s}^{-1}$

8 a ${}^{224}_{88}\text{Ra} \rightarrow {}^{220}_{86}\text{Rn} + {}^{4}_{2}\text{He}$
c $3.0 \times 10^{5}\ \text{m s}^{-1}$

9 b 3 and 0 for superscript values, 2 and –1 for subscript values
c $4.5 \times 10^{7}\ \text{m s}^{-1}$

10 a i proton/neutron
ii electron
b i up, up, down
ii down, down, up
c i proton + electron + antineutrino
ii down changes to an up

11 b i ${}^{0}_{1}\text{e}+$ *or* ${}^{0}_{1}\beta + {}^{0}_{0}\nu_{(e)}$
ii weak (force/interaction)
iii mass-energy
momentum
proton number
nucleon number
charge
c $\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = (+)e$

12 a electron and quark
b i 60, 28
ii (electron)antineutrino

13 b

	proton	β^- particle	antineutrino
charge	$+1.6\times10^{-19}$ (C) / +e	-1.6×10^{-19} (C)/$-e$	zero/0
mass	1.67×10^{-27} (kg)/1.7×10^{-27} (kg)	$9.1(1)\times10^{-31}$ (kg)	very small/zero/0

14 a i Q plotted at (82, 210)
ii R plotted at (83, 210)
b lepton(s)
c up down down → up up down / udd → uud or down → up / d → u
15 a i alpha, neutron and proton
ii neutron
iii β^+
b up/u
16 b i $-\frac{1}{3}e$
17 b i 1 to 3×10^{-10} m
ii 1 to 7×10^{-15} m

AS Level review exercise

1 a i 0.51 J
b 390 m s^{-1}
c i 150 J
2 b ii 1800 Ω
3 a i 800 N m^{-1}
b ii 0.96 m s^{-1}
4 a i 0.38 m s^{-2}
b 19 W
c i 20 N
ii 8.5 s
5 a i 1 0.16 Ω
2 14 – E
ii 7.3 V
b i 1.8×10^5 C
ii 2.5×10^6 J
iii 3.6×10^5 J
c 86%
6 a i 7.5 m s^{-1}
ii 13 m s^{-1}
b 8.6 m
c i 0.73 N s
7 a 250 Hz
c i 630 nm
8 b i 0.018 m
9 a kg m^{-2} s^{-3}
b ii 9.4 m s^{-1}
10 b i 4.0 Ω
ii 2.0 Ω
c 2.0 V
d i 72 W
ii 192 W
11 b i 1 0.68 m s^{-2}
2 8.6×10^5 J
iii 9.1 kW

A Level

12 Motion in a circle

Questions

1 60 m
2 4.5 rad s^{-1}
3 a 0.20 m s^{-1}
b 0.52 rad s^{-1}
c 0.10 m s^{-2}
4 4.7 N
5 a 660 m
b 5000 N

END OF TOPIC QUESTIONS

2 a 3.32 rad s^{-1}
b 0.53 revolutions per second
3 a 7.8 km s^{-1}
b 88.7 minutes
4 a ii 1 3 mg
2 4 mg
4 b i 6.4 rad s^{-1}
ii 4.6 m s^{-1}

13 Gravitational fields

Questions

1 4.6×10^{16} N
2 1.4 hours
3 25 N kg^{-1}
4 a -1.4×10^{10} J
b 1.1×10^4 m s^{-1}

END OF TOPIC QUESTIONS

1 3900 kg m^{-3}
2 7.78×10^8 km
3 –0.50%; –0.25%
4 6.2×10^{-11} N
5 b ii 8.85×10^4 km
6 c 1.0×10^{26} kg
7 c 1330 kg m^{-3}

Topic 14 Temperature

Questions

1 a 423 K
b 123 K
2 a 8.3×10^4 J
b 87 J
3 4200 J kg^{-1} K^{-1}

4 a i 25 kJ
ii 172 kJ
b 6.9
5 480°C
6 44 g

END OF TOPIC QUESTIONS

1 −18°C
2 a 22.6 g
3 105 g
4 b 400 W

15 Ideal gases

Questions

1 a 2.33×10^{-26} kg
b 6.24×10^{24}
2 3.3×10^{-9} m
3 1.29 kg m^{-3}
4 1.1 mol; 6.65×10^{23}; 2.1×10^{25} m^{-3}
5 8.0×10^{-5} m^3
6 8.7×10^{-6} m^3
7 1.1×10^{-20} J
8 630 m s^{-1}

END OF TOPIC QUESTIONS

1 212 kPa
2 1.28
3 6100 m s^{-1}
4 c i 500 m s^{-1}
ii 4.0×10^5 m^2 s^{-2}
5 b ii 360 K
6 c 580 m s^{-1}

16 Thermodynamics

Questions

1 1800 J
2 75 J
3 350 J by the system

END OF TOPIC QUESTIONS

1 0, 250 J
2 30 J increase
3 7 kJ
4 b i 0
ii 240 J
iii

change	work done on gas/J	heating supplied to gas/J	increase in internal energy/J
P→Q	240 J	−600	−360 J
Q→R	0	+720	720 J
R→P	−840 J	+480	−360 J

5 b iii 48 J increase
iv 1.2×10^{22}
6 b ii 2270 kJ kg^{-1}

17 Oscillations

Questions

1 a $x = 0.20 \sin 4.2t$
b 0.24 s
2 a 0.166 J
b 0.542 J
c 0.708 J

END OF TOPIC QUESTIONS

1 a 400 Hz
b 2.51×10^3 rad s^{-1}
c 10.1 m s^{-1}
d 2.53×10^4 m s^{-2}
e 3.6 mm
f 8 .1 m s^{-1}
2 a 6.4 N m^{-1}
b 15 mm
c 0.53 s
d 10.8 mm above equilibrium point
3 3.2
4 a i 1 1.7 cm
2 2.8 Hz
c 1.2 mm
5 a ii 3.8 Hz
b ii 17 mm

18 Electric fields

Questions

2 a right
b left
c left
d right
3 8.8×10^{13} m s^{-2}
4 a 5.0×10^4 V m^{-1}
b 2.4×10^{-12} C
5 5.5 mm upwards
6 3.7×10^{-13} N repulsion
7 7.2×10^{-8} C
8 0.45 J

END OF TOPIC QUESTIONS

1 a 3.7×10^{-14} m
2 b ii 2 8.3×10^5 m s^{-1}
4 a i 17 N
ii 7.8×10^{-13} J
b 1.5×10^7 m s^{-1}

19 Capacitance

Questions

1 450 μF

2 1.43×10^{-9} F
3 a 1.5 mC
b ii 500 μF
iii 3.0 V
iv 0.75 mC
4 45 mC
5 a 2.4×10^{-4} C
b 1.5×10^{-4} C
c 12.5 V

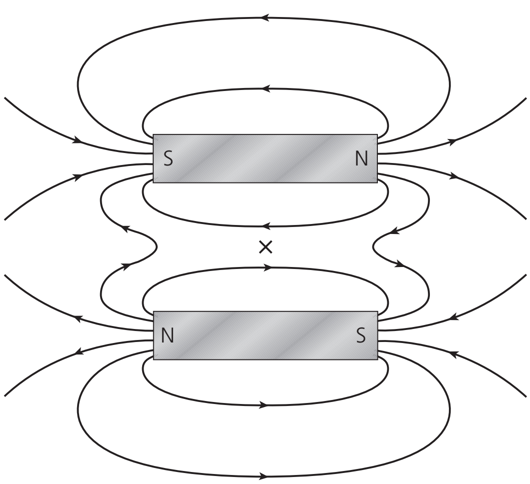
END OF TOPIC QUESTIONS

1 a 2.0 μF
b 1.2 μF
2 a i 2.7×10^{-4} J
ii 9.0×10^{-5} C
b i 3.0×10^{-5} A
ii 3.0 s
iii 1.2 V
3 b i 8 μF
ii 6.0
iii 36 μC
4 a ii 7.0 s
iii 2.0×10^{4} Ω
b i 14 s

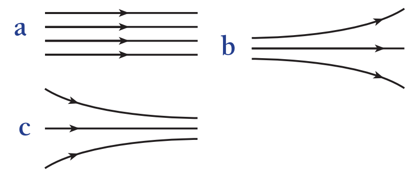
20 Magnetic fields

Questions

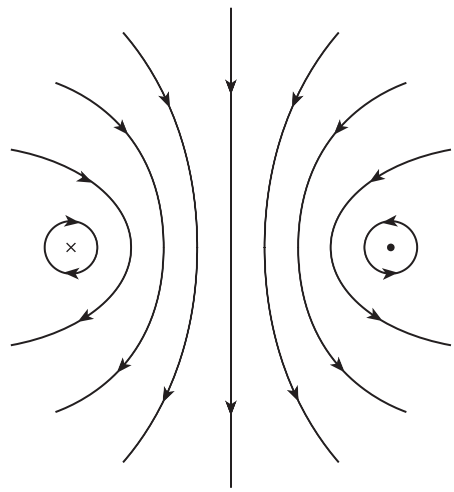
1 Diagram should be as in Fig. 20.4 but with the direction of field lines reversed
2



3 5.0×10^{-7} N
4 a 0.030 N m^{-1}
b 0.026 N m^{-1}
5 a 9.4×10^{6} m s^{-1}
b 9.2 cm
6 5.0×10^{6} m s^{-1}
7 0.40 μV
8



9



10 opposite directions with radii ratio α/e^{-} = 3600
11 a 0
b 15 mWb
c 8.6 mWb
12 a i 1360 m^2
ii 5.44×10^{-2} Wb
iii 5.44×10^{-2} V
13 0.17 V
14 a 2.0 m^2
b 0.30 Wb
c 0.30 V

END OF TOPIC QUESTIONS

1 a ii 3.1×10^{4} A
2 a i 8.0×10^{-5} T
ii 1.3×10^{-3} N m^{-1}
3 a 0.15 T
5 b i 53 mV
ii 0 V; 53 mV; –0 V
0 V; –0.20 V

21 Alternating currents

Questions

1 $V = 325 \sin 314t$
2 10
3 28 mV
4 19.8 mV
5 17 V
6 a i 325 V
ii 1.5 kW
b 3.0 kW; 0 kW

END OF TOPIC QUESTIONS

1 20 V; 2.4 A
2 a 340 V
b 14 mA

22 Quantum physics

Questions

1 a 290 nm
b 2.3×10^{-27} N s
2 345 nm
3 5.9×10^{14} Hz
4 a 1.4×0^{15} Hz
b sodium and zinc
5 9.7×10^{-12} m
6 4.8×10^{-11} m
7 2.4×10^{3} m s^{-1}
8 488 nm
9 3.2×10^{15} Hz
10 9.1×10^{-8} m

END OF TOPIC QUESTIONS

2 a 1.0×10^{11}
b 1.0×10^{-27} N s
c 1.0×10^{-16} N
3 a 510 nm
b i 4.1×10^{-19} J
ii 9.4×10^{5} m s^{-1}
4 a 4.4×10^{-32} m
b 2.7×10^{-35} m
c 2.4×10^{-11} m
d 1.3×10^{-13} m
6 b 4.56×10^{-19} J
c i 890 nm
ii 280 nm

23 Nuclear physics

Questions

1 0.108517 u
2 101 MeV
3 7.21 MeV per nucleon
4 a 0.6899 u
b 1.03×10^{-10} J
c 1.06×10^{-12} J
5 c 5.0 MeV
6 a $\frac{1}{4}$
b 1/1024
c $\frac{3}{4}$
d 15/16
7 a 2.0×10^{6} Bq
b 3200 Bq
8 2.8 days
9 a 19.8 min
b 5400 years
10 1.3×10^{-5} s^{-1}

END OF TOPIC QUESTIONS

1 b 6.0 hours
2 0.687u; 642 MeV; 6.62 MeV per nucleon
3 a 0.214 u
b 199 MeV
c 8.2×10^{10} J
d 74 g
4 a 4.96 MeV
b 14 MeV
5 7.8×10^{-11} kg
6 a 2.6 hours
b 9.1×10^{20}
7 b i 3.6×10^{4} Bq
ii 8.2×10^{-4} s^{-1}

24 Medical physics

Questions

1 b 9.4×10^{-3}
2 0.2
3 1.66×10^{-11} m
5 20 cm
6 0.019
9 1.4 MeV

END OF TOPIC QUESTIONS

2 0.908I incident on fat–muscle boundary
0.904I transmitted through fat–muscle boundary
0.332I incident on muscle–bone boundary
0.211 transmitted into bone
5 b P: 5, Q: 9, R:7, S:13
6 c 0.512 MeV

25 Astronomy and cosmology

Questions

1 600 W m^{-2}
2 a 5.5×10^{29} W
b 1.8×10^{19} m
3 Rigel 12100 K; Betelgeuse 3300 K
4 3.3×10^{7} km
5 6500 km s^{-1} away from Earth
6 2.9×10^{8} years
8 2.9 cm

END OF TOPIC QUESTIONS

1 c 490 W
2 b i 3000 K
ii 2.6×10^{11} m
3 b 13.7 km s^{-1} radially towards Earth
4 b ii 7.5×10^{8} years
6 a *L* luminosity, *A* surface area (of star), *T* surface temperature
7 5800 K

26 Practical work

END OF TOPIC QUESTIONS

1 a x measured and recorded with an appropriate unit to the nearest millimetre and V_1 and V_2 measured and recorded with the unit 'V' (volt) to the nearest 0.01 V.
magnitude of V_2 > magnitude of V_1

b Six sets of results for x, V_1 and V_2. Each column heading should be labelled with a quantity and a unit, as the example shows, there should not be units in the body of the table.

x/cm	V_1/V	V_2/V	$(V_2 - V_1)$/V	$\frac{V_1}{x}$/V cm^{-1}

All values of x should be given to the nearest millimetre and all values of V_1 and V_2 given to the nearest 0.01 V. At least one value of x should be less than 10.0 cm and one value greater than 70.0 cm.
Each calculated value of $\frac{V_1}{x}$ should have the same number of significant figures (or one more) than the number of significant figures in the quantity with the least significant figures of V_1 and x.
For example, if x is measured to three significant figures and V is measured to two significant figures, then $\frac{V_1}{x}$ should be recorded to two (or three) significant figures (but not one or four or more).
$\frac{V_1}{x}$ calculated correctly for each row.

c i Each axis should be labelled with a quantity and a unit, for example $(V_2 - V_1)$/V on the y-axis and $\frac{V_1}{x}$/V cm^{-1} on the x-axis. Scales added to each axis every 2 cm. Scales should be simple e.g. 0, 10.0, 20.0, 30.0, 40.0, etc. The scales on each axis should allow the plotted points to occupy at least half the graph grid in both the x-direction and the y-direction.
Points should be indicated by a fine cross or an encircled dot. The diameter of the point should be less than 1 mm. All the data should be plotted to better than 1 mm. The quality of the experiment is assessed by checking that all the plotted points are within 0.040 V of a straight line.

ii The straight line of best fit should show an even distribution of points on either side of the line along its whole length. The thickness of the line should be less than 1 mm. If one point does not follow the trend, then the point should be circled and labelled anomalous.

iii Gradient calculated by substituting the co ordinates of two data points that are on the straight line of best fit line into
gradient = $\frac{y_2 - y_1}{x_2 - x_1}$
The two points on the line chosen for the calculation should be separated by more than half the length of the line drawn. y-intercept determined either by reading off the y-axis if $\frac{V_1}{x}$ is zero or by substituting the coordinates of a point on the straight line of best fit and the gradient should be substituted into the equation of a straight line $y = mx + c$, i.e. $c = y - mx$.

d P = gradient and Q = y-intercept. Unit for P: m, cm or mm or equivalent and unit for Q: V.

2 a h_0 measured and recorded to the nearest millimetre and an appropriate unit given.

b i h measured to the nearest millimetre.

ii y calculated with consistent unit.

c Percentage uncertainty = $\frac{\Delta y}{y} \times 100$ where Δy is 3 or 4 mm.

d Time for at least five oscillations.
Timings repeated.
Average T determined.

e New value of h and y.
New value of T determined.
New value of T greater.

f i c calculated for twice.

ii Percentage difference between the two values calculated and compared with the percentage uncertainty.

g g determined with correct unit and correct power of ten.

h

sources of uncertainty and limitations	improvements
two sets of readings are not enough to get a valid conclusion	take many readings and plot a graph of T against $\sqrt{y}$
when measuring h, the rule may not be vertical	use a set square positioned against the floor and the rule
initially the blade is not horizontal so h_0 is not exact	mark the position of the centre of the mass
difficult to judge the centre of the mass to measure h	measure h at both sides of the mass and average
difficult to judge the start or end of the oscillations	video oscillations and timer and play back frame by frame

3 Stage 1: Understanding the problem.
The independent variable is p and the dependent variable is B. Keep the current in the coil constant. Keep the number of turns of the coil constant.

Stage 2: Method
Diagram with all the apparatus placed on a bench. All the apparatus labelled, with a Hall probe positioned at point P with a ruler supported by a retort stand. Circuit diagram showing the coil connected to a d.c. power supply and ammeter. Include a variable resistor in the circuit so that the variable resistor may be adjusted so that the current is constant. The use of a ruler to measure distance p. Detailed information about measuring the distance p, for example placing a wooded ruler across the top of the iron core, recording the reading of the rule at the top of the iron core and at point P.
Detail on using the Hall probe, for example, rotate the Hall probe until maximum reading is obtained, repeat each experiment for the same value of p and reverse the direction of the current/Hall probe and average the reading from the Hall probe and calibrate the Hall probe in a known magnetic field. To increase the size of the magnetic field, use a large number of turns on the coil and use a large current.
Stage 3: Analysis
Since $B = kNIe^{-\alpha p}$
$\ln B = -\alpha p + \ln kNI$
Plot a graph of ln B against p. If the relationships is valid then the graph will be a straight line with gradient = $-\alpha$ and y-intercept = ln (kNI).
$\alpha = -$gradient
$k = \frac{e^{y\text{-intercept}}}{NI}$
Stage 4: Safety
With a large current in the coil, the coil may become hot so do not touch coil or switch off current when not in use.

4 **a** gradient = $\frac{R}{E}$

y-intercept = $\frac{r}{E}$

b

$\frac{1}{I}$ / A^{-1}
29 or 29.4 ± 1.7
22 or 21.7 ± 0.9
18 or 17.9 ± 0.6
15 or 15.2 ± 0.5
13 or 13.2 ± 0.3
12 or 11.9 ± 0.3

Notes: since all the current values are given to two significant figures, then the number of significant figures in $\frac{1}{I}$ should be either two or three.

c **i** All six data points plotted on the graph grid to an accuracy of better than 1 mm. All six error bars plotted to an accuracy of better than 1 mm.

ii Straight line of best fit with points balanced. Steepest or shallowest worst acceptable line drawn through the error bars of all the data points. Worst acceptable line labelled or dashed. If dashed, the dashes must clearly pass through each error bar.

iii Gradient of the line of best fit determined using points from the line of best fit substituted into $\frac{y_2 - y_1}{x_2 - x_1}$ with the points on the line chosen for the calculation should be separated by more than half of the length of the line drawn. The reading of the data point should be better than 1 mm. If the calculation of $\frac{1}{I}$ and the graph are correct then the gradient should be about 50.
Gradient of the worst acceptable line determined using points from the worst acceptable line substituted into $\frac{y_2 - y_1}{x_2 - x_1}$ with the points on the line chosen for the calculation should be separated by more than half of the length of the line drawn. The reading of the data point should be better than 1 mm.
Uncertainty in gradient = gradient of line of best fit – gradient of worst acceptable line

iv y-intercept of the line of best fit determined using points from the line of best fit and the gradient substituted into $y = mx + c$. The reading of the data point should be better than 1 mm. If the calculation of $\frac{1}{I}$ the graph and the gradient are correct then the y-intercept should be about 5.
Intercept of the worst acceptable line determined using points from the worst acceptable line and the gradient of the worst acceptable line substituted into $y = mx + c$. The reading of the data point should be better than 1 mm.
absolute uncertainty = y-intercept of line of best fit – y-intercept of worst acceptable line

d **i** $E = \frac{470}{\text{gradient}} = \frac{470}{50} = 9.4\,\text{V}$

$r = E \times y\text{-intercept} = 9.4 \times 5 = 47\,\Omega$ OR

$r = \frac{470}{\text{gradient}} \times y\text{-intercept} = \frac{470}{50} \times 5 = 47\,\Omega$

ii percentage uncertainty in r = percentage uncertainty in gradient + percentage uncertainty in y-intercept

Glossary

absolute uncertainty The size of the range of values within which the 'true value' of a measurement is likely to lie.

absolute zero The lowest possible temperature: zero kelvin on the thermodynamic temperature scale.

absorption spectrum A spectrum of light transmitted through a low pressure gas, showing dark lines due to absorption at specific wavelengths.

acceleration of free fall The same uniform acceleration with which all objects fall near the surface of the Earth, due to Earth's uniform gravitational field.

accuracy The closeness of a measured value to the 'true' or 'known' value; it depends on the equipment and techniques used and the skill of the experimenter.

activity The activity of a radioactive source is the number of nuclear decays occurring per unit time in the source.

adiabatic (change) Change that takes place where no thermal energy can enter or leave the system

air resistance The forces that oppose the motion of an object as it passes through the air.

alternating current A current or voltage that reverses its direction regularly and is usually sinusoidal.

ampere The SI base unit of current.

amplitude The maximum displacement of an oscillation.

analyser Acts as a second polariser after an initial polariser has filtered light waves from a normal source to generate plane-polarised light.

angular frequency The constant ω in the defining equation for simple harmonic motion; $\omega = 2\pi/T$.

angular speed For an object moving in a circle, the angular speed is defined as the angle swept out by the radius of the circle per unit time.

angular velocity The angular speed in a given direction.

annihilation Occurs when a particle interacts with its antiparticle, releasing their combined mass as energy in the form of photons.

antineutrino The antimatter equivalent of the neutrino, it has no electrical charge and little or no mass and is emitted from the nucleus at the same time as the β^- particle.

antinode A point of maximum amplitude.

antiparticle All fundamental particles have a corresponding antimatter particle with the same mass but opposite charge.

antiphase When a crest and a trough of two waves are aligned, so that the waves are exactly out of phase.

antiquarks The antiparticles of quarks, which are identical to their corresponding quarks except that they have the opposite values of charge.

Archimedes' principle The rule that the upthrust acting on an object immersed in a fluid is equal to the weight of the fluid displaced.

A-scan A technique used for the display of an ultrasound scan.

atomic mass units (u) One unified atomic mass unit (1 u) is equal to 1.66×10^{-27} kg.

atomic number The number of protons in the nucleus of an atom.

avalanche effect Effect in a Geiger-Müller tube where particles accelerated by the potential difference between the central wire anode and the cylindrical cathode then cause further ionisation.

average acceleration (change in velocity)/(time taken) or $\Delta v/\Delta t$.

average velocity (displacement)/(time taken) or $\Delta x/\Delta t$.

Avogadro constant The number of elementary entities in 1 mole of any substance, 6.02×10^{23}.

background radiation Radiation from both natural and human-made sources that is present in the environment all the time, rather than due to the deliberate introduction of a radioactive source.

balance point Position of a sliding contact on a wire where a centre-zero galvanometer reads zero; the current through the cell is zero and the p.d. across the length of wire is 'balanced' with the p.d. across the cell.

baryons Particles made up of three quarks or three antiquarks, such as protons and neutrons.

base units The seven fundamental units upon which the SI system is founded.

becquerels Unit used to measure the activity of a radioactive source, where 1 becquerel is 1 decay per second.

binding energy The energy equivalent of the mass defect of a nucleus. It is the energy required to separate to infinity all the nucleons of a nucleus.

binding energy per nucleon The total energy needed to completely separate all the nucleons in a nucleus divided by the number of nucleons in the nucleus.

blueshift Effect where the spectral lines in an absorption spectrum from a star are observed to have a decrease in wavelength compared to their known values as measured in a laboratory.

Boltzmann constant Constant that relates the average kinetic energy of particles in a gas with the temperature of the gas; has the value $1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.

bottom (b) One of the six types or flavours of quark.

Boyle's law Law stating that for a given mass of a gas, the volume V of the gas is inversely proportional to its pressure p, provided that the temperature is held constant: $p_1V_1 = p_2V_2$.

bridge rectifier Circuit used for full-wave rectification that uses four diodes arranged in a diamond pattern.

B-scan A technique used for the display of an ultrasound scan that consists of a series of A-scans, all taken from different angles, so that a two-dimensional image is formed.

capacitance The ratio of charge Q to potential V for a conductor.

capacitors Circuit components that store charge and, therefore, have capacitance.

cathode-ray oscilloscope (CRO) Instrument used to display, measure and analyse various waveforms of electrical circuits.

centre of gravity The point at which the whole weight of an object may be considered to act.

centre-zero galvanometer A sensitive current-measuring analogue meter with a centre-zero scale that shows negative currents when the needle is to the left-hand side of the zero mark and positive currents when it is to the right.

centripetal acceleration The acceleration of any object that is travelling in a circle, which is always towards the centre of the circle.

centripetal force Force acting on any object that is travelling in a circle to cause the centripetal acceleration; acts towards the centre of the circle.

Cepheid variables Stars whose radius varies periodically; this causes the temperature of the star to change and so the luminosity also varies periodically.

chain reaction Situation where a single nuclear reaction causes at least one subsequent nuclear reaction, potentially giving a self-propagating chain of reactions.

Charles' law Law stating that the volume of an ideal gas at constant pressure is directly proportional to the absolute temperature: $V_1/T_1 = V_2/T_2$.

charm (c) One of the six types or flavours of quark.

components A single vector may be split up, or resolved, into two components with a combined effect that is the same as the original vector.

compressive Describes a deformation (or force) that occurs (or acts) when an object is squeezed or compressed.

computed tomography Technique whereby a three-dimensional image through the body may be obtained using a CT scanner, by combining data from X-ray images of individual slices taken from different angles.

conservation of kinetic energy Total kinetic energy of colliding bodies before collision is the same as the total kinetic energy afterwards.

constant phase difference Wave sources which maintain a constant phase difference are described as coherent sources. Two or more waves are coherent if they have a constant phase difference.

constructive Type of interference when two waves arrive at a point in phase and give a resultant wave with a greater amplitude.

continuous spectrum Spectrum that has all colours (and wavelengths) between two limits.

contrast An X-ray image having a wide range of degrees of blackening in different regions is said to have good contrast.

conventional current Idea that electric current is a flow of positive charge from positive to negative, based on early studies of electricity. In reality the electric current in metals is the flow of electrons in the opposite direction.

coplanar All in the same plane.

corrected count rate Count-rate due to a radioactive source where the background count-rate has been subtracted from the total measured count-rate.

Cosmological Principle Idea that, on a large enough scale, the Universe is both homogeneous and isotropic, which means that the Universe would have the same general appearance from anywhere else in the Universe as it appears from Earth.

coulomb Unit of electric charge.

Coulomb's law Law stating that the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

couple Two forces, equal in magnitude but opposite in direction, whose lines of action do not coincide.

critical damping Damping that causes the displacement to decrease to zero in the shortest time possible, without any oscillation.

cycle The motion of any particle in a wave from the maximum positive displacement (a crest) to a maximum negative displacement (a trough) back to a maximum positive displacement.

damped Oscillations are said to be damped when frictional and other resistive forces cause the oscillator's energy to be dissipated, and this energy is converted eventually into thermal energy.

daughter nuclide The new nuclide formed when one element changes into another due to radioactive decay.

de Broglie wavelength The wavelength associated with a moving particle.

decay constant λ For radioactive decay, the decay constant λ is the probability per unit time of the decay of a nucleus.

decay curve A graph, such as that seen in radioactive decay, that shows an exponential decrease – the value decreases by the same fraction over equal time intervals.

deformation Change in shape or size of a solid object as the result of a force.

derived units Units consisting of some combination of the seven fundamental base units.

dielectric The insulating material placed between the plates of a capacitor to increase its capacitance.

diffraction The spreading of a wave into regions where it would not be seen if it moved only in straight lines after passing through a narrow slit or past an edge.

diffraction grating A plate on which there is a very large number of parallel, identical, very closely spaced slits. If monochromatic light is incident on this plate, a pattern of narrow bright fringes is produced.

direct current A steady current in one direction, such as that in a circuit with a battery.

discrete energy levels Certain specific energy levels that the electrons in an atom can have; they cannot have energies in between these levels.

displacement Change of position; the length travelled in a straight line in a specified direction from the starting point to the finishing point.

distance The length along the actual path travelled from the starting point to the finishing point.

Doppler effect The frequency change due to the relative motion between a source of sound or light and an observer.

down (d) One of the six types or flavours of quark.

drag force The frictional force in a fluid.

drift speed Speed of the charge carriers as they move through a conductor.

e.m.f. The electromotive force; it measures, in volts, the energy transferred per unit of charge that passes through the power supply.

eddy current damping The process by which eddy currents induced in a conductor by a varying magnetic field cause heating and dissipate energy.

eddy currents Currents of varying magnitude and direction that are induced in a conductor by a varying magnetic field.

efficiency The ratio of useful energy output from a system to the total energy input; a measure of how much of the total energy may be considered useful and is not 'lost'.

elastic deformation Deformation where the object returns to its original shape and size when the load on it is removed.

elastic limit The maximum force that can be applied to a wire/spring such that the wire/spring returns to its original length when the force is removed.

elastic potential energy (strain energy) Energy stored in an object due to change of shape or size, which is completely recovered when the force causing the deformation is removed.

electric current A flow of charge carriers.

electric field A region of space where a stationary electric charge experiences a force.

electric field strength The force per unit charge acting on a small stationary positive charge placed at a certain point.

electric potential energy Energy due to the position of a charge in an electric field.

electrical potential The work done per unit positive charge in bringing a small test charge from infinity to the point.

electrolytic capacitor Type of capacitor where the dielectric is deposited by an electrochemical reaction. These capacitors must be connected with the correct polarity for their plates, or they will be damaged.

electromagnet Solenoid wound around a soft-iron core to increase the strength of the magnetic field.

electromagnetic force The force on a current carrying conductor at an angle to a magnetic field. The direction of the force may be predicted using Fleming's left-hand rule.

electromagnetic induction Induction of an e.m.f. by a magnetic field.

electromagnetic spectrum Continuous range of frequencies (or wavelengths) of electromagnetic radiation.

electromagnetic waves Waves consisting of electric and magnetic fields that oscillate at right angles to each other and to the direction in which the wave is travelling.

electromotive force The energy transferred per unit of charge that passes through a power supply, measured in volts.

electron energy levels Certain specific energy levels that the electrons in an atom can have; they cannot have energies in between these levels.

electron transition Movement of an electron between energy levels.

electrons One of the three types of particle that make up the atoms of all elements; they have a negative charge, a mass of about 1/2000 of 1 u, and are found orbiting the nucleus.

electronvolt (eV) The work done (energy gained) by an electron when accelerated through a potential difference of one volt. One eV is equivalent to 1.60×10^{-19} J.

emission line spectrum A characteristic spectrum of electromagnetic radiation for a particular element, emitted when electrons in an excited atom return to lower energy levels.

emission spectrum A spectrum of the electromagnetic radiation emitted by a source.

empirical scale of temperature A temperature scale derived by experiment for each particular type of thermometer, using the changes of state of substances at fixed temperatures to define reference temperatures, or fixed points.

equilibrium The state of a system or object when there is no resultant force and no resultant torque.

excited state The state of an electron that has absorbed energy and been promoted to a higher energy level.

exponential Changes by the same fraction over equal time intervals.

exponential decay An exponential decrease – the value decreases by the same fraction over equal time intervals.

extension The increase in length or deformation of a spring caused by a tensile force; equal to the extended length – natural/original length.

farad The unit of capacitance (symbol F). One farad is one coulomb per volt.

Faraday's law of electromagnetic induction Law stating that the e.m.f. induced is proportional to the rate of change of magnetic flux linkage.

first harmonic The first resonant frequency of a vibration, with the simplest standing wave pattern: a single loop.

first law of motion Another term used for Newton's first law.

first law of thermodynamics Law stating that the increase in internal energy of a system is equal to the sum of the thermal energy added to the system and the work done on it: $\Delta U = q + w$.

first overtone The second resonant frequency of a vibration, where the stationary wave pattern has two loops.

fixed points Fixed temperatures at which substances change state, which can be used as reference temperatures when establishing a temperature scale.

Fleming's left-hand rule Method of determining the direction of the force relative to the directions of the current and the magnetic field in a current-carrying conductor that is at angle to a magnetic field, by using the first two fingers and thumb of the left hand held at right angles to one another.

Fleming's right-hand rule Method of determining the direction of the induced e.m.f. or current in a wire moving through a magnetic field at right angles to the field, by using the first two fingers and thumb of the right hand held at right angles to one another.

forced vibrations Vibrations in an object that has periodic forces acting on it, which make the object vibrate at the frequency of the applied force, rather than at the natural frequency of the system.

forward bias Condition in a diode where the application of a voltage allows it to conduct a current in the direction of the arrowhead on the symbol.

fractional uncertainty The ratio of the absolute uncertainty in a measurement to the measured value.

free electrons Outer electrons that are not held tightly to the nucleus within an atom and are therefore free to move through a material, such as a metal.

free oscillations A particle is said to be undergoing free oscillations when the only external force acting on it is the restoring force.

free-body diagram Labelled diagram showing all of the forces acting on a body, with their sizes and directions.

frequency *f* The number of oscillations (cycles) of a wave per unit time.

fringes An interference pattern where the maxima and minima disturbances are produced by the superposition of overlapping waves.

full-wave rectification Process used to convert alternating current into direct current by reversing the polarity of the negative half-cycles of the input.

fundamental mode Another term for the first harmonic.

fundamental particle A particle that is not formed from other particles; examples include quarks and leptons.

galaxy A group of hundreds of millions of stars, stellar remnants, gas and dark matter, held together by gravity.

Gay-Lussac's law The relation between the pressure and thermodynamic temperature of a fixed mass of gas at constant volume: $p_1/T_1 = p_2/T_2$.

geostationary orbit Equatorial orbit with exactly the same period of rotation as the Earth (24 hours), in the same direction as the Earth (west to east), so that the orbiting satellite is always above the same point on the Equator.

geostationary satellites Satellites with equatorial orbits with exactly the same period of rotation as the Earth (24 hours), which move in the same direction as the Earth (west to east), so that they are always above the same point on the Equator.

gravitational constant Constant of proportionality G in Newton's law of gravitation; the value of G is $6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

gravitational field A region of space where a mass experiences a force.

gravitational field line The direction of the gravitational force acting on a point mass.

gravitational field strength The force per unit mass acting on a small mass placed at a point.

gravitational potential The work done per unit mass in bringing a small test mass from infinity to a point in a gravitational field.

gravitational potential energy Energy possessed by a mass due to its position in a gravitational field.

ground state The state of an atom and its electrons when the electrons occupy the lowest energy levels available.

hadrons Subatomic particles that are affected by the strong force, for example, protons and neutrons.

half-life $t_{1/2}$ The time taken for the number of undecayed nuclei in a sample of a radioactive isotope to be reduced to half its original number.

half-wave rectification Process used to convert alternating current into direct current by using a diode to reject the negative part of the input; the output voltage consists only of the positive half-cycles of the input voltage.

Hall voltage A potential difference that develops across a conductor when there is a current in the conductor that is normal to a magnetic field, due to charge carriers moving at right angles to the magnetic field and so experiencing a force that tends to make them move to one side of the conductor.

harmonic oscillator A system that, when displaced from its equilibrium position, experiences a restoring force F proportional to the displacement x and therefore follows simple harmonic motion.

heavy damping Damping that causes an exponential reduction in the amplitude of vibration of an oscillation, but over a greater time than for critical damping.

Helmholtz coils Two identical flat coils, with the same current in each, positioned so that their planes are parallel and separated by a distance equal to the radius of either coil.

hertz (Hz) Unit of frequency, where 1 Hz = 1 cycle per second.

Hooke's law Law stating that, provided the limit of proportionality is not exceeded, the extension of an object is proportional to the applied load.

Hubble constant The constant of proportionality in Hubble's law, H_0; the present accepted value is $75\,\mathrm{km\,s^{-1}}$ per Mpc or $2.4 \times 10^{-18}\,\mathrm{m\,s^{-1}}$ per m.

Hubble's law Conclusion that the radial speed at which galaxies are moving away from Earth is proportional to their distance from the Earth.

ideal gas A gas that obeys the equation of state $pV \propto T$ at all pressures p, volumes V and thermodynamic temperatures T.

impulse The product of a force acting on an object and the time for which it acts: impulse = $F\Delta t$.

in phase Describes particles or waves that have the same displacements at the same times, i.e. the crests and troughs of the two waves are aligned.

instantaneous velocity The average velocity measured over an infinitesimally short time interval.

intensity The power per unit area of a wave.

intensity reflection coefficient Ratio of the reflected intensity I_R to the incident intensity I_0 for a wave incident normally on a boundary between two media having specific acoustic impedances of Z_1 and Z_2: $I_R/I_0 = (Z_2 - Z_1)^2/(Z_2 + Z_1)^2$.

interference Where two or more waves meet or overlap, causing a resultant wave that is the net effect of the overlapping waves.

interference pattern The collection of maxima and minima fringes produced by the superposition of overlapping waves.

internal energy Random kinetic and potential energy of the molecules in an object.

internal resistance The resistance between the terminals of a power supply.

inverse square law Law stating that a quantity is inversely proportional to the square of the distance from the source of that quantity.

ion Charged particle formed when an atom gains or loses one or more electrons, so that it does not contain an equal number of protons and electrons.

isothermal (change) Change that takes place at constant temperature; the change in internal energy is zero.

isotopes Different forms of the same element that have the same number of protons but different numbers of neutrons in their nuclei.

Joule heating The heating effect caused by an electric current passing through a resistor.

kelvin The unit of thermodynamic temperature; one kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Kepler's third law of planetary motion Law stating that for planets or satellites describing circular orbits about the same central body, the square of the period is proportional to the cube of the radius of the orbit.

kinetic theory of an ideal gas Theory that relates the large-scale (macroscopic) quantities *p*, *V* and *T* to the small-scale (microscopic) behaviour of the particles of a gas by making some very simple assumptions about the atoms or molecules that make up the gas.

Kirchhoff's first law Law stating that the sum of the currents entering a junction in a circuit is always equal to the sum of the currents leaving the junction.

Kirchhoff's second law Law stating that the sum of the electromotive forces in a closed circuit is equal to the sum of the potential differences.

latent heat of fusion The latent heat required to melt (fuse) a solid.

latent heat of vaporisation The latent heat required to vaporise a liquid without any change of temperature.

law of conservation of energy Law stating that energy cannot be created or destroyed, it can only be converted from one form to another.

law of pressures Another term for Gay-Lussac's law.

Lenz's law Law stating that the direction of an induced e.m.f. is such as to cause effects to oppose the change producing it.

leptons Fundamental, subatomic particles that are not affected by the strong force, for example, electrons and positrons.

light-dependent resistor (LDR) A type of resistor where the resistance decreases as the intensity of light on it increases.

limit of proportionality The point up to which the load (applied force) is proportional to the extension.

linear absorption coefficient A constant used in calculating the transmitted X-ray intensity that is dependent on the medium and on the energy of the X-ray photons.

linear attenuation coefficient Another term for the linear absorption coefficient.

linear momentum The product of the mass *m* and velocity *v* of an object: $p = mv$.

load The tensile force that causes an extension.

logarithmic scales Nonlinear scales where each interval is increased by a factor of the base of the logarithm, rather than increasing in equal increments as with a linear scale.

longitudinal (wave) Wave in which the direction of the vibrations of the particles in the wave is along or parallel to the direction in which the energy of the wave is travelling.

luminosity The total power (the total energy emitted per unit time) of an object.

magnetic field A region of space where a moving charge or a magnetic material experiences a (magnetic) force.

magnetic field lines Lines of magnetic force; their direction is the direction in which a free magnetic north pole would move if placed in the field and their closeness indicates the strength of the magnetic field.

magnetic flux The product of the magnetic flux density and the area normal to the lines of flux: $\Phi = BA$.

magnetic flux density The force per unit current per unit length on a wire placed at right angles to a uniform magnetic field.

magnetic flux linkage The product of the magnetic flux through a coil and the number of turns *N* on the coil: $N\Phi = BAN$.

Malus's law The intensity I of plane-polarised electromagnetic waves after transmission through a polarising filter or series of filters is equal to $I_0 \cos^2 \theta$, where I_0 is the maximum intensity and θ is the angle between the polarising directions of the two sheets.

mass defect The difference between the total mass of the separate nucleons in a nucleus and the combined mass of the nucleus.

mass number The total number of protons plus number of neutrons in a nucleus.

meson A type of hadron made up of a quark and an antiquark; pions (π) and kappas (K) are examples of mesons.

mol Abbreviation of mole.

molar gas constant Constant of proportionality *R* in the universal gas equation; it has the value $8.3\,J\,K^{-1}\,mol^{-1}$ for all gases.

molar mass The mass of 1 mole of substance.

mole The amount of substance that contains $6.02214076 \times 10^{23}$ elementary entities, usually atoms or molecules but they could also be ions or electrons.

momentum The product of the mass *m* and velocity *v* of an object $p = mv$; its complete name is linear momentum.

motor effect Phenomenon when a current-carrying conductor placed in a magnetic field experiences a force, due to the interaction of the magnetic field with that produced by the current-carrying conductor.

natural frequency The frequency of vibration at which an object will vibrate when allowed to do so freely.

neutral point Point in the magnetic field pattern between two magnets where there is no resultant magnetic field because the two fields are equal in magnitude but opposite in direction.

neutrons One of the three types of particle that make up the atoms of all elements; they have no charge, a mass of about 1 u, and are found in the nucleus.

Newton's first law Newton's first law states that every object continues in its state of rest, or with uniform velocity, unless acted on by a resultant force.

Newton's law of gravitation Law stating that two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation.

Newton's second law Law stating that the resultant force acting on an object is proportional to the rate of change of its momentum.

Newton's third law Law stating that whenever one object exerts a force on another, the second object exerts an equal and opposite force on the first.

node A point of zero amplitude.

nuclear atom Model of the atom based on the results of Rutherford's α-particle scattering experiment, in which there is a small, positively charged nucleus at the centre of the atom that contains most of its mass, with negative electrons orbiting it; most of the atom is empty space.

nuclear fission The splitting of a heavy nucleus into two lighter nuclei of approximately the same mass.

nuclear fusion When two light nuclei combine to form a nucleus of greater mass.

nucleon The name given to either a proton or a neutron in the nucleus.

nucleon number Another term for mass number.

nucleus The positively charged central part of an atom, which contains the protons and neutrons; almost all of the mass of the atom is concentrated here.

nuclide A class of nuclei that have a particular nucleon number and a particular proton number.

ohm The unit of resistance, which has the symbol Ω.

Ohm's law Law stating that, for a metallic conductor at constant temperature, the current in the conductor is proportional to the potential difference across it.

order of magnitude The power of ten to which a number is raised.

oscillation The motion of any particle in a wave from the maximum positive displacement (a crest) to a maximum negative displacement (a trough) back to a maximum positive displacement.

overdamping Another term for heavy damping.

parabola The curve traced out by a particle subject to a constant force in one direction.

parallax error Error introduced into a measurement by reading a scale from different angles.

parallel circuit A circuit where the current can take alternative routes in different loops.

parent nuclide The original nuclide when one element changes into another, the daughter nuclide, due to radioactive decay.

path difference The difference between the distances travelled by two waves meeting at a point.

peak value The maximum value (amplitude) of the current or voltage from an alternating current supply.

percentage uncertainty The ratio of the absolute uncertainty in a measurement to the measured value, expressed as a percentage.

period *T* The time for a particle in a wave to complete one oscillation or one cycle.

permittivity of free space A physical constant used in calculating the force between charged particles in a vacuum; it has the symbol ε_0 and a value of $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

phase difference The relative positions of two points on the same wave, or of two waves of the same frequency, can be stated as a phase difference in degrees.

photoelectric emission The release of electrons from the surface of a metal when electromagnetic radiation is incident on its surface.

photoelectrons The electrons emitted by photoelectric emission.

photons Name given to a quantum of energy when the energy is in the form of electromagnetic radiation.

physical quantity A feature of something that can be measured, for example, length, mass or the time interval for a particular event; every physical quantity has a numerical value and a unit.

piezo-electric transducer A device that converts electrical energy into ultrasound energy by means of a piezo-electric crystal such as quartz.

pixel In CT scanning, a pixel is the number given to the intensity transmitted through each voxel, which are together used to build up an image from measurements of the X-ray intensity along different directions through the section or slice.

plane polarised A polarised wave is a transverse wave in which vibrations occur in only one of the directions at right angles to the direction in which the wave energy is travelling.

plastic deformation In a plastic deformation, an object does not return to its original shape and size when the load on it is removed.

polariser Filter, such as a sheet of Polaroid, that can be used to produce plane-polarised light from unpolarised light.

positive ion Positively charged particle formed when an atom loses one or more electrons.

positron The antiparticle of an electron, which has the same mass as an electron but opposite charge.

Positron emission tomography (PET) Technique using radioactive tracers that decay by emitting a positron, which then annihilates when it meets an electron and emits photons; these are detected by PET scanners, which are used in medical imaging and diagnosis to determine how well certain body functions are operating and to identify abnormalities.

potential difference (p.d.) The energy transferred per unit of charge that passes through the resistor; measured in volts.

potential energy The ability of an object to do work as a result of its position or shape.

potentiometer A continuously variable potential divider.

power The rate of converting energy or using energy: power = work done/time taken.

precision Depends on the range of a set of measured values; the smaller the range, the better the precision.

principle of moments States that, for an object to be in rotational equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about that same point.

principle of superposition of waves States that, when two or more waves meet at a point, the resultant displacement at that point is equal to the sum of the displacements of the individual waves at that point.

progressive waves Waves that transfer energy from place to place without the transfer of matter.

proton number Another term for the atomic number.

protons One of the three types of particle that make up the atoms of all elements; they have a positive charge, a mass of about 1 u, and are found in the nucleus.

quanta Discrete packets or quantities of energy of electromagnetic radiation.

quantised Exists only in discrete amounts, rather than being continuous.

quantum theory Theory in which electromagnetic radiation is thought of as consisting of packets of energy called photons.

quark Fundamental particle, different combinations of which make up other particles such as hadrons.

quark model Model in which the hadrons are made up of fundamental particles called quarks; there are six 'flavours' of quark, each with a characteristic charge and strangeness.

quasar Very distant objects in the Universe that have very large redshifts and a huge luminosity.

radial All the lines (of force) appear to converge towards the centre.

radians One radian (rad) is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

radiant flux intensity The radiant power per unit area passing normally through unit area.

radioactive Nuclei that are unstable and therefore emit particles and/or electromagnetic radiation in order to increase their stability.

radioactive decay series Sequence of radioactive decay from parent nuclide through succeeding unstable daughter nuclides, until a stable nuclide is eventually reached.

radioactivity The particles and/or electromagnetic radiation emitted by a radioactive nucleus.

random process Process such as radioactive decay or the throwing of dice, where there is a constant probability that a nucleus will decay in a certain time or a number will be thrown, but it is impossible to predict which nucleus will decay or which of the dice will show the number.

random error Random error in measurements due to the scatter of readings around the true value, which may be reduced by repeating a reading and averaging, or by plotting a graph and taking a best-fit line.

reciprocal The quantity obtained by dividing the number one by a given quantity.

rectification Process used to convert an alternating current into a direct current.

rectifiers Devices that use diodes to change alternating current into direct current.

redshift Effect where the spectral lines in the absorption spectrum from a star are seen to have an increase in wavelength from their known values as measured in a laboratory.

relative permittivity ε_r The capacitance of a parallel-plate capacitor with the dielectric between the plates divided by the capacitance of the same capacitor with a vacuum between the plates.

resistance The ratio of the potential difference V across a conductor to the current I in it; good conductors have low resistance because they offer little opposition to the movement of electrons.

resistivity Property of a particular material that indicates how strongly it resists the flow of electrical current; it is a constant for that material at a particular temperature.

resistor Device that has resistance to the flow of electric current.

resolved (a vector) Split up into two vectors or components that have a combined effect that is the same as the original vector.

resonance When the natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration.

resonance curve Graph showing the variation with driving frequency of the amplitude of vibration of an object.

resonant frequency Frequency at which resonance occurs.

restoring force Force acting on a particle that is undergoing simple harmonic motion; this force always acts towards the fixed point about which the particle is moving.

resultant The combined effect of two different vectors.

reverse bias Condition where a diode will not conduct because of its very high resistance when the voltage (potential difference, high to low) is in the opposite direction to the arrowhead on the symbol (potential difference is reversed compared with the forward bias condition).

rheostat A type of resistor that can produce a continuously variable voltage.

right-hand grip rule Method used to find the direction of the field in a solenoid, where the solenoid is grasped in the right hand with the fingers pointing in the direction of the conventional current; the thumb then gives the direction of the magnetic field.

root-mean-square r.m.s. The r.m.s. value of the alternating current or voltage is that value of the direct current or direct voltage that would produce thermal energy at the same rate in a resistor.

root-mean-square speed or r.m.s. speed A measure of the speed of the molecules in a gas, equal to the square root of the average velocity squared of the molecules.

scalar quantity A quantity that can be described fully by giving its magnitude and unit; it does not have a direction.

scintillation A tiny pulse of light.

second harmonic Another term for the first overtone.

series circuit Circuit in which the components are connected one after another, forming one complete loop.

simple harmonic motion (s.h.m.) The motion of a particle about a fixed point such that its acceleration a is proportional to its displacement x from the fixed point, and is in the opposite direction.

smoothing Use of a capacitor to reduce the fluctuations in the unidirectional output produced after rectification and to give a steady direct current.

solenoid A coil of wire used as an electromagnet.

specific acoustic impedance Z The product of the density ρ of a medium and the speed c of a wave in the medium: $Z = \rho c$.

specific heat capacity The quantity of thermal energy per unit mass required to raise the temperature of the substance by one degree.

specific latent heat of fusion The quantity of thermal energy per unit mass required to convert solid to liquid without any change in temperature.

specific latent heat of vaporisation The quantity of thermal energy per unit mass required to convert liquid to vapour without any change in temperature.

spectrometers Instruments used to investigate spectra and measure their wavelengths.

spectroscopy The study of spectra.

speed Scalar quantity that shows how fast an object is moving: average speed = distance moved along actual path/time taken.

spring constant The force per unit extension, $k = F/x$.

standing waves The result of the overlapping and hence interference of two waves of equal frequency and amplitude, travelling along the same line with the same speed but in opposite directions.

stationary waves Another term for standing waves.

strain A measure of the extent of the deformation when an object has its shape or size changed by forces acting on it.

strange (s) One of the six types or flavours of quark.

strangeness One of the properties of quarks.

stress The force acting per unit area so as to cause a strain.

strong force The force that holds the nucleons in a nucleus together.

strong nuclear force Another term for the strong force.

supernovae Stellar objects used as standard candles for galaxies that are a very long distance from Earth.

systematic error An error that results in all readings being either above or below the true value, by a fixed amount and in the same direction each time the measurement is taken.

Système Internationale (SI) System of units founded upon seven fundamental or base units and based on the metric system of measurement, which is used by scientists around the world.

tensile Relating to stretching.

tensile strain Extension/original length

tensile stress Stress per unit area caused by a tensile force: tensile stress = force/cross-sectional area.

terminal potential difference The potential difference across the terminals of a cell or power supply when a current is being delivered.

terminal velocity The maximum velocity of an object moving through a resistive fluid (a liquid or a gas).

thermal equilibrium When different regions in thermal contact are at the same temperature.

thermistors Negative temperature coefficient devices, which have a resistance that decreases significantly with rise in temperature.

thermocouple Device where one end of each of two wires of different metals are twisted together and the other ends are connected to the terminals of a sensitive voltmeter; an e.m.f. is produced and the reading on the voltmeter depends on the temperature of the junction of the wires.

thermodynamic temperature scale Temperature scale based on the theoretical behaviour of a so-called ideal gas, which starts with zero at absolute zero (−273.15°C).

thermonuclear reactions Reactions requiring conditions of extremely high temperature and pressure, similar to those found at the centre of the Sun.

threshold frequency The minimum frequency of incident radiation required to cause photoelectron emission from the surface of a particular metal: $hf_0 = \varphi$.

threshold wavelength The maximum wavelength corresponding to the threshold frequency to give rise to photoelectric emission.

top (t) One of the six types or flavours of quark.

tracers Chemical compounds in which one or more of the atoms have been replaced by radioactive nuclei of the same element that can then be used to locate or follow the progress of the compound in living tissues.

transformer An electrical device consisting of two or more coils wound around the same core that allows the transfer of electrical energy between circuits.

transmutation The process by which one element turns into another as a result of radioactive decay.

transverse (wave) Wave in which the vibrations of the particles in the wave are at right angles to the direction in which the energy of the wave is travelling.

ultrasound Sound with frequencies above the range of human hearing, typically above about 20 kHz.

unified atomic mass units (u) One unified atomic mass unit (1 u) is equal to 1.66×10^{-27} kg.

universal gas constant Another term for the molar gas constant.

unpolarised Describes waves where the vibrations take place in many directions in a plane at right angles to the direction of the wave energy.

up (u) One of the six types or flavours of quark.

upthrust Upward force on an object immersed in a fluid due to the pressure of the fluid on it.

variable resistor Another term for a rheostat.

vector quantity A quantity that has magnitude, unit and direction; they may not be added algebraically.

vector triangle Diagram used to find the resultant of two vectors by representing them in magnitude and direction by the sides of a triangle; the third side gives the magnitude and direction of the resultant.

velocity A vector quantity representing the rate of change of distance with time, and the direction in which it is moving.

viscous force The frictional force in a fluid (a liquid or a gas).

volt Unit used to measure potential difference.

voxels Series of small units that an object undergoing CT scanning is divided into in order to produce an image; each voxel will absorb the X-ray beam to a different extent.

watt The unit of power (symbol W), equal to a rate of working of 1 joule per second.

wavefront Imaginary line joining points on a wave that are in phase.

wavelength The minimum distance between particles which are vibrating in phase with each other, i.e. between two crests or two troughs of a wave.

weak force Force responsible for β-decay in a nucleus.

weak interaction Another term for the weak force.

weber (Wb) Unit of magnetic flux; one weber is equal to one tesla metre-squared, i.e. $T\,m^2$.

weber per square metre ($Wb\,m^{-2}$) An alternative name for the tesla (T), which is the unit of magnetic flux density.

weight The force of gravity that acts on an object: $W = mg$.

weightlessness Apparent weightlessness occurs when an object is in free fall and there is no external contact force acting upon it.

work Work is done when a force moves the point at which it acts (the point of application) in the direction of the force: work done = force × displacement in the direction of the force.

work function energy Φ The minimum amount of energy necessary for an electron to escape from the surface of the metal during photoelectric emission.

Young modulus Property of a material defined as Young modulus = stress/strain.

β^+ emission Type of radioactive decay used in PET scans, where a proton in the nucleus forms a neutron, a positive electron and a neutrino.

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